The Parma Polyhedra Library Configured Prolog Language Interface User's Manual* (version 0.11.2)

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February 27, 2011

*This work has been partly supported by: University of Parma's FIL scientific research project (ex 60%) "Pure and Applied Mathematics"; MURST project "Automatic Program Certification by Abstract Interpretation"; MURST project "Abstract Interpretation, Type Systems and Control-Flow Analysis"; MURST project "Automatic Aggregate- and Number-Reasoning for Computing: from Decision Algorithms to Constraint Programming with Multisets, Sets, and Maps"; MURST project "Constraint Based Verification of Reactive Systems"; MURST project "Abstract Interpretation: Design and Applications"; EPSRC project "Numerical Domains for Software Analysis"; EPSRC project "Geometric Abstractions for Scalable Program Analyzers".

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1 Prolog Language Interface

The Parma Polyhedra Library comes equipped with a Prolog interface. Despite the lack of standardization of Prolog's foreign language interfaces, the PPL Prolog interface supports several Prolog systems and, to the extent this is possible, provides a uniform view of the library from each such system.

The system-independent features of the library are described in Section System-Independent Features. Section Compilation and Installation explains how the Prolog interface is compiled and installed. Section System-Dependent Features illustrates the system-dependent features of the interface for all the supported systems.

The structure of this section is as follows:

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- Compilation and Installation
- System-Dependent Features
  - GNU Prolog
  - CIAO Prolog
  - SICStus Prolog
  - SWI Prolog
  - XSB
  - YAP

In the sequel, `prefix` is the prefix under which you have installed the library (typically `/usr` or `/usr/local`). Copyright (C) 2001-2010 Roberto Bagnara <bagnara@cs.unipr.it> Copyright (C) 2010-2011 BUGSENG srl (http://bugseng.com)

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System-Independent Features

The Prolog interface provides access to the numerical abstractions (convex polyhedra, BD shapes, octagonal shapes, etc.) implemented by the PPL library. A general introduction to the numerical abstractions, their representation in the PPL and the operations provided by the PPL is given in the main *PPL user manual*. Here we just describe those aspects that are specific to the Prolog interface.

Overview

First, here is a list of notes with general information and advice on the use of the interface.

- The numerical abstract domains available to the Prolog user consist of the *simple* domains, *powersets* of a simple domain and *products* of simple domains.
  - The simple domains are:
    - convex polyhedra, which consist of C_Polyhedron and NNC_Polyhedron;
    - weakly relational, which consist of BD_Shape_N and Octagonal_Shape_N where N is one of the numeric types int8, int16, int32, int64, mpz_class, mpq_class, float, double, long_double;
    - boxes which consist of Int8_Box, Int16_Box, Int32_Box, Int64_Box, Uint8_Box, Uint16_Box, Uint32_Box, Uint64_Box, Double_Box, Long_Double_Box, Z_Box, Rational_Box, Float_Box; and
    - the Grid domain.
  - The powerset domains are Pointset_Powerset_S where S is a simple domain.
  - The product domains consist of Direct_Product_S_T, Smash_Product_S_T and Constraints_Product_S_T where S and T are simple domains.
- In the following, any of the above numerical abstract domains is called a PPL *domain* and any element of a PPL domain is called a *PPL object*.
- The Prolog interface to the PPL is initialized and finalized by the predicates `ppl_initialize/0` and `ppl_finalize/0`. Thus the only interface predicates callable after `ppl_finalize/0` are `ppl_finalize/0` itself (this further call has no effect) and `ppl_initialize/0`, after which the interface's services are usable again. Some Prolog systems allow the specification of initialization and deinitialization functions in their foreign language interfaces. The corresponding incarnations of the Prolog interface have been written so that `ppl_initialize/0` and/or `ppl_finalize/0` are called automatically. Section System-Dependent Features will detail in which cases initialization and finalization is automatically performed or is left to the Prolog programmer's responsibility. However, for portable applications, it is best to invoke `ppl_initialize/0` and `ppl_finalize/0` explicitly: since they can be called multiple times without problems, this will result in enhanced portability at a cost that is, by all means, negligible.

- A PPL object such as a polyhedron can only be accessed by means of a Prolog term called a *handle*. Note, however, that the data structure of a handle, is implementation-dependent, system-dependent and version-dependent, and, for this reason, deliberately left unspecified. What we do guarantee is that the handle requires very little memory.

- A Prolog term can be bound to a valid handle for a PPL object by using predicates such as

```
ppl_new_C_Polyhedron_from_space_dimension/3,
ppl_new_C_Polyhedron_from_C_Polyhedron/2,
ppl_new_C_Polyhedron_from_constraints/2,
ppl_new_C_Polyhedron_from_generators/2,
```

  These predicates will create or copy a PPL polyhedron and construct a valid handle for referencing it. The last argument is a Prolog term that is unified with a new valid handle for accessing this polyhedron.

- As soon as a PPL object is no longer required, the memory occupied by it should be released using the PPL predicate such as `ppl_delete_Polyhedron/1`. To understand why this is important, consider a Prolog program and a variable that is bound to a Herbrand term. When the variable dies (goes out of scope) or is uninstantiated (on backtracking), the term it is bound to is amenable to garbage collection. But this only applies for the standard domain of the language: Herbrand terms. In Prolog+PPL, when, for example, a variable bound to a handle for a Polyhedron dies or is uninstantiated, the handle can be garbage-collected, but the polyhedron to which the handle refers will not be released. Once a handle has been used as an argument in `ppl_delete_Polyhedron/1`, it becomes invalid.

- For a PPL object with space dimension `k`, the identifiers used for the PPL variables must lie between 0 and $k - 1$ and correspond to the indices of the associated Cartesian axes. For example, when using the predicates that combine PPL polyhedra or add constraints or generators to a representation of a PPL polyhedron, the polyhedra referenced and any constraints or generators in the call should follow all the (space) dimension-compatibility rules stated in Section *Representations of Convex Polyhedra* of the main *PPL user manual*.

- As explained above, a polyhedron has a fixed topology C or NNC, that is determined at the time of its initialization. All subsequent operations on the polyhedron must respect all the topological compatibility rules stated in Section *Representations of Convex Polyhedra* of the main *PPL user manual*.

- Any application using the PPL should make sure that only the intended version(s) of the library are ever used. Predicates

```
ppl_version_major/1,
ppl_version_minor/1,
ppl_version_revision/1,
ppl_version_beta/1,
ppl_version/1,
ppl_banner.
```

  allow run-time checking of information about the version being used.

Predicate Specifications

The PPL predicates provided by the Prolog interface are specified below. The specification uses the following grammar rules:

```
Number      --> unsigned integer       ranging from 0 to an upper bound
                                       depending on the actual Prolog system.

C_int       --> Number | - Number      C integer

C_unsigned  --> Number                 C unsigned integer

Coeff       --> Number                 used in linear expressions;
                                       the upper bound will depend on how
                                       the PPL has been configured

Dimension_Type
            --> Number                 used for the number of affine and
                                       space dimensions and the names of
                                       the dimensions;
                                       the upper bound will depend on
                                       the maximum number of dimensions
                                       allowed by the PPL
                                       (see ppl_max_space_dimensions/1)

Boolean     --> true | false

Handle      --> Prolog term            used to identify a Polyhedron

Topology    --> c | nnc                Polyhedral kind;
                                       c is closed and nnc is NNC

VarId       --> Dimension_Type         variable identifier

PPL_Var     --> '$VAR'(VarId)          PPL variable

Lin_Expr    --> PPL_Var                PPL variable
            | Coeff
            | Lin_Expr                 unary plus
            | - Lin_Expr               unary minus
            | Lin_Expr + Lin_Expr      addition
            | Lin_Expr - Lin_Expr      subtraction
            | Coeff * Lin_Expr         multiplication
            | Lin_Expr * Coeff         multiplication

Relation_Symbol --> =                  equals
            | =<                       less than or equal
            | >=                       greater than or equal
            | <                        strictly less than
            | >                        strictly greater than

Constraint  --> Lin_Expr Relation_Symbol Lin_Expr
                                       constraint

Constraint_System                      list of constraints
            --> []
            | [Constraint | Constraint_System]

Modulus     --> Coeff | - Coeff

Congruence  --> Lin_Expr =:= Lin_Expr  congruence with modulo 1
            | (Lin_Expr =:= Lin_Expr) / Modulus
                                       congruence with modulo Modulus

Congruence_System                      list of congruences
            --> []
            | [Congruence | Congruence_System]

Generator_Denominator --> Coeff        must be non-zero
            | - Coeff

Generator   --> point(Lin_Expr)        point
```

```
            | point(Lin_Expr, Generator_Denominator)
                                        point
            | closure_point(Lin_Expr)   closure point
            | closure_point(Lin_Expr, Generator_Denominator)
                                        closure point
            | ray(Lin_Expr)             ray
            | line(Lin_Expr)            line

Generator_System                        list of generators
            --> []
            | [Generator | Generator_System]

Grid_Generator
            --> grid_point(Lin_Expr)    grid point
            | grid_point(Lin_Expr, Generator_Denominator)
                                        grid point
            | parameter(Lin_Expr)       parameter
            | parameter(Lin_Expr, Generator_Denominator)
                                        parameter
            | grid_line(Lin_Expr)       grid line

Grid_Generator_System                   list of grid generators
            --> []
            | [Grid_Generator | Grid_Generator_System]

Atom        --> Prolog atom

Universe_or_Empty                       PPL object
            --> universe | empty

Poly_Relation --> is_disjoint           with a constraint or congruence
            | strictly_intersects       with a constraint or congruence
            | is_included               with a constraint or congruence
            | saturates                 with a constraint or congruence
            | subsumes                  with a (grid) generator

Relation_List --> []
            | [Poly_Relation | Relation_List]

Complexity  --> polynomial | simplex | any

Vars_Pair   --> PPLVar - PPLVar         map relation

P_Func      --> []                      list of map relations
            | [Vars_Pair | P_Func].

Optimization_Mode --> max | min

MIP_Problem_Status --> unfeasible
            | unbounded
            | optimized

Control_Parameter_Name --> pricing      for MIP problems

Control_Parameter_Value                 for MIP problems
            --> pricing_steepest_edge_float
            | pricing_steepest_edge_exact
            | pricing_textbook

Vars_List   --> []                      list of PPL variables
            | [PPL_Var | Vars_List].
```

Predicate Descriptions

Below is a short description of many of the interface predicates. For full definitions of terminology used here, see the main *PPL user manual*.

Domain Independent Predicates

First we describe the domain independent predicates that are included with all instantiations of the Prolog interfaces.

`ppl_version_major(?C_int)`

Unifies `C_int` with the major number of the PPL version.

`ppl_version_minor(?C_int)`

Unifies `C_int` with the minor number of the PPL version.

`ppl_version_revision(?C_int)`

Unifies `C_int` with the revision number of the PPL version.

`ppl_version_beta(?C_int)`

Unifies `C_int` with the beta number of the PPL version.

`ppl_version(?Atom)`

Unifies `Atom` with the PPL version.

`ppl_banner(?Atom)`

Unifies `Atom` with information about the PPL version, the licensing, the lack of any warranty whatsoever, the C++ compiler used to build the library, where to report bugs and where to look for further information.

`ppl_Coefficient_bits(?Bits)`

Unifies `Bits` with the number of bits used to encode a Coefficient in the C++ interface; 0 if unbounded.

`ppl_Coefficient_is_bounded`

Succeeds if and only if the Coefficients in the C++ interface are bounded.

`ppl_Coefficient_max(Max)`

If the Coefficients in the C++ interface are bounded, then the maximum coefficient the C++ interface can handle is unified with `Max`. If the Prolog system cannot handle this coefficient, then an exception is thrown. It fails if the Coefficients in the C++ interface are unbounded.

`ppl_Coefficient_min(Min)`

If the Coefficients in the C++ interface are bounded, then the minimum coefficient the C++ interface can handle is unified with `Min`. If the Prolog system cannot handle this coefficient, then an exception is thrown. It fails if the Coefficients in the C++ interface are unbounded.

`ppl_max_space_dimension(?Dimension_Type)`

Unifies `Dimension_Type` with the maximum space dimension this library can handle.

`ppl_initialize`

Initializes the PPL interface. Multiple calls to `ppl_initialize` does no harm.

`ppl_finalize`

Finalizes the PPL interface. Once this is executed, the next call to an interface predicate must either be to `ppl_initialize` *or to* `ppl_finalize`*. Multiple calls to* `ppl_finalize` *does no harm.*

`ppl_set_timeout_exception_atom(+Atom)`

Sets the atom to be thrown by timeout exceptions to `Atom`*. The default value is* `time_out`*.*

`ppl_timeout_exception_atom(?Atom)`

The atom to be thrown by timeout exceptions is unified with `Atom`*.*

`ppl_set_timeout(+C_unsigned)`

Computations taking exponential time will be interrupted some time after `C_unsigned` *ms after that call. If the computation is interrupted that way, the current timeout exception atom will be thrown.* `C_unsigned` *must be strictly greater than zero.*

`ppl_reset_timeout`

Resets the timeout time so that the computation is not interrupted.

`ppl_set_rounding_for_PPL`

Sets the FPU rounding mode so that the PPL abstractions based on floating point numbers work correctly. This is performed automatically at initialization-time. Calling this function is needed only if restore_pre_PPL_rounding() has previously been called.

`ppl_restore_rounding_for_PPL`

Sets the FPU rounding mode as it was before initialization of the PPL. After calling this function it is absolutely necessary to call set_rounding_for_PPL() before using any PPL abstractions based on floating point numbers. This is performed automatically at finalization-time.

MIP Predicates

Here we describe the predicates available for PPL objects defining mixed integer (linear) programming problems.

`ppl_new_MIP_Problem_from_space_dimension(+Dimension_Type, -Handle)`

Creates an MIP Problem MIP *with the feasible region the vector space of dimension* `Dimension_Type`*, objective function* 0 *and optimization mode* `max`*.* `Handle` *is unified with the handle for* MIP*.*

`ppl_new_MIP_Problem(+Constraint_System, +Lin_Expr, +Optimization_Mode, -Handle)`

Creates an MIP Problem MIP *with the feasible region represented by* `Constraint_System`*, objective function* `Lin_Expr` *and optimization mode* `Optimization_Mode`*.* `Handle` *is unified with the handle for* MIP*.*

`ppl_new_MIP_Problem_from_MIP_Problem(+Handle_1, -Handle_2)`

Creates an MIP Problem MIP *from the MIP Problem referenced by* `Handle_1`*.* `Handle_2` *is unified with the handle for* MIP*.*

`ppl_MIP_Problem_swap(+Handle_1, +Handle_2)`

Swaps the MIP Problem referenced by `Handle_1` *with the one referenced by* `Handle_2`*.*

`ppl_delete_MIP_Problem(+Handle)`

Deletes the MIP Problem referenced by `Handle`*. After execution,* `Handle` *is no longer a valid handle for a PPL MIP Problem.*

`ppl_MIP_Problem_space_dimension(+Handle, ?Dimension_Type)`

Unifies the dimension of the vector space in which the MIP Problem referenced by `Handle` *is embedded with* `Dimension_Type`*.*

`ppl_MIP_Problem_integer_space_dimensions(+Handle, ?Vars_List)`

Unifies `Vars_List` *with a list of variables representing the integer space dimensions of the MIP Problem referenced by* `Handle`*.*

`ppl_MIP_Problem_constraints(+Handle, -Constraint_System)`

Unifies `Constraint_System` *with a list of the constraints in the constraints system representing the feasible region for the MIP Problem referenced by* `Handle`*.*

`ppl_MIP_Problem_objective_function(+Handle, ?Lin_Expr)`

Unifies `Lin_Expr` *with the objective function for the MIP Problem referenced by* `Handle`*.*

`ppl_MIP_Problem_optimization_mode(+Handle, ?Optimization_Mode)`

Unifies `Optimization_Mode` *with the optimization mode for the MIP Problem referenced by* `Handle`*.*

`ppl_MIP_Problem_clear(+Handle)`

Resets the MIP problem referenced by `Handle` *to be the trivial problem with the feasible region the* 0*-dimensional universe, objective function* 0 *and optimization mode* `max`*.*

`ppl_MIP_Problem_add_space_dimensions_and_embed( +Handle, +Dimension_Type)`

Embeds the MIP problem referenced by `Handle` *in a space that is enlarged by* `Dimension_Type` *dimensions.*

`ppl_MIP_Problem_add_to_integer_space_dimensions(+Handle, +Vars_List)`

Updates the MIP Problem referenced by `Handle` *so that the variables in* `Vars_List` *are added to the set of integer space dimensions.*

`ppl_MIP_Problem_add_constraint(+Handle, +Constraint)`

Updates the MIP Problem referenced by `Handle` *so that the feasible region is represented by the original constraint system together with the constraint* `Constraint`*.*

`ppl_MIP_Problem_add_constraints(+Handle, +Constraint_System)`

Updates the MIP Problem referenced by `Handle` *so that the feasible region is represented by the original constraint system together with all the constraints in* `Constraint_System`*.*

`ppl_MIP_Problem_set_objective_function(+Handle, +Lin_Expr)`

Updates the MIP Problem referenced by `Handle` *so that the objective function is changed to* `Lin_Expr`*.*

`ppl_MIP_Problem_set_control_parameter(+Handle, +Control_Parameter_Value)`

Updates the MIP Problem referenced by `Handle` *so that the value for the relevant control parameter name is changed to* `Control_Parameter_Value`*.*

`ppl_MIP_Problem_get_control_parameter(+Handle, +Control_Parameter_Name, ?Control_Parameter_Value)`

Unifies `Control_Parameter_Value` *with the value of the control parameter* `Control_Parameter_Name`*.*

`ppl_MIP_Problem_set_optimization_mode(+Handle, +Optimization_Mode)`

Updates the MIP Problem referenced by `Handle` *so that the optimization mode is changed to* `Optimization_Mode`*.*

`ppl_MIP_Problem_is_satisfiable(+Handle)`

Succeeds if and only if the MIP Problem referenced by `Handle` is satisfiable.

`ppl_MIP_Problem_solve(+Handle, ?MIP_Problem_Status)`

Solves the MIP problem referenced by `Handle` and unifies `MIP_Problem_Status` with: `unfeasible`, if the MIP problem is not satisfiable; `unbounded`, if the MIP problem is satisfiable but there is no finite bound to the value of the objective function; `optimized`, if the MIP problem admits an optimal solution.

`ppl_MIP_Problem_feasible_point(+Handle, ?Generator)`

Unifies `Generator` with a feasible point for the MIP problem referenced by `Handle`.

`ppl_MIP_Problem_optimizing_point(+Handle, ?Generator)`

Unifies `Generator` with an optimizing point for the MIP problem referenced by `Handle`.

`ppl_MIP_Problem_optimal_value(+Handle, ?Coeff_1, ?Coeff_2)`

Unifies `Coeff_1` and `Coeff_2` with the numerator and denominator, respectively, for the optimal value for the MIP problem referenced by `Handle`.

`ppl_MIP_Problem_evaluate_objective_function(+Handle, +Generator, ?Coeff_1, ?Coeff_2)`

Evaluates the objective function of the MIP problem referenced by `Handle` at point `Generator`. `Coefficient_1` is unified with the numerator and `Coefficient_2` is unified with the denominator of the objective function value at `Generator`.

`ppl_MIP_Problem_OK(+Handle)`

Succeeds only if the MIP Problem referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

Predicates for the Int8_Box Domain

Here we provide a short description for each of the predicates available for the domain of int8 boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for int8 boxes

The constructor predicates build a int8 box from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Int8_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new int8 box $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Int8_Box_from_space_dimension(3, universe, X).
```

creates the int8 box defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Int8_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new int8 box `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Int8_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new int8 box `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Int8_Box_from_generators(+Generator_System, -Handle)`

Builds a new int8 box `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new int8 boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Int8_Box_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Grid(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new int8 box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_Int8_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int8 box `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_Int16_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int8 box `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_Int32_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int8 box `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_Int64_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int8 box `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_Uint8_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int8 box `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_Uint16_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int8 box `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_Uint32_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int8 box `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_Uint64_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int8 box `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_Z_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int8 box `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_Rational_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int8 box `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_BD_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int8 box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_BD_Shape_int16_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int8 box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_BD_Shape_int32_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int8 box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_BD_Shape_int64_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int8 box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_BD_Shape_mpz_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int8 box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_BD_Shape_mpq_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int8 box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_Octagonal_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int8 box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_Octagonal_Shape_int16_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int8 box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_Octagonal_Shape_int32_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int8 box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int8_Box_from_Octagonal_Shape_int64_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int8 box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int8 box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int8 box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int8 box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int8 box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int8 box `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int8 box `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int8 box `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int8 box `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int8 box `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int8 box `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int8 box `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int8 box `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int8 box `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int8_Box_from_Octagonal_Shape_long_double_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new int8 box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Int8_Box domain.

`ppl_delete_Int8_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the int8 box

Test Predicates

These predicates test the int8 box for different properties and succeed or fail depending on the outcome.

`ppl_Int8_Box_is_empty(+Handle)`

Succeeds if and only if the int8 box referenced by `Handle` is empty.

`ppl_Int8_Box_is_universe(+Handle)`

Succeeds if and only if the int8 box referenced by `Handle` is the universe.

`ppl_Int8_Box_is_bounded(+Handle)`

Succeeds if and only if the int8 box referenced by `Handle` is bounded.

`ppl_Int8_Box_contains_integer_point(+Handle)`

Succeeds if and only if the int8 box referenced by `Handle` contains an integer point.

`ppl_Int8_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the int8 box referenced by `Handle` is topologically closed.

`ppl_Int8_Box_is_discrete(+Handle)`

Succeeds if and only if the int8 box referenced by `Handle` *is discrete.*

```
ppl_Int8_Box_bounds_from_above(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from above in the int8 box referenced by* `Handle`*.*

```
ppl_Int8_Box_bounds_from_below(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from below in the int8 box referenced by* `Handle`*.*

```
ppl_Int8_Box_contains_Int8_Box(+Handle_1, +Handle_2)
```

Succeeds if and only if the int8 box referenced by `Handle_2` *is included in or equal to the int8 box referenced by* `Handle_1`*.*

```
ppl_Int8_Box_strictly_contains_Int8_Box(+Handle_1, +Handle_2)
```

Succeeds if and only if the int8 box referenced by `Handle_2` *is included in but not equal to the int8 box referenced by* `Handle_1`*.*

```
ppl_Int8_Box_is_disjoint_from_Int8_Box(+Handle_1, +Handle_2)
```

Succeeds if and only if the int8 box referenced by `Handle_2` *is disjoint from the int8 box referenced by* `Handle_1`*.*

```
ppl_Int8_Box_equals_Int8_Box(+Handle_1, +Handle_2)
```

Succeeds if and only if the int8 box referenced by `Handle_1` *is equal to the int8 box referenced by* `Handle_2`*.*

```
ppl_Int8_Box_OK(+Handle)
```

Succeeds only if the int8 box referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_Int8_Box_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the int8 box referenced by `Handle` *constrains the dimension* `PPL_Var`*.*

Predicates that return information about the int8 box

These predicates will obtain more detailed information about the int8 box unifying some of their arguments with the results.

```
ppl_Int8_Box_space_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the int8 box referenced by* `Handle`*.*

```
ppl_Int8_Box_affine_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` *with the affine dimension of the int8 box referenced by* `Handle`*.*

```
ppl_Int8_Box_relation_with_constraint(+Handle, +Constraint,
?Relation_List)
```

Unifies `Relation_List` *with the list of relations the int8 box referenced by* `Handle` *has with* `Constraint`*. The possible relations are listed in the grammar rules above.*

```
ppl_Int8_Box_relation_with_generator(+Handle, +Generator, ?Relation_-
List)
```

Unifies `Relation_List` *with the list of relations the int8 box referenced by* `Handle` *has with* `Generator`*. The possible relations are listed in the grammar rules above.*

`ppl_Int8_Box_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the int8 box referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Int8_Box_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the int8 box referenced by `Handle`.

`ppl_Int8_Box_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the int8 box referenced by `Handle`.

`ppl_Int8_Box_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the int8 box referenced by `Handle`.

`ppl_Int8_Box_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the int8 box referenced by `Handle`.

`ppl_Int8_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if int8 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Int8_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if int8 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Int8_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if int8 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Int8_Box_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if int8 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the

infinum is also the minimum, `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

`ppl_Int8_Box_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the total memory in bytes occupied by the int8 box referenced by* `Handle`*.*

`ppl_Int8_Box_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the external memory in bytes occupied by the int8 box referenced by* `Handle`*.*

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Int8_Box_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the int8 box referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the int8 box

These predicates may modify the int8 box referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the int8 box by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Int8_Box_add_constraint(+Handle, +Constraint)`

Updates the int8 box referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Int8_Box_add_congruence(+Handle, +Congruence)`

Updates the int8 box referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Int8_Box_add_constraints( +Handle, +Constraint_System)`

Updates the int8 box referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Int8_Box_add_congruences( +Handle, +Congruence_System)`

Updates the int8 box referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Int8_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the int8 box referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Int8_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the int8 box referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`.

`ppl_Int8_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the int8 box referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`.

`ppl_Int8_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the int8 box referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`.

Predicates that transform the int8 box

These predicates enable transformations such as taking the topological closure (which for the domain of int8 box is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Int8_Box_topological_closure_assign(+Handle)`

Assigns to the int8 box referenced by `Handle` *its topological closure.*

`ppl_Int8_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the int8 box `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`.

`ppl_Int8_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the int8 box `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Int8_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the int8 box referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

`ppl_Int8_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the int8 box referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

`ppl_Int8_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to int8 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Int8_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to int8 box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Int8_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to int8 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Int8_Box_generalized_affine_preimage(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to int8 box `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` ⋈ `Lin_Expr/Coeff`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Int8_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1,
+Relation_Symbol, +Lin_Expr_2)
```

Assigns to int8 box `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Int8_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1,
+Relation_Symbol, +Lin_Expr_2)
```

Assigns to int8 box `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Int8_Box_wrap_assign(+Handle, +List_of_PPL_Var, +Width,
+Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)
```

Transforms the int8 box `P` referenced by `Handle` by wrapping the dimensions given by `List_of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one int8 box

These predicates include the binary operators which will assign to the int8 box referred to by the first argument its combination with the int8 box referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Int8_Box_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the int8 box `P` referenced by `Handle_1` the intersection of `P` and the int8 box referenced by `Handle_2`.

```
ppl_Int8_Box_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the int8 box `P` referenced by `Handle_1` the upper bound of `P` and the int8 box referenced by `Handle_2`.

```
ppl_Int8_Box_difference_assign(+Handle_1, +Handle_2)
```

Assigns to the int8 box `P` referenced by `Handle_1` the difference of `P` and the int8 box referenced by `Handle_2`.

```
ppl_Int8_Box_time_elapse_assign(+Handle_1, +Handle_2)
```

Assigns to the int8 box `P` referenced by `Handle_1` the time elapse of `P` and the int8 box referenced by `Handle_2`.

```
ppl_Int8_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the int8 box `P_1` referenced by `Handle_1` with the int8 box referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_Int8_Box_simplify_using_context_assign(+Handle_1, +Handle_2,
?Boolean)
```

Succeeds if and only if the intersection of int8 box `P_1` referenced by `Handle_1` and the int8 box `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

```
ppl_Int8_Box_linear_partition(+Handle_1, +Handle_2, -Handle_3,
-Handle_4)
```

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Int8_Box domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the int8 box referred to by the second argument has to be contained in (or equal to) the int8 box referred to by the first argument.

```
ppl_Int8_Box_CC76_widening_assign_with_tokens(+Handle_1, +Handle_2,
+C_unsigned_1, ?C_unsigned_2)
```

Assigns to the int8 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the int8 box referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Int8_Box_CC76_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the int8 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the int8 box referenced by `Handle_2`.

```
ppl_Int8_Box_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_-
unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_Int8_Box_CC76_widening_assign_with_tokens/4`

```
ppl_Int8_Box_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_Int8_Box_CC76_widening_assign/2`

```
ppl_Int8_Box_limited_CC76_extrapolation_assign_with_tokens(+Handle_1,
+Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the int8 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the int8 box referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Int8_Box_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2,
+Constraint_System)
```

Assigns to the int8 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the int8 box referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all

the points of `P_1`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the int8 box referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Int8_Box_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the int8 box `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the int8 box referenced by* `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the int8 box referred to in the first argument. These predicates enable the modification of the vector space of the int8 box referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Int8_Box_add_space_dimensions_and_embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the int8 box* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Int8_Box_add_space_dimensions_and_project(+Handle, +Dimension_-
Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the int8 box* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

```
ppl_Int8_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the int8 box `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Int8_Box_remove_higher_space_dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the int8 box `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`.

```
ppl_Int8_Box_expand_space_dimension(+Handle, +PPL_Var, +Dimension_-
Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the int8 box referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

```
ppl_Int8_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_-
Var)
```

Modifies the int8 box referenced by `Handle` *by folding the space dimensions contained in* `List_of_-PPL_Vars` *into dimension* `PPL_Var`. *The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

`ppl_Int8_Box_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Int16_Box Domain

Here we provide a short description for each of the predicates available for the domain of int16 boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for int16 boxes

The constructor predicates build a int16 box from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Int16_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new int16 box $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Int16_Box_from_space_dimension(3, universe, X).
```

creates the int16 box defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Int16_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new int16 box `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Int16_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new int16 box `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Int16_Box_from_generators(+Generator_System, -Handle)`

Builds a new int16 box `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new int16 boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Int16_Box_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Grid(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new int16 box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Uint8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Uint16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Uint32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Uint64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Z_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Rational_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for

P_1.

`ppl_new_Int16_Box_from_BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int16_Box_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int16 box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Int16_Box domain.

`ppl_delete_Int16_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the int16 box

Test Predicates

These predicates test the int16 box for different properties and succeed or fail depending on the outcome.

`ppl_Int16_Box_is_empty(+Handle)`

Succeeds if and only if the int16 box referenced by `Handle` is empty.

`ppl_Int16_Box_is_universe(+Handle)`

Succeeds if and only if the int16 box referenced by `Handle` is the universe.

`ppl_Int16_Box_is_bounded(+Handle)`

Succeeds if and only if the int16 box referenced by `Handle` is bounded.

`ppl_Int16_Box_contains_integer_point(+Handle)`

Succeeds if and only if the int16 box referenced by `Handle` contains an integer point.

`ppl_Int16_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the int16 box referenced by `Handle` is topologically closed.

`ppl_Int16_Box_is_discrete(+Handle)`

Succeeds if and only if the int16 box referenced by `Handle` is discrete.

`ppl_Int16_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the int16 box referenced by `Handle`.

`ppl_Int16_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the int16 box referenced by `Handle`.

`ppl_Int16_Box_contains_Int16_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the int16 box referenced by `Handle_2` is included in or equal to the int16 box referenced by `Handle_1`.

`ppl_Int16_Box_strictly_contains_Int16_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the int16 box referenced by `Handle_2` is included in but not equal to the int16 box referenced by `Handle_1`.

`ppl_Int16_Box_is_disjoint_from_Int16_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the int16 box referenced by `Handle_2` is disjoint from the int16 box referenced by `Handle_1`.

`ppl_Int16_Box_equals_Int16_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the int16 box referenced by `Handle_1` is equal to the int16 box referenced by `Handle_2`.

`ppl_Int16_Box_OK(+Handle)`

Succeeds only if the int16 box referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Int16_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the int16 box referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the int16 box

These predicates will obtain more detailed information about the int16 box unifying some of their arguments with the results.

`ppl_Int16_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the int16 box referenced by `Handle`.

`ppl_Int16_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the int16 box referenced by `Handle`.

`ppl_Int16_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the int16 box referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Int16_Box_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the int16 box referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Int16_Box_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the int16 box referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Int16_Box_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the int16 box referenced by `Handle`.

`ppl_Int16_Box_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the int16 box referenced by `Handle`.

`ppl_Int16_Box_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the int16 box referenced by `Handle`.

`ppl_Int16_Box_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the int16 box referenced by `Handle`.

`ppl_Int16_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if int16 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Int16_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean)
```

Succeeds if and only if int16 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Int16_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if int16 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Int16_Box_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if int16 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Int16_Box_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the total memory in bytes occupied by the int16 box referenced by `Handle`.

```
ppl_Int16_Box_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the external memory in bytes occupied by the int16 box referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Int16_Box_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the int16 box referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the int16 box

These predicates may modify the int16 box referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the int16 box by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Int16_Box_add_constraint(+Handle, +Constraint)`

Updates the int16 box referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Int16_Box_add_congruence(+Handle, +Congruence)`

Updates the int16 box referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Int16_Box_add_constraints( +Handle, +Constraint_System)`

Updates the int16 box referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Int16_Box_add_congruences( +Handle, +Congruence_System)`

Updates the int16 box referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Int16_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the int16 box referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Int16_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the int16 box referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Int16_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the int16 box referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Int16_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the int16 box referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the int16 box

These predicates enable transformations such as taking the topological closure (which for the domain of int16 box is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Int16_Box_topological_closure_assign(+Handle)`

Assigns to the int16 box referenced by `Handle` *its topological closure.*

`ppl_Int16_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the int16 box `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

`ppl_Int16_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the int16 box `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Int16_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the int16 box referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_Int16_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the int16 box referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_Int16_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1,
+Lin_Expr_2, +Coeff)
```

Assigns to int16 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_Int16_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1,
+Lin_Expr_2, +Coeff)
```

Assigns to int16 box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_Int16_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_-
Symbol, +Lin_Expr, +Coeff)
```

Assigns to int16 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`*.*

```
ppl_Int16_Box_generalized_affine_preimage(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to int16 box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Int16_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1,
+Relation_Symbol, +Lin_Expr_2)
```

Assigns to int16 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`*.*

```
ppl_Int16_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_-
1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to int16 box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Int16_Box_wrap_assign(+Handle, +List_of_PPL_Var, +Width,
+Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)
```

Transforms the int16 box `P` *referenced by* `Handle` *by wrapping the dimensions given by* `List_of_-PPL_Vars` *while respecting the specified* `Width`*,* `Representation` *and* `Overflow` *behavior of all these variables. The* `Constraint_System` *represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer* `C_unsigned` *and* `Boolean` *allow control of the complexity/precision ratio; higher values for* `C_unsigned` *will lead to possibly greater precision while a true value for* `Boolean` *indicates that the space dimensions should be wrapped individ-*

ually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one int16 box

These predicates include the binary operators which will assign to the int16 box referred to by the first argument its combination with the int16 box referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Int16_Box_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the int16 box `P` referenced by `Handle_1` the intersection of `P` and the int16 box referenced by `Handle_2`.

`ppl_Int16_Box_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the int16 box `P` referenced by `Handle_1` the upper bound of `P` and the int16 box referenced by `Handle_2`.

`ppl_Int16_Box_difference_assign(+Handle_1, +Handle_2)`

Assigns to the int16 box `P` referenced by `Handle_1` the difference of `P` and the int16 box referenced by `Handle_2`.

`ppl_Int16_Box_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the int16 box `P` referenced by `Handle_1` the time elapse of `P` and the int16 box referenced by `Handle_2`.

`ppl_Int16_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the int16 box `P_1` referenced by `Handle_1` with the int16 box referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Int16_Box_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of int16 box `P_1` referenced by `Handle_1` and the int16 box `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

`ppl_Int16_Box_linear_partition(+Handle_1, +Handle_2, -Handle_3, -Handle_4)`

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Int16_Box domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the int16 box referred to by the second argument has to be contained in (or equal to) the int16 box referred to by the first argument.

`ppl_Int16_Box_CC76_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the int16 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the int16 box referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Int16_Box_CC76_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the int16 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the int16 box referenced by `Handle_2`.

```
ppl_Int16_Box_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_-
unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_Int16_Box_CC76_widening_assign_with_tokens/4`

```
ppl_Int16_Box_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_Int16_Box_CC76_widening_assign/2`

```
ppl_Int16_Box_limited_CC76_extrapolation_assign_with_tokens(+Handle_1,
+Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the int16 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the int16 box referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Int16_Box_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2,
+Constraint_System)
```

Assigns to the int16 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the int16 box referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the int16 box referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Int16_Box_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the int16 box `P` referenced by `Handle_1` the concatenation of `P` and the int16 box referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the int16 box referred to in the first argument. These predicates enable the modification of the vector space of the int16 box referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Int16_Box_add_space_dimensions_and_embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the int16 box `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Int16_Box_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the int16 box `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Int16_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the int16 box `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Int16_Box_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the int16 box `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Int16_Box_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the int16 box referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Int16_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the int16 box referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Int16_Box_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Int32_Box Domain

Here we provide a short description for each of the predicates available for the domain of int32 boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for int32 boxes

The constructor predicates build a int32 box from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Int32_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new int32 box $\mathcal{P}$ with `Dimension_Type` *dimensions; it is empty or the universe depending on whether* `Atom` *is* `empty` *or* `universe`*, respectively.* `Handle` *is unified with the handle for $\mathcal{P}$. Thus the query*

```
?- ppl_new_Int32_Box_from_space_dimension(3, universe, X).
```

creates the int32 box defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` *bound to a valid handle for accessing it.*

`ppl_new_Int32_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new int32 box `P` *from* `Constraint_System`*.* `Handle` *is unified with the handle for* `P`*.*

`ppl_new_Int32_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new int32 box `P` *from* `Congruence_System`*.* `Handle` *is unified with the handle for* `P`*.*

`ppl_new_Int32_Box_from_generators(+Generator_System, -Handle)`

Builds a new int32 box `P` *from* `Generator_System`*.* `Handle` *is unified with the handle for* `P`*.*

Predicates that build new int32 boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Int32_Box_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` *from the int8_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Int32_Box_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` *from the int16_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Int32_Box_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` *from the int32_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Int32_Box_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` *from the int64_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Int32_Box_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` *from the uint8_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Int32_Box_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Grid(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new int32 box `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_Octagonal_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new int32 box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_Int8_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int32 box `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_Int16_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int32 box `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_Int32_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int32 box `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_Int64_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int32 box `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_Uint8_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int32 box `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_Uint16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int32 box `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_Uint32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int32 box `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_Uint64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int32 box `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_Z_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int32 box `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_Rational_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int32 box `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_BD_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int32 box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_BD_Shape_int16_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int32 box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_BD_Shape_int32_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int32 box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_BD_Shape_int64_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int32 box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_BD_Shape_mpz_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int32 box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_BD_Shape_mpq_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int32 box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_Octagonal_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int32 box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_Octagonal_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new int32 box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int32_Box_from_Octagonal_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new int32 box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int32 box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int32 box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int32 box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int32 box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int32 box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int32 box `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int32 box `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int32 box `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int32 box `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int32 box `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int32 box `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int32 box `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int32 box `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int32 box `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int32_Box_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int32 box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Int32_Box domain.

`ppl_delete_Int32_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the int32 box

Test Predicates

These predicates test the int32 box for different properties and succeed or fail depending on the outcome.

`ppl_Int32_Box_is_empty(+Handle)`

Succeeds if and only if the int32 box referenced by `Handle` is empty.

`ppl_Int32_Box_is_universe(+Handle)`

Succeeds if and only if the int32 box referenced by `Handle` is the universe.

`ppl_Int32_Box_is_bounded(+Handle)`

Succeeds if and only if the int32 box referenced by `Handle` is bounded.

`ppl_Int32_Box_contains_integer_point(+Handle)`

Succeeds if and only if the int32 box referenced by `Handle` *contains an integer point.*

`ppl_Int32_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the int32 box referenced by `Handle` *is topologically closed.*

`ppl_Int32_Box_is_discrete(+Handle)`

Succeeds if and only if the int32 box referenced by `Handle` *is discrete.*

`ppl_Int32_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from above in the int32 box referenced by* `Handle`*.*

`ppl_Int32_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from below in the int32 box referenced by* `Handle`*.*

`ppl_Int32_Box_contains_Int32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the int32 box referenced by `Handle_2` *is included in or equal to the int32 box referenced by* `Handle_1`*.*

`ppl_Int32_Box_strictly_contains_Int32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the int32 box referenced by `Handle_2` *is included in but not equal to the int32 box referenced by* `Handle_1`*.*

`ppl_Int32_Box_is_disjoint_from_Int32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the int32 box referenced by `Handle_2` *is disjoint from the int32 box referenced by* `Handle_1`*.*

`ppl_Int32_Box_equals_Int32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the int32 box referenced by `Handle_1` *is equal to the int32 box referenced by* `Handle_2`*.*

`ppl_Int32_Box_OK(+Handle)`

Succeeds only if the int32 box referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

`ppl_Int32_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the int32 box referenced by `Handle` *constrains the dimension* `PPL_Var`*.*

Predicates that return information about the int32 box

These predicates will obtain more detailed information about the int32 box unifying some of their arguments with the results.

`ppl_Int32_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the int32 box referenced by* `Handle`*.*

`ppl_Int32_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` *with the affine dimension of the int32 box referenced by* `Handle`*.*

`ppl_Int32_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` *with the list of relations the int32 box referenced by* `Handle` *has with* `Constraint`*. The possible relations are listed in the grammar rules above.*

`ppl_Int32_Box_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the int32 box referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Int32_Box_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the int32 box referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Int32_Box_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the int32 box referenced by `Handle`.

`ppl_Int32_Box_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the int32 box referenced by `Handle`.

`ppl_Int32_Box_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the int32 box referenced by `Handle`.

`ppl_Int32_Box_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the int32 box referenced by `Handle`.

`ppl_Int32_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if int32 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Int32_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if int32 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Int32_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if int32 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Int32_Box_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if int32 box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Int32_Box_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the int32 box referenced by* `Handle`*.*

```
ppl_Int32_Box_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the int32 box referenced by* `Handle`*.*

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Int32_Box_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the int32 box referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the int32 box

These predicates may modify the int32 box referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the int32 box by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Int32_Box_add_constraint(+Handle, +Constraint)
```

Updates the int32 box referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

```
ppl_Int32_Box_add_congruence(+Handle, +Congruence)
```

Updates the int32 box referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

```
ppl_Int32_Box_add_constraints( +Handle, +Constraint_System)
```

Updates the int32 box referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

```
ppl_Int32_Box_add_congruences( +Handle, +Congruence_System)
```

Updates the int32 box referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

```
ppl_Int32_Box_refine_with_constraint( +Handle, +Constraint)
```

Updates the int32 box referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`.

```
ppl_Int32_Box_refine_with_congruence( +Handle, +Congruence)
```

Updates the int32 box referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`.

```
ppl_Int32_Box_refine_with_constraints( +Handle, +Constraint_System)
```

Updates the int32 box referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`.

```
ppl_Int32_Box_refine_with_congruences( +Handle, +Congruence_System)
```

Updates the int32 box referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`.

Predicates that transform the int32 box

These predicates enable transformations such as taking the topological closure (which for the domain of int32 box is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Int32_Box_topological_closure_assign(+Handle)
```

Assigns to the int32 box referenced by `Handle` *its topological closure.*

```
ppl_Int32_Box_unconstrain_space_dimension(+Handle, +PPL_Var)
```

Modifies the int32 box `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`.

```
ppl_Int32_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)
```

Modifies the int32 box `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Int32_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the int32 box referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Int32_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the int32 box referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Int32_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1,
+Lin_Expr_2, +Coeff)
```

Assigns to int32 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Int32_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1,
+Lin_Expr_2, +Coeff)
```

Assigns to int32 box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Int32_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_-
Symbol, +Lin_Expr, +Coeff)
```

Assigns to int32 box `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `PPL_Var` ⋈ `Lin_Expr/Coeff`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Int32_Box_generalized_affine_preimage(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to int32 box `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` ⋈ `Lin_Expr/Coeff`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Int32_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1,
+Relation_Symbol, +Lin_Expr_2)
```

Assigns to int32 box `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Int32_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_
1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to int32 box `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Int32_Box_wrap_assign(+Handle, +List_of_PPL_Var, +Width,
+Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)
```

Transforms the int32 box `P` referenced by `Handle` by wrapping the dimensions given by `List_of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one int32 box

These predicates include the binary operators which will assign to the int32 box referred to by the first argument its combination with the int32 box referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Int32_Box_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the int32 box `P` referenced by `Handle_1` the intersection of `P` and the int32 box referenced by `Handle_2`.

```
ppl_Int32_Box_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the int32 box `P` referenced by `Handle_1` the upper bound of `P` and the int32 box referenced by `Handle_2`.

```
ppl_Int32_Box_difference_assign(+Handle_1, +Handle_2)
```

Assigns to the int32 box `P` referenced by `Handle_1` the difference of `P` and the int32 box referenced by `Handle_2`.

```
ppl_Int32_Box_time_elapse_assign(+Handle_1, +Handle_2)
```

Assigns to the int32 box `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the int32 box referenced by* `Handle_2`*.*

```
ppl_Int32_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the int32 box `P_1` *referenced by* `Handle_1` *with the int32 box referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Int32_Box_simplify_using_context_assign(+Handle_1, +Handle_2,
?Boolean)
```

Succeeds if and only if the intersection of int32 box `P_1` *referenced by* `Handle_1` *and the int32 box* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

```
ppl_Int32_Box_linear_partition(+Handle_1, +Handle_2, -Handle_3,
-Handle_4)
```

`Handle_1` *and* `Handle_2` *are handles for elements* `P_1` *and* `P_2` *in the Int32_Box domain. The predicate unifies handle* `Handle_3` *to a reference to the intersection of* `P_1` *and* `P_2` *and* `Handle_4` *to a reference to a pointset powerset of nnc polyhedra* `P_4`*; where* `P_4` *is the linear partition of* `P_1` *with respect to* `P_2`*. This predicate is only provided if the class* `Pointset_Powerset_NNC_Polyhedron` *has been enabled when configuring the library.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the int32 box referred to by the second argument has to be contained in (or equal to) the int32 box referred to by the first argument.

```
ppl_Int32_Box_CC76_widening_assign_with_tokens(+Handle_1, +Handle_2,
+C_unsigned_1, ?C_unsigned_2)
```

Assigns to the int32 box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the int32 box referenced by* `Handle_2`*. The widening with tokens delay technique is applied with* `C_unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Int32_Box_CC76_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the int32 box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the int32 box referenced by* `Handle_2`*.*

```
ppl_Int32_Box_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_-
unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_Int32_Box_CC76_widening_assign_with_tokens/4`

```
ppl_Int32_Box_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_Int32_Box_CC76_widening_assign/2`

```
ppl_Int32_Box_limited_CC76_extrapolation_assign_with_tokens(+Handle_1,
+Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the int32 box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the int32 box referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*. The widening with tokens delay technique is applied with* `C_unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

`ppl_Int32_Box_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the int32 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the int32 box referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the int32 box referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Int32_Box_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the int32 box `P` referenced by `Handle_1` the concatenation of `P` and the int32 box referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the int32 box referred to in the first argument. These predicates enable the modification of the vector space of the int32 box referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Int32_Box_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the int32 box `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Int32_Box_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the int32 box `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Int32_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the int32 box `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Int32_Box_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the int32 box `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Int32_Box_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the int32 box referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Int32_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the int32 box referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Int32_Box_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Int64_Box Domain

Here we provide a short description for each of the predicates available for the domain of int64 boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for int64 boxes

The constructor predicates build a int64 box from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Int64_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new int64 box $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Int64_Box_from_space_dimension(3, universe, X).
```

creates the int64 box defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Int64_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new int64 box `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Int64_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new int64 box `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Int64_Box_from_generators(+Generator_System, -Handle)`

Builds a new int64 box `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new int64 boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Int64_Box_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Grid(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new int64 box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int64 box `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int64 box `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int64 box `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Int64_Box_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new int64 box `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Uint8_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int64 box `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Uint16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Uint32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Uint64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Z_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int64 box `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Rational_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_BD_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_BD_Shape_int16_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_BD_Shape_int32_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_BD_Shape_int64_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_BD_Shape_mpz_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_BD_Shape_mpq_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Octagonal_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Octagonal_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new int64 box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Octagonal_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new int64 box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Octagonal_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new int64 box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Octagonal_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new int64 box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Octagonal_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new int64 box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_C_Polyhedron_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_NNC_Polyhedron_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Grid_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int64 box `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Float_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new int64 box `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_BD_Shape_float_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Octagonal_Shape_float_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Double_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_BD_Shape_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Octagonal_Shape_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Long_Double_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_BD_Shape_long_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new int64 box `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Int64_Box_from_Octagonal_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new int64 box `P_1` *from the octagonal_shape_long_double referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

Destructor predicate

Below is the destructor predicate for the Int64_Box domain.

`ppl_delete_Int64_Box(+Handle)`

Invalidates the handle referenced by `Handle`*: this makes sure the corresponding resources will eventually be released.*

Predicates that do not change the int64 box

Test Predicates

These predicates test the int64 box for different properties and succeed or fail depending on the outcome.

`ppl_Int64_Box_is_empty(+Handle)`

Succeeds if and only if the int64 box referenced by `Handle` *is empty.*

`ppl_Int64_Box_is_universe(+Handle)`

Succeeds if and only if the int64 box referenced by `Handle` *is the universe.*

`ppl_Int64_Box_is_bounded(+Handle)`

Succeeds if and only if the int64 box referenced by `Handle` *is bounded.*

`ppl_Int64_Box_contains_integer_point(+Handle)`

Succeeds if and only if the int64 box referenced by `Handle` *contains an integer point.*

`ppl_Int64_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the int64 box referenced by `Handle` *is topologically closed.*

`ppl_Int64_Box_is_discrete(+Handle)`

Succeeds if and only if the int64 box referenced by `Handle` *is discrete.*

`ppl_Int64_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from above in the int64 box referenced by* `Handle`*.*

`ppl_Int64_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from below in the int64 box referenced by* `Handle`*.*

`ppl_Int64_Box_contains_Int64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the int64 box referenced by `Handle_2` *is included in or equal to the int64 box referenced by* `Handle_1`*.*

`ppl_Int64_Box_strictly_contains_Int64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the int64 box referenced by `Handle_2` *is included in but not equal to the int64 box referenced by* `Handle_1`*.*

`ppl_Int64_Box_is_disjoint_from_Int64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the int64 box referenced by `Handle_2` *is disjoint from the int64 box referenced by* `Handle_1`*.*

`ppl_Int64_Box_equals_Int64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the int64 box referenced by `Handle_1` is equal to the int64 box referenced by `Handle_2`.

`ppl_Int64_Box_OK(+Handle)`

Succeeds only if the int64 box referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Int64_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the int64 box referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the int64 box

These predicates will obtain more detailed information about the int64 box unifying some of their arguments with the results.

`ppl_Int64_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the int64 box referenced by `Handle`.

`ppl_Int64_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the int64 box referenced by `Handle`.

`ppl_Int64_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the int64 box referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Int64_Box_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the int64 box referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Int64_Box_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the int64 box referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Int64_Box_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the int64 box referenced by `Handle`.

`ppl_Int64_Box_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the int64 box referenced by `Handle`.

`ppl_Int64_Box_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the int64 box referenced by `Handle`.

`ppl_Int64_Box_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the int64 box referenced by `Handle`.

`ppl_Int64_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if int64 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Int64_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if int64 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Int64_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if int64 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Int64_Box_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if int64 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Int64_Box_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the total memory in bytes occupied by the int64 box referenced by `Handle`.

`ppl_Int64_Box_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the int64 box referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Int64_Box_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the int64 box referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the int64 box

These predicates may modify the int64 box referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the int64 box by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Int64_Box_add_constraint(+Handle, +Constraint)`

Updates the int64 box referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Int64_Box_add_congruence(+Handle, +Congruence)`

Updates the int64 box referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Int64_Box_add_constraints( +Handle, +Constraint_System)`

Updates the int64 box referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Int64_Box_add_congruences( +Handle, +Congruence_System)`

Updates the int64 box referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Int64_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the int64 box referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Int64_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the int64 box referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Int64_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the int64 box referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Int64_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the int64 box referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the int64 box

These predicates enable transformations such as taking the topological closure (which for the domain of int64 box is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Int64_Box_topological_closure_assign(+Handle)`

Assigns to the int64 box referenced by `Handle` *its topological closure.*

```
ppl_Int64_Box_unconstrain_space_dimension(+Handle, +PPL_Var)
```

Modifies the int64 box `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

```
ppl_Int64_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)
```

Modifies the int64 box `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Int64_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the int64 box referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_Int64_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the int64 box referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_Int64_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1,
+Lin_Expr_2, +Coeff)
```

Assigns to int64 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_Int64_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1,
+Lin_Expr_2, +Coeff)
```

Assigns to int64 box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_Int64_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_-
Symbol, +Lin_Expr, +Coeff)
```

Assigns to int64 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`*.*

```
ppl_Int64_Box_generalized_affine_preimage(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to int64 box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Int64_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1,
+Relation_Symbol, +Lin_Expr_2)
```

Assigns to int64 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`*.*

```
ppl_Int64_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_-
1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to int64 box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Int64_Box_wrap_assign(+Handle, +List_of_PPL_Var, +Width,
+Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)
```

Transforms the int64 box `P` *referenced by* `Handle` *by wrapping the dimensions given by* `List_of_PPL_Vars` *while respecting the specified* `Width`, `Representation` *and* `Overflow` *behavior of all these variables. The* `Constraint_System` *represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer* `C_unsigned` *and* `Boolean` *allow control of the complexity/precision ratio; higher values for* `C_unsigned` *will lead to possibly greater precision while a true value for* `Boolean` *indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.*

Predicates whose results depend on more than one int64 box

These predicates include the binary operators which will assign to the int64 box referred to by the first argument its combination with the int64 box referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Int64_Box_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the int64 box `P` *referenced by* `Handle_1` *the intersection of* `P` *and the int64 box referenced by* `Handle_2`.

```
ppl_Int64_Box_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the int64 box `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the int64 box referenced by* `Handle_2`.

```
ppl_Int64_Box_difference_assign(+Handle_1, +Handle_2)
```

Assigns to the int64 box `P` *referenced by* `Handle_1` *the difference of* `P` *and the int64 box referenced by* `Handle_2`.

```
ppl_Int64_Box_time_elapse_assign(+Handle_1, +Handle_2)
```

Assigns to the int64 box `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the int64 box referenced by* `Handle_2`.

```
ppl_Int64_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the int64 box `P_1` *referenced by* `Handle_1` *with the int64 box referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Int64_Box_simplify_using_context_assign(+Handle_1, +Handle_2,
?Boolean)
```

Succeeds if and only if the intersection of int64 box `P_1` *referenced by* `Handle_1` *and the int64 box* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`.

```
ppl_Int64_Box_linear_partition(+Handle_1, +Handle_2, -Handle_3,
-Handle_4)
```

`Handle_1` *and* `Handle_2` *are handles for elements* `P_1` *and* `P_2` *in the Int64_Box domain. The predicate unifies handle* `Handle_3` *to a reference to the intersection of* `P_1` *and* `P_2` *and* `Handle_4` *to a reference to a pointset powerset of nnc polyhedra* `P_4`*; where* `P_4` *is the linear partition of* `P_1` *with respect to* `P_2`*. This predicate is only provided if the class* `Pointset_Powerset_NNC_Polyhedron` *has been enabled when configuring the library.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with*

Tokens and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the int64 box referred to by the second argument has to be contained in (or equal to) the int64 box referred to by the first argument.

`ppl_Int64_Box_CC76_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the int64 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the int64 box referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_Int64_Box_CC76_widening_assign(+Handle_1, +Handle_2)`

Assigns to the int64 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the int64 box referenced by `Handle_2`.

`ppl_Int64_Box_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Same as predicate `ppl_Int64_Box_CC76_widening_assign_with_tokens/4`

`ppl_Int64_Box_widening_assign(+Handle_1, +Handle_2)`

Same as predicate `ppl_Int64_Box_CC76_widening_assign/2`

`ppl_Int64_Box_limited_CC76_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the int64 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the int64 box referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_Int64_Box_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the int64 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the int64 box referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the int64 box referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Int64_Box_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the int64 box `P` referenced by `Handle_1` the concatenation of `P` and the int64 box referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the int64 box referred to in the first argument. These predicates enable the modification of the vector space of the int64 box referred to in the

first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Int64_Box_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the int64 box `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Int64_Box_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the int64 box `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Int64_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the int64 box `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Int64_Box_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the int64 box `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Int64_Box_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the int64 box referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Int64_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the int64 box referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Int64_Box_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Uint8_Box Domain

Here we provide a short description for each of the predicates available for the domain of uint8 boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for uint8 boxes

The constructor predicates build a uint8 box from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.

- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Uint8_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new uint8 box $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Uint8_Box_from_space_dimension(3, universe, X).
```

creates the uint8 box defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Uint8_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new uint8 box `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Uint8_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new uint8 box `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Uint8_Box_from_generators(+Generator_System, -Handle)`

Builds a new uint8 box `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new uint8 boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Uint8_Box_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Grid(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new uint8 box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Uint8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Uint16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Uint32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Uint64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Z_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Rational_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_int16_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_int32_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_int64_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_mpz_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_mpq_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for

P_1.

`ppl_new_Uint8_Box_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint8_Box_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint8 box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Uint8_Box domain.

`ppl_delete_Uint8_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the uint8 box

Test Predicates

These predicates test the uint8 box for different properties and succeed or fail depending on the outcome.

`ppl_Uint8_Box_is_empty(+Handle)`

Succeeds if and only if the uint8 box referenced by `Handle` is empty.

`ppl_Uint8_Box_is_universe(+Handle)`

Succeeds if and only if the uint8 box referenced by `Handle` *is the universe.*

`ppl_Uint8_Box_is_bounded(+Handle)`

Succeeds if and only if the uint8 box referenced by `Handle` *is bounded.*

`ppl_Uint8_Box_contains_integer_point(+Handle)`

Succeeds if and only if the uint8 box referenced by `Handle` *contains an integer point.*

`ppl_Uint8_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the uint8 box referenced by `Handle` *is topologically closed.*

`ppl_Uint8_Box_is_discrete(+Handle)`

Succeeds if and only if the uint8 box referenced by `Handle` *is discrete.*

`ppl_Uint8_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from above in the uint8 box referenced by* `Handle`*.*

`ppl_Uint8_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from below in the uint8 box referenced by* `Handle`*.*

`ppl_Uint8_Box_contains_Uint8_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the uint8 box referenced by `Handle_2` *is included in or equal to the uint8 box referenced by* `Handle_1`*.*

`ppl_Uint8_Box_strictly_contains_Uint8_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the uint8 box referenced by `Handle_2` *is included in but not equal to the uint8 box referenced by* `Handle_1`*.*

`ppl_Uint8_Box_is_disjoint_from_Uint8_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the uint8 box referenced by `Handle_2` *is disjoint from the uint8 box referenced by* `Handle_1`*.*

`ppl_Uint8_Box_equals_Uint8_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the uint8 box referenced by `Handle_1` *is equal to the uint8 box referenced by* `Handle_2`*.*

`ppl_Uint8_Box_OK(+Handle)`

Succeeds only if the uint8 box referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

`ppl_Uint8_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the uint8 box referenced by `Handle` *constrains the dimension* `PPL_Var`*.*

Predicates that return information about the uint8 box

These predicates will obtain more detailed information about the uint8 box unifying some of their arguments with the results.

`ppl_Uint8_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the uint8 box referenced by* `Handle`*.*

`ppl_Uint8_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` *with the affine dimension of the uint8 box referenced by* `Handle`*.*

`ppl_Uint8_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the uint8 box referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Uint8_Box_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the uint8 box referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Uint8_Box_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the uint8 box referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Uint8_Box_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the uint8 box referenced by `Handle`.

`ppl_Uint8_Box_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the uint8 box referenced by `Handle`.

`ppl_Uint8_Box_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the uint8 box referenced by `Handle`.

`ppl_Uint8_Box_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the uint8 box referenced by `Handle`.

`ppl_Uint8_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if uint8 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Uint8_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if uint8 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Uint8_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if uint8 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

Coeff_1 is unified with the numerator of the supremum value and Coeff_2 with the denominator of the supremum value and Point with a point or closure point where Lin_Expr reaches this value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Uint8_Box_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if uint8 box P referenced by Handle is not empty and Lin_Expr is bounded from below in P.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value and Point with a point or closure point where Lin_Expr reaches this value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Uint8_Box_external_memory_in_bytes(+Handle, ?Number)
```

Unifies Number with the size of the total memory in bytes occupied by the uint8 box referenced by Handle.

```
ppl_Uint8_Box_total_memory_in_bytes(+Handle, ?Number)
```

Unifies Number with the size of the external memory in bytes occupied by the uint8 box referenced by Handle.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Uint8_Box_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the uint8 box referenced by Handle on the standard output.

Space-dimension preserving predicates that may change the uint8 box

These predicates may modify the uint8 box referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the uint8 box by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Uint8_Box_add_constraint(+Handle, +Constraint)
```

Updates the uint8 box referenced by Handle to one obtained by adding Constraint to its constraint system. For a C polyhedron, Constraint must be an equality or a non-strict inequality.

```
ppl_Uint8_Box_add_congruence(+Handle, +Congruence)
```

Updates the uint8 box referenced by Handle to one obtained by adding Congruence to its congruence system. For a C polyhedron, Congruence must be an equality.

```
ppl_Uint8_Box_add_constraints( +Handle, +Constraint_System)
```

Updates the uint8 box referenced by Handle to one obtained by adding to its constraint system the constraints in Constraint_System. For a C polyhedron, Constraints must be a list of equalities and

non-strict inequalities.

`ppl_Uint8_Box_add_congruences( +Handle, +Congruence_System)`

Updates the uint8 box referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Uint8_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the uint8 box referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Uint8_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the uint8 box referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Uint8_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the uint8 box referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Uint8_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the uint8 box referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the uint8 box

These predicates enable transformations such as taking the topological closure (which for the domain of uint8 box is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Uint8_Box_topological_closure_assign(+Handle)`

Assigns to the uint8 box referenced by `Handle` *its topological closure.*

`ppl_Uint8_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the uint8 box `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

`ppl_Uint8_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the uint8 box `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Uint8_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the uint8 box referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

`ppl_Uint8_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the uint8 box referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

`ppl_Uint8_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to uint8 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Uint8_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to uint8 box `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

`ppl_Uint8_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to uint8 box `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Uint8_Box_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to uint8 box `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Uint8_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to uint8 box `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Uint8_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to uint8 box `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Uint8_Box_wrap_assign(+Handle, +List_of_PPL_Var, +Width, +Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)`

Transforms the uint8 box `P` referenced by `Handle` by wrapping the dimensions given by `List_of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one uint8 box

These predicates include the binary operators which will assign to the uint8 box referred to by the first argument its combination with the uint8 box referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Uint8_Box_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the uint8 box `P` referenced by `Handle_1` the intersection of `P` and the uint8 box referenced by `Handle_2`.

`ppl_Uint8_Box_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the uint8 box `P` referenced by `Handle_1` the upper bound of `P` and the uint8 box referenced by `Handle_2`.

`ppl_Uint8_Box_difference_assign(+Handle_1, +Handle_2)`

Assigns to the uint8 box `P` referenced by `Handle_1` the difference of `P` and the uint8 box referenced by `Handle_2`.

`ppl_Uint8_Box_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the uint8 box `P` referenced by `Handle_1` the time elapse of `P` and the uint8 box referenced by `Handle_2`.

`ppl_Uint8_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the uint8 box `P_1` referenced by `Handle_1` with the uint8 box referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Uint8_Box_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of uint8 box `P_1` referenced by `Handle_1` and the uint8 box `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

`ppl_Uint8_Box_linear_partition(+Handle_1, +Handle_2, -Handle_3, -Handle_4)`

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Uint8_Box domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the uint8 box referred to by the second argument has to be contained in (or equal to) the uint8 box referred to by the first argument.

`ppl_Uint8_Box_CC76_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the uint8 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the uint8 box referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_Uint8_Box_CC76_widening_assign(+Handle_1, +Handle_2)`

Assigns to the uint8 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the uint8 box referenced by `Handle_2`.

`ppl_Uint8_Box_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Same as predicate `ppl_Uint8_Box_CC76_widening_assign_with_tokens/4`

`ppl_Uint8_Box_widening_assign(+Handle_1, +Handle_2)`

Same as predicate `ppl_Uint8_Box_CC76_widening_assign/2`

`ppl_Uint8_Box_limited_CC76_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the uint8 box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the uint8 box referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*. The widening with tokens delay technique is applied with* `C_unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Uint8_Box_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2,
+Constraint_System)
```

Assigns to the uint8 box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the uint8 box referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*.*

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the uint8 box referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Uint8_Box_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the uint8 box `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the uint8 box referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the uint8 box referred to in the first argument. These predicates enable the modification of the vector space of the uint8 box referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Uint8_Box_add_space_dimensions_and_embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the uint8 box* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Uint8_Box_add_space_dimensions_and_project(+Handle, +Dimension_-
Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the uint8 box* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

```
ppl_Uint8_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the uint8 box `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Uint8_Box_remove_higher_space_dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the uint8 box `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`*.*

`ppl_Uint8_Box_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the uint8 box referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Uint8_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the uint8 box referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Uint8_Box_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Uint16_Box Domain

Here we provide a short description for each of the predicates available for the domain of uint16 boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for uint16 boxes

The constructor predicates build a uint16 box from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Uint16_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new uint16 box $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Uint16_Box_from_space_dimension(3, universe, X).
```

creates the uint16 box defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Uint16_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new uint16 box `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Uint16_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new uint16 box `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Uint16_Box_from_generators(+Generator_System, -Handle)`

Builds a new uint16 box `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new uint16 boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Uint16_Box_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_-2)`

Builds a new uint16 box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_-2)`

Builds a new uint16 box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Grid(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new uint16 box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Uint8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Uint16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Uint32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Uint64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Z_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Rational_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint16_Box_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint16 box `P_1` *from the bd_shape_long_double referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

```
ppl_new_Uint16_Box_from_Octagonal_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new uint16 box `P_1` *from the octagonal_shape_long_double referenced by handle* `Handle_-1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

Destructor predicate

Below is the destructor predicate for the Uint16_Box domain.

```
ppl_delete_Uint16_Box(+Handle)
```

Invalidates the handle referenced by `Handle`*: this makes sure the corresponding resources will eventually be released.*

Predicates that do not change the uint16 box

Test Predicates

These predicates test the uint16 box for different properties and succeed or fail depending on the outcome.

```
ppl_Uint16_Box_is_empty(+Handle)
```

Succeeds if and only if the uint16 box referenced by `Handle` *is empty.*

```
ppl_Uint16_Box_is_universe(+Handle)
```

Succeeds if and only if the uint16 box referenced by `Handle` *is the universe.*

```
ppl_Uint16_Box_is_bounded(+Handle)
```

Succeeds if and only if the uint16 box referenced by `Handle` *is bounded.*

```
ppl_Uint16_Box_contains_integer_point(+Handle)
```

Succeeds if and only if the uint16 box referenced by `Handle` *contains an integer point.*

```
ppl_Uint16_Box_is_topologically_closed(+Handle)
```

Succeeds if and only if the uint16 box referenced by `Handle` *is topologically closed.*

```
ppl_Uint16_Box_is_discrete(+Handle)
```

Succeeds if and only if the uint16 box referenced by `Handle` *is discrete.*

```
ppl_Uint16_Box_bounds_from_above(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from above in the uint16 box referenced by* `Handle`*.*

```
ppl_Uint16_Box_bounds_from_below(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from below in the uint16 box referenced by* `Handle`*.*

```
ppl_Uint16_Box_contains_Uint16_Box(+Handle_1, +Handle_2)
```

Succeeds if and only if the uint16 box referenced by `Handle_2` *is included in or equal to the uint16 box referenced by* `Handle_1`*.*

```
ppl_Uint16_Box_strictly_contains_Uint16_Box(+Handle_1, +Handle_2)
```

Succeeds if and only if the uint16 box referenced by `Handle_2` is included in but not equal to the uint16 box referenced by `Handle_1`.

```
ppl_Uint16_Box_is_disjoint_from_Uint16_Box(+Handle_1, +Handle_2)
```

Succeeds if and only if the uint16 box referenced by `Handle_2` is disjoint from the uint16 box referenced by `Handle_1`.

```
ppl_Uint16_Box_equals_Uint16_Box(+Handle_1, +Handle_2)
```

Succeeds if and only if the uint16 box referenced by `Handle_1` is equal to the uint16 box referenced by `Handle_2`.

```
ppl_Uint16_Box_OK(+Handle)
```

Succeeds only if the uint16 box referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

```
ppl_Uint16_Box_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the uint16 box referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the uint16 box

These predicates will obtain more detailed information about the uint16 box unifying some of their arguments with the results.

```
ppl_Uint16_Box_space_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` with the dimension of the vector space enclosing the uint16 box referenced by `Handle`.

```
ppl_Uint16_Box_affine_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` with the affine dimension of the uint16 box referenced by `Handle`.

```
ppl_Uint16_Box_relation_with_constraint(+Handle, +Constraint,
?Relation_List)
```

Unifies `Relation_List` with the list of relations the uint16 box referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

```
ppl_Uint16_Box_relation_with_generator(+Handle, +Generator,
?Relation_List)
```

Unifies `Relation_List` with the list of relations the uint16 box referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

```
ppl_Uint16_Box_relation_with_congruence(+Handle, +Congruence,
?Relation_List)
```

Unifies `Relation_List` with the list of relations the uint16 box referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

```
ppl_Uint16_Box_get_constraints(+Handle, ?Constraint_System)
```

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the uint16 box referenced by `Handle`.

```
ppl_Uint16_Box_get_congruences(+Handle, ?Congruence_System)
```

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the uint16 box referenced by `Handle`.

```
ppl_Uint16_Box_get_minimized_constraints(+Handle, ?Constraint_System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the minimized constraint system satisfied by the uint16 box referenced by* `Handle`.

```
ppl_Uint16_Box_get_minimized_congruences(+Handle, ?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the minimized congruence system satisfied by the uint16 box referenced by* `Handle`.

```
ppl_Uint16_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean)
```

Succeeds if and only if uint16 box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Uint16_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean)
```

Succeeds if and only if uint16 box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Uint16_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if uint16 box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Uint16_Box_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if uint16 box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Uint16_Box_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the uint16 box referenced by* `Handle`.

```
ppl_Uint16_Box_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the uint16 box referenced by* `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Uint16_Box_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the uint16 box referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the uint16 box

These predicates may modify the uint16 box referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the uint16 box by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Uint16_Box_add_constraint(+Handle, +Constraint)`

Updates the uint16 box referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Uint16_Box_add_congruence(+Handle, +Congruence)`

Updates the uint16 box referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Uint16_Box_add_constraints( +Handle, +Constraint_System)`

Updates the uint16 box referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Uint16_Box_add_congruences( +Handle, +Congruence_System)`

Updates the uint16 box referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Uint16_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the uint16 box referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Uint16_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the uint16 box referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Uint16_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the uint16 box referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Uint16_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the uint16 box referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the uint16 box

These predicates enable transformations such as taking the topological closure (which for the domain of uint16 box is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Uint16_Box_topological_closure_assign(+Handle)`

Assigns to the uint16 box referenced by `Handle` *its topological closure.*

`ppl_Uint16_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the uint16 box `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

`ppl_Uint16_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the uint16 box `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Uint16_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the uint16 box referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff to PPL_Var`*.*

`ppl_Uint16_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the uint16 box referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff to PPL_Var`*.*

`ppl_Uint16_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to uint16 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Uint16_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to uint16 box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Uint16_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to uint16 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Uint16_Box_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to uint16 box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Uint16_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to uint16 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Uint16_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to uint16 box `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

`ppl_Uint16_Box_wrap_assign(+Handle, +List_of_PPL_Var, +Width, +Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)`

Transforms the uint16 box `P` referenced by `Handle` by wrapping the dimensions given by `List_of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one uint16 box

These predicates include the binary operators which will assign to the uint16 box referred to by the first argument its combination with the uint16 box referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Uint16_Box_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the uint16 box `P` referenced by `Handle_1` the intersection of `P` and the uint16 box referenced by `Handle_2`.

`ppl_Uint16_Box_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the uint16 box `P` referenced by `Handle_1` the upper bound of `P` and the uint16 box referenced by `Handle_2`.

`ppl_Uint16_Box_difference_assign(+Handle_1, +Handle_2)`

Assigns to the uint16 box `P` referenced by `Handle_1` the difference of `P` and the uint16 box referenced by `Handle_2`.

`ppl_Uint16_Box_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the uint16 box `P` referenced by `Handle_1` the time elapse of `P` and the uint16 box referenced by `Handle_2`.

`ppl_Uint16_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the uint16 box `P_1` referenced by `Handle_1` with the uint16 box referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Uint16_Box_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of uint16 box `P_1` referenced by `Handle_1` and the uint16 box `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

`ppl_Uint16_Box_linear_partition(+Handle_1, +Handle_2, -Handle_3, -Handle_4)`

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Uint16_Box domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to

a reference to a pointset powerset of nnc polyhedra `P_4`*; where* `P_4` *is the linear partition of* `P_1` *with respect to* `P_2`*. This predicate is only provided if the class* `Pointset_Powerset_NNC_Polyhedron` *has been enabled when configuring the library.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the uint16 box referred to by the second argument has to be contained in (or equal to) the uint16 box referred to by the first argument.

```
ppl_Uint16_Box_CC76_widening_assign_with_tokens(+Handle_1, +Handle_2,
+C_unsigned_1, ?C_unsigned_2)
```

Assigns to the uint16 box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the uint16 box referenced by* `Handle_2`*. The widening with tokens delay technique is applied with* `C_unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Uint16_Box_CC76_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the uint16 box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the uint16 box referenced by* `Handle_2`*.*

```
ppl_Uint16_Box_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_-
unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_Uint16_Box_CC76_widening_assign_with_tokens/4`

```
ppl_Uint16_Box_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_Uint16_Box_CC76_widening_assign/2`

```
ppl_Uint16_Box_limited_CC76_extrapolation_assign_with_tokens(+Handle_-
1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the uint16 box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the uint16 box referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*. The widening with tokens delay technique is applied with* `C_unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Uint16_Box_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2,
+Constraint_System)
```

Assigns to the uint16 box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the uint16 box referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*.*

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the uint16 box referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Uint16_Box_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the uint16 box `P` referenced by `Handle_1` the concatenation of `P` and the uint16 box referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the uint16 box referred to in the first argument. These predicates enable the modification of the vector space of the uint16 box referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Uint16_Box_add_space_dimensions_and_embed(+Handle, +Dimension_-Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the uint16 box `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Uint16_Box_add_space_dimensions_and_project(+Handle, +Dimension_-Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the uint16 box `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Uint16_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the uint16 box `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Uint16_Box_remove_higher_space_dimensions(+Handle, +Dimension_-Type)`

Removes the higher dimensions from the vector space enclosing the uint16 box `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Uint16_Box_expand_space_dimension(+Handle, +PPL_Var, +Dimension_-Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the uint16 box referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Uint16_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the uint16 box referenced by `Handle` by folding the space dimensions contained in `List_of_-PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Uint16_Box_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Uint32_Box Domain

Here we provide a short description for each of the predicates available for the domain of uint32 boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for uint32 boxes

The constructor predicates build a uint32 box from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Uint32_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new uint32 box $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Uint32_Box_from_space_dimension(3, universe, X).
```

creates the uint32 box defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Uint32_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new uint32 box `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Uint32_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new uint32 box `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Uint32_Box_from_generators(+Generator_System, -Handle)`

Builds a new uint32 box `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new uint32 boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Uint32_Box_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new uint32 box `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint32_Box_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new uint32 box `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint32_Box_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new uint32 box `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint32_Box_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new uint32 box `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint32_Box_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new uint32 box `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint32_Box_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new uint32 box `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint32_Box_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new uint32 box `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint32_Box_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new uint32 box `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint32_Box_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new uint32 box `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint32_Box_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new uint32 box `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint32_Box_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new uint32 box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint32_Box_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new uint32 box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint32_Box_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new uint32 box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint32_Box_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new uint32 box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint32_Box_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new uint32 box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint32_Box_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new uint32 box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)
```

Builds a new uint32 box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)
```

Builds a new uint32 box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)
```

Builds a new uint32 box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)
```

Builds a new uint32 box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_-
2)
```

Builds a new uint32 box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_-
2)
```

Builds a new uint32 box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_C_Polyhedron(+Handle_1, -Handle_2)
```

Builds a new uint32 box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_NNC_Polyhedron(+Handle_1, -Handle_2)
```

Builds a new uint32 box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Grid(+Handle_1, -Handle_2)
```

Builds a new uint32 box `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Float_Box(+Handle_1, -Handle_2)
```

Builds a new uint32 box `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_BD_Shape_float(+Handle_1, -Handle_2)
```

Builds a new uint32 box `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_float(+Handle_1, -Handle_2)
```

Builds a new uint32 box `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Double_Box(+Handle_1, -Handle_2)
```

Builds a new uint32 box `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_BD_Shape_double(+Handle_1, -Handle_2)
```

Builds a new uint32 box `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_double(+Handle_1, -Handle_2)
```

Builds a new uint32 box `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Long_Double_Box(+Handle_1, -Handle_2)
```

Builds a new uint32 box `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_BD_Shape_long_double(+Handle_1, -Handle_2)
```

Builds a new uint32 box `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new uint32 box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Int8_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new uint32 box `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Int16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Int32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Int64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Uint8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Uint16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Uint32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Uint64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Z_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new uint32 box `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Rational_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_BD_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_BD_Shape_int16_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_BD_Shape_int32_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_BD_Shape_int64_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_BD_Shape_mpz_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_BD_Shape_mpq_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_C_Polyhedron_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_NNC_Polyhedron_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Grid_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new uint32 box `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Float_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_BD_Shape_float_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_float_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Double_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_BD_Shape_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Long_Double_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_BD_Shape_long_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint32_Box_from_Octagonal_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new uint32 box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Uint32_Box domain.

```
ppl_delete_Uint32_Box(+Handle)
```

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the uint32 box

Test Predicates

These predicates test the uint32 box for different properties and succeed or fail depending on the outcome.

`ppl_Uint32_Box_is_empty(+Handle)`

Succeeds if and only if the uint32 box referenced by `Handle` is empty.

`ppl_Uint32_Box_is_universe(+Handle)`

Succeeds if and only if the uint32 box referenced by `Handle` is the universe.

`ppl_Uint32_Box_is_bounded(+Handle)`

Succeeds if and only if the uint32 box referenced by `Handle` is bounded.

`ppl_Uint32_Box_contains_integer_point(+Handle)`

Succeeds if and only if the uint32 box referenced by `Handle` contains an integer point.

`ppl_Uint32_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the uint32 box referenced by `Handle` is topologically closed.

`ppl_Uint32_Box_is_discrete(+Handle)`

Succeeds if and only if the uint32 box referenced by `Handle` is discrete.

`ppl_Uint32_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the uint32 box referenced by `Handle`.

`ppl_Uint32_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the uint32 box referenced by `Handle`.

`ppl_Uint32_Box_contains_Uint32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the uint32 box referenced by `Handle_2` is included in or equal to the uint32 box referenced by `Handle_1`.

`ppl_Uint32_Box_strictly_contains_Uint32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the uint32 box referenced by `Handle_2` is included in but not equal to the uint32 box referenced by `Handle_1`.

`ppl_Uint32_Box_is_disjoint_from_Uint32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the uint32 box referenced by `Handle_2` is disjoint from the uint32 box referenced by `Handle_1`.

`ppl_Uint32_Box_equals_Uint32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the uint32 box referenced by `Handle_1` is equal to the uint32 box referenced by `Handle_2`.

`ppl_Uint32_Box_OK(+Handle)`

Succeeds only if the uint32 box referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Uint32_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the uint32 box referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the uint32 box

These predicates will obtain more detailed information about the uint32 box unifying some of their arguments with the results.

`ppl_Uint32_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the uint32 box referenced by `Handle`.

`ppl_Uint32_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the uint32 box referenced by `Handle`.

`ppl_Uint32_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the uint32 box referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Uint32_Box_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the uint32 box referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Uint32_Box_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the uint32 box referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Uint32_Box_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the uint32 box referenced by `Handle`.

`ppl_Uint32_Box_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the uint32 box referenced by `Handle`.

`ppl_Uint32_Box_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the uint32 box referenced by `Handle`.

`ppl_Uint32_Box_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the uint32 box referenced by `Handle`.

`ppl_Uint32_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if uint32 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Uint32_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if uint32 box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Uint32_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if uint32 box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Uint32_Box_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if uint32 box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Uint32_Box_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the uint32 box referenced by* `Handle`.

```
ppl_Uint32_Box_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the uint32 box referenced by* `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Uint32_Box_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the uint32 box referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the uint32 box

These predicates may modify the uint32 box referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the uint32 box by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Uint32_Box_add_constraint(+Handle, +Constraint)`

Updates the uint32 box referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_Uint32_Box_add_congruence(+Handle, +Congruence)`

Updates the uint32 box referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Uint32_Box_add_constraints( +Handle, +Constraint_System)`

Updates the uint32 box referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Uint32_Box_add_congruences( +Handle, +Congruence_System)`

Updates the uint32 box referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Uint32_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the uint32 box referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Uint32_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the uint32 box referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Uint32_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the uint32 box referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Uint32_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the uint32 box referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the uint32 box

These predicates enable transformations such as taking the topological closure (which for the domain of uint32 box is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Uint32_Box_topological_closure_assign(+Handle)`

Assigns to the uint32 box referenced by `Handle` its topological closure.

`ppl_Uint32_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the uint32 box `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Uint32_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the uint32 box `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Uint32_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the uint32 box referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff to PPL_Var`.

```
ppl_Uint32_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the uint32 box referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff to PPL_Var`.

```
ppl_Uint32_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1,
+Lin_Expr_2, +Coeff)
```

Assigns to uint32 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Uint32_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1,
+Lin_Expr_2, +Coeff)
```

Assigns to uint32 box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Uint32_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_
Symbol, +Lin_Expr, +Coeff)
```

Assigns to uint32 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Uint32_Box_generalized_affine_preimage(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to uint32 box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Uint32_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1,
+Relation_Symbol, +Lin_Expr_2)
```

Assigns to uint32 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Uint32_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_
Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to uint32 box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Uint32_Box_wrap_assign(+Handle, +List_of_PPL_Var, +Width,
+Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)
```

Transforms the uint32 box `P` *referenced by* `Handle` *by wrapping the dimensions given by* `List_of_PPL_Vars` *while respecting the specified* `Width`, `Representation` *and* `Overflow` *behavior of all these variables. The* `Constraint_System` *represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer* `C_unsigned` *and* `Boolean` *allow control of the complexity/precision ratio; higher values for* `C_unsigned` *will lead to possibly greater precision while a true value for* `Boolean` *indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.*

Predicates whose results depend on more than one uint32 box

These predicates include the binary operators which will assign to the uint32 box referred to by the first argument its combination with the uint32 box referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Uint32_Box_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the uint32 box `P` referenced by `Handle_1` the intersection of `P` and the uint32 box referenced by `Handle_2`.

`ppl_Uint32_Box_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the uint32 box `P` referenced by `Handle_1` the upper bound of `P` and the uint32 box referenced by `Handle_2`.

`ppl_Uint32_Box_difference_assign(+Handle_1, +Handle_2)`

Assigns to the uint32 box `P` referenced by `Handle_1` the difference of `P` and the uint32 box referenced by `Handle_2`.

`ppl_Uint32_Box_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the uint32 box `P` referenced by `Handle_1` the time elapse of `P` and the uint32 box referenced by `Handle_2`.

`ppl_Uint32_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the uint32 box `P_1` referenced by `Handle_1` with the uint32 box referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Uint32_Box_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of uint32 box `P_1` referenced by `Handle_1` and the uint32 box `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

`ppl_Uint32_Box_linear_partition(+Handle_1, +Handle_2, -Handle_3, -Handle_4)`

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Uint32_Box domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the uint32 box referred to by the second argument has to be contained in (or equal to) the uint32 box referred to by the first argument.

`ppl_Uint32_Box_CC76_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the uint32 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the uint32 box referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1`

tokens; `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

`ppl_Uint32_Box_CC76_widening_assign(+Handle_1, +Handle_2)`

Assigns to the uint32 box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the uint32 box referenced by* `Handle_2`*.*

`ppl_Uint32_Box_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Same as predicate `ppl_Uint32_Box_CC76_widening_assign_with_tokens/4`

`ppl_Uint32_Box_widening_assign(+Handle_1, +Handle_2)`

Same as predicate `ppl_Uint32_Box_CC76_widening_assign/2`

`ppl_Uint32_Box_limited_CC76_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the uint32 box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the uint32 box referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*. The widening with tokens delay technique is applied with* `C_unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

`ppl_Uint32_Box_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the uint32 box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the uint32 box referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*.*

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the uint32 box referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Uint32_Box_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the uint32 box `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the uint32 box referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the uint32 box referred to in the first argument. These predicates enable the modification of the vector space of the uint32 box referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Uint32_Box_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the uint32 box `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Uint32_Box_add_space_dimensions_and_project(+Handle, +Dimension_-
Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the uint32 box `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Uint32_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the uint32 box `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Uint32_Box_remove_higher_space_dimensions(+Handle, +Dimension_-
Type)
```

Removes the higher dimensions from the vector space enclosing the uint32 box `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Uint32_Box_expand_space_dimension(+Handle, +PPL_Var, +Dimension_-
Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the uint32 box referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Uint32_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars,
+PPL_Var)
```

Modifies the uint32 box referenced by `Handle` by folding the space dimensions contained in `List_of_-PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Uint32_Box_map_space_dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Uint64_Box Domain

Here we provide a short description for each of the predicates available for the domain of uint64 boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for uint64 boxes

The constructor predicates build a uint64 box from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Uint64_Box_from_space_dimension(+Dimension_Type, +Universe_-
or_Empty, -Handle)
```

Builds a new uint64 box $\mathcal{P}$ with `Dimension_Type` *dimensions; it is empty or the universe depending on whether* `Atom` *is* `empty` *or* `universe`*, respectively.* `Handle` *is unified with the handle for $\mathcal{P}$. Thus the query*

```
?- ppl_new_Uint64_Box_from_space_dimension(3, universe, X).
```

creates the uint64 box defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` *bound to a valid handle for accessing it.*

`ppl_new_Uint64_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new uint64 box `P` *from* `Constraint_System`*.* `Handle` *is unified with the handle for* `P`*.*

`ppl_new_Uint64_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new uint64 box `P` *from* `Congruence_System`*.* `Handle` *is unified with the handle for* `P`*.*

`ppl_new_Uint64_Box_from_generators(+Generator_System, -Handle)`

Builds a new uint64 box `P` *from* `Generator_System`*.* `Handle` *is unified with the handle for* `P`*.*

Predicates that build new uint64 boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Uint64_Box_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` *from the int8_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Uint64_Box_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` *from the int16_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Uint64_Box_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` *from the int32_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Uint64_Box_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` *from the int64_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Uint64_Box_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` *from the uint8_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Uint64_Box_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Grid(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new uint64 box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Uint8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Uint16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Uint32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Uint64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Z_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Rational_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_BD_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_BD_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_BD_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_BD_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_BD_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for

P_1.

`ppl_new_Uint64_Box_from_Octagonal_Shape_int64_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Octagonal_Shape_mpz_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Octagonal_Shape_mpq_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Uint64_Box_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new uint64 box `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint64_Box_from_BD_Shape_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint64 box `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint64_Box_from_Octagonal_Shape_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new uint64 box `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint64_Box_from_Long_Double_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint64 box `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint64_Box_from_BD_Shape_long_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new uint64 box `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Uint64_Box_from_Octagonal_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new uint64 box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Uint64_Box domain.

```
ppl_delete_Uint64_Box(+Handle)
```

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the uint64 box

Test Predicates

These predicates test the uint64 box for different properties and succeed or fail depending on the outcome.

```
ppl_Uint64_Box_is_empty(+Handle)
```

Succeeds if and only if the uint64 box referenced by `Handle` is empty.

```
ppl_Uint64_Box_is_universe(+Handle)
```

Succeeds if and only if the uint64 box referenced by `Handle` is the universe.

```
ppl_Uint64_Box_is_bounded(+Handle)
```

Succeeds if and only if the uint64 box referenced by `Handle` is bounded.

`ppl_Uint64_Box_contains_integer_point(+Handle)`

Succeeds if and only if the uint64 box referenced by `Handle` contains an integer point.

`ppl_Uint64_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the uint64 box referenced by `Handle` is topologically closed.

`ppl_Uint64_Box_is_discrete(+Handle)`

Succeeds if and only if the uint64 box referenced by `Handle` is discrete.

`ppl_Uint64_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the uint64 box referenced by `Handle`.

`ppl_Uint64_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the uint64 box referenced by `Handle`.

`ppl_Uint64_Box_contains_Uint64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the uint64 box referenced by `Handle_2` is included in or equal to the uint64 box referenced by `Handle_1`.

`ppl_Uint64_Box_strictly_contains_Uint64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the uint64 box referenced by `Handle_2` is included in but not equal to the uint64 box referenced by `Handle_1`.

`ppl_Uint64_Box_is_disjoint_from_Uint64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the uint64 box referenced by `Handle_2` is disjoint from the uint64 box referenced by `Handle_1`.

`ppl_Uint64_Box_equals_Uint64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the uint64 box referenced by `Handle_1` is equal to the uint64 box referenced by `Handle_2`.

`ppl_Uint64_Box_OK(+Handle)`

Succeeds only if the uint64 box referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Uint64_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the uint64 box referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the uint64 box

These predicates will obtain more detailed information about the uint64 box unifying some of their arguments with the results.

`ppl_Uint64_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the uint64 box referenced by `Handle`.

`ppl_Uint64_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the uint64 box referenced by `Handle`.

`ppl_Uint64_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` *with the list of relations the uint64 box referenced by* `Handle` *has with* `Constraint`*. The possible relations are listed in the grammar rules above.*

```
ppl_Uint64_Box_relation_with_generator(+Handle, +Generator,
?Relation_List)
```

Unifies `Relation_List` *with the list of relations the uint64 box referenced by* `Handle` *has with* `Generator`*. The possible relations are listed in the grammar rules above.*

```
ppl_Uint64_Box_relation_with_congruence(+Handle, +Congruence,
?Relation_List)
```

Unifies `Relation_List` *with the list of relations the uint64 box referenced by* `Handle` *has with* `Congruence`*. The possible relations are listed in the grammar rules above.*

```
ppl_Uint64_Box_get_constraints(+Handle, ?Constraint_System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the constraint system satisfied by the uint64 box referenced by* `Handle`*.*

```
ppl_Uint64_Box_get_congruences(+Handle, ?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the congruence system satisfied by the uint64 box referenced by* `Handle`*.*

```
ppl_Uint64_Box_get_minimized_constraints(+Handle, ?Constraint_System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the minimized constraint system satisfied by the uint64 box referenced by* `Handle`*.*

```
ppl_Uint64_Box_get_minimized_congruences(+Handle, ?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the minimized congruence system satisfied by the uint64 box referenced by* `Handle`*.*

```
ppl_Uint64_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean)
```

Succeeds if and only if uint64 box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Uint64_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean)
```

Succeeds if and only if uint64 box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Uint64_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if uint64 box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

`ppl_Uint64_Box_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if uint64 box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Uint64_Box_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the total memory in bytes occupied by the uint64 box referenced by `Handle`.

`ppl_Uint64_Box_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the uint64 box referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Uint64_Box_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the uint64 box referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the uint64 box

These predicates may modify the uint64 box referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the uint64 box by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Uint64_Box_add_constraint(+Handle, +Constraint)`

Updates the uint64 box referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_Uint64_Box_add_congruence(+Handle, +Congruence)`

Updates the uint64 box referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Uint64_Box_add_constraints( +Handle, +Constraint_System)`

Updates the uint64 box referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Uint64_Box_add_congruences( +Handle, +Congruence_System)`

Updates the uint64 box referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Uint64_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the uint64 box referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Uint64_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the uint64 box referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Uint64_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the uint64 box referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Uint64_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the uint64 box referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the uint64 box

These predicates enable transformations such as taking the topological closure (which for the domain of uint64 box is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Uint64_Box_topological_closure_assign(+Handle)`

Assigns to the uint64 box referenced by `Handle` *its topological closure.*

`ppl_Uint64_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the uint64 box `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

`ppl_Uint64_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the uint64 box `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Uint64_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the uint64 box referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff to PPL_Var`*.*

`ppl_Uint64_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the uint64 box referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff to PPL_Var`*.*

`ppl_Uint64_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to uint64 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Uint64_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to uint64 box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

```
ppl_Uint64_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_-
Symbol, +Lin_Expr, +Coeff)
```

Assigns to uint64 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, *where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`.

```
ppl_Uint64_Box_generalized_affine_preimage(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to uint64 box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Uint64_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1,
+Relation_Symbol, +Lin_Expr_2)
```

Assigns to uint64 box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, *where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`.

```
ppl_Uint64_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_-
Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to uint64 box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Uint64_Box_wrap_assign(+Handle, +List_of_PPL_Var, +Width,
+Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)
```

Transforms the uint64 box `P` *referenced by* `Handle` *by wrapping the dimensions given by* `List_of_-PPL_Vars` *while respecting the specified* `Width`, `Representation` *and* `Overflow` *behavior of all these variables. The* `Constraint_System` *represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer* `C_unsigned` *and* `Boolean` *allow control of the complexity/precision ratio; higher values for* `C_unsigned` *will lead to possibly greater precision while a true value for* `Boolean` *indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.*

Predicates whose results depend on more than one uint64 box

These predicates include the binary operators which will assign to the uint64 box referred to by the first argument its combination with the uint64 box referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Uint64_Box_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the uint64 box `P` *referenced by* `Handle_1` *the intersection of* `P` *and the uint64 box referenced by* `Handle_2`.

```
ppl_Uint64_Box_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the uint64 box `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the uint64 box referenced by* `Handle_2`.

```
ppl_Uint64_Box_difference_assign(+Handle_1, +Handle_2)
```

Assigns to the uint64 box `P` referenced by `Handle_1` the difference of `P` and the uint64 box referenced by `Handle_2`.

```
ppl_Uint64_Box_time_elapse_assign(+Handle_1, +Handle_2)
```

Assigns to the uint64 box `P` referenced by `Handle_1` the time elapse of `P` and the uint64 box referenced by `Handle_2`.

```
ppl_Uint64_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the uint64 box `P_1` referenced by `Handle_1` with the uint64 box referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_Uint64_Box_simplify_using_context_assign(+Handle_1, +Handle_2,
?Boolean)
```

Succeeds if and only if the intersection of uint64 box `P_1` referenced by `Handle_1` and the uint64 box `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

```
ppl_Uint64_Box_linear_partition(+Handle_1, +Handle_2, -Handle_3,
-Handle_4)
```

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Uint64_Box domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the uint64 box referred to by the second argument has to be contained in (or equal to) the uint64 box referred to by the first argument.

```
ppl_Uint64_Box_CC76_widening_assign_with_tokens(+Handle_1, +Handle_2,
+C_unsigned_1, ?C_unsigned_2)
```

Assigns to the uint64 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the uint64 box referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Uint64_Box_CC76_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the uint64 box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the uint64 box referenced by `Handle_2`.

```
ppl_Uint64_Box_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_-
unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_Uint64_Box_CC76_widening_assign_with_tokens/4`

```
ppl_Uint64_Box_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_Uint64_Box_CC76_widening_assign/2`

```
ppl_Uint64_Box_limited_CC76_extrapolation_assign_with_tokens(+Handle_-
1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the uint64 box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the uint64 box referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*. The widening with tokens delay technique is applied with* `C_unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Uint64_Box_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2,
+Constraint_System)
```

Assigns to the uint64 box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the uint64 box referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*.*

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the uint64 box referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Uint64_Box_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the uint64 box `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the uint64 box referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the uint64 box referred to in the first argument. These predicates enable the modification of the vector space of the uint64 box referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Uint64_Box_add_space_dimensions_and_embed(+Handle, +Dimension_-
Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the uint64 box* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Uint64_Box_add_space_dimensions_and_project(+Handle, +Dimension_-
Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the uint64 box* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

```
ppl_Uint64_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the uint64 box `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Uint64_Box_remove_higher_space_dimensions(+Handle, +Dimension_-
Type)
```

Removes the higher dimensions from the vector space enclosing the uint64 box `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`*.*

```
ppl_Uint64_Box_expand_space_dimension(+Handle, +PPL_Var, +Dimension_-
Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the uint64 box referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

```
ppl_Uint64_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars,
+PPL_Var)
```

Modifies the uint64 box referenced by `Handle` *by folding the space dimensions contained in* `List_of_-PPL_Vars` *into dimension* `PPL_Var`*. The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_Uint64_Box_map_space_dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for the Z_Box Domain

Here we provide a short description for each of the predicates available for the domain of z boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for z boxes

The constructor predicates build a z box from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Z_Box_from_space_dimension(+Dimension_Type, +Universe_or_-
Empty, -Handle)
```

Builds a new z box $\mathcal{P}$ *with* `Dimension_Type` *dimensions; it is empty or the universe depending on whether* `Atom` *is* `empty` *or* `universe`*, respectively.* `Handle` *is unified with the handle for* $\mathcal{P}$*. Thus the query*

```
?- ppl_new_Z_Box_from_space_dimension(3, universe, X).
```

creates the z box defining the 3-dimensional vector space $\mathbb{R}^3$ *with* `X` *bound to a valid handle for accessing it.*

```
ppl_new_Z_Box_from_constraints(+Constraint_System, -Handle)
```

Builds a new z box `P` *from* `Constraint_System`*.* `Handle` *is unified with the handle for* `P`*.*

```
ppl_new_Z_Box_from_congruences(+Congruence_System, -Handle)
```

Builds a new z box `P` *from* `Congruence_System`*.* `Handle` *is unified with the handle for* `P`*.*

```
ppl_new_Z_Box_from_generators(+Generator_System, -Handle)
```

Builds a new z box `P` *from* `Generator_System`*.* `Handle` *is unified with the handle for* `P`*.*

Predicates that build new z boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Z_Box_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new z box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Grid(+Handle_1, -Handle_2)
```

Builds a new z box `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Float_Box(+Handle_1, -Handle_2)
```

Builds a new z box `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_BD_Shape_float(+Handle_1, -Handle_2)
```

Builds a new z box `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Octagonal_Shape_float(+Handle_1, -Handle_2)
```

Builds a new z box `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Double_Box(+Handle_1, -Handle_2)
```

Builds a new z box `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_BD_Shape_double(+Handle_1, -Handle_2)
```

Builds a new z box `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Octagonal_Shape_double(+Handle_1, -Handle_2)
```

Builds a new z box `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Long_Double_Box(+Handle_1, -Handle_2)
```

Builds a new z box `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_BD_Shape_long_double(+Handle_1, -Handle_2)
```

Builds a new z box `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)
```

Builds a new z box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Int8_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new z box `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Int16_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new z box `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Int32_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new z box `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Int64_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new z box `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Uint8_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new z box `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Uint16_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new z box `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Uint32_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new z box `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Uint64_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new z box `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Z_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new z box `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Rational_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new z box `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_BD_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new z box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_BD_Shape_int16_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new z box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_BD_Shape_int32_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new z box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_BD_Shape_int64_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new z box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_BD_Shape_mpz_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new z box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_BD_Shape_mpq_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new z box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Octagonal_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new z box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Octagonal_Shape_int16_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new z box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Octagonal_Shape_int32_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new z box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Octagonal_Shape_int64_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new z box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Octagonal_Shape_mpz_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new z box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_Octagonal_Shape_mpq_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new z box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Z_Box_from_C_Polyhedron_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new z box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new z box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new z box `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new z box `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new z box `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new z box `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new z box `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new z box `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new z box `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new z box `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new z box `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Z_Box_from_Octagonal_Shape_long_double_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new z box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for

P_1.

Destructor predicate

Below is the destructor predicate for the Z_Box domain.

`ppl_delete_Z_Box(+Handle)`

Invalidates the handle referenced by `Handle`*: this makes sure the corresponding resources will eventually be released.*

Predicates that do not change the z box

Test Predicates

These predicates test the z box for different properties and succeed or fail depending on the outcome.

`ppl_Z_Box_is_empty(+Handle)`

Succeeds if and only if the z box referenced by `Handle` *is empty.*

`ppl_Z_Box_is_universe(+Handle)`

Succeeds if and only if the z box referenced by `Handle` *is the universe.*

`ppl_Z_Box_is_bounded(+Handle)`

Succeeds if and only if the z box referenced by `Handle` *is bounded.*

`ppl_Z_Box_contains_integer_point(+Handle)`

Succeeds if and only if the z box referenced by `Handle` *contains an integer point.*

`ppl_Z_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the z box referenced by `Handle` *is topologically closed.*

`ppl_Z_Box_is_discrete(+Handle)`

Succeeds if and only if the z box referenced by `Handle` *is discrete.*

`ppl_Z_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from above in the z box referenced by* `Handle`*.*

`ppl_Z_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from below in the z box referenced by* `Handle`*.*

`ppl_Z_Box_contains_Z_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the z box referenced by `Handle_2` *is included in or equal to the z box referenced by* `Handle_1`*.*

`ppl_Z_Box_strictly_contains_Z_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the z box referenced by `Handle_2` *is included in but not equal to the z box referenced by* `Handle_1`*.*

`ppl_Z_Box_is_disjoint_from_Z_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the z box referenced by `Handle_2` *is disjoint from the z box referenced by* `Handle_1`*.*

`ppl_Z_Box_equals_Z_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the z box referenced by `Handle_1` *is equal to the z box referenced by* `Handle_2`.

```
ppl_Z_Box_OK(+Handle)
```

Succeeds only if the z box referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_Z_Box_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the z box referenced by `Handle` *constrains the dimension* `PPL_Var`.

Predicates that return information about the z box

These predicates will obtain more detailed information about the z box unifying some of their arguments with the results.

```
ppl_Z_Box_space_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the z box referenced by* `Handle`.

```
ppl_Z_Box_affine_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` *with the affine dimension of the z box referenced by* `Handle`.

```
ppl_Z_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_-
List)
```

Unifies `Relation_List` *with the list of relations the z box referenced by* `Handle` *has with* `Constraint`. *The possible relations are listed in the grammar rules above.*

```
ppl_Z_Box_relation_with_generator(+Handle, +Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the z box referenced by* `Handle` *has with* `Generator`. *The possible relations are listed in the grammar rules above.*

```
ppl_Z_Box_relation_with_congruence(+Handle, +Congruence, ?Relation_-
List)
```

Unifies `Relation_List` *with the list of relations the z box referenced by* `Handle` *has with* `Congruence`. *The possible relations are listed in the grammar rules above.*

```
ppl_Z_Box_get_constraints(+Handle, ?Constraint_System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the constraint system satisfied by the z box referenced by* `Handle`.

```
ppl_Z_Box_get_congruences(+Handle, ?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the congruence system satisfied by the z box referenced by* `Handle`.

```
ppl_Z_Box_get_minimized_constraints(+Handle, ?Constraint_System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the minimized constraint system satisfied by the z box referenced by* `Handle`.

```
ppl_Z_Box_get_minimized_congruences(+Handle, ?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the minimized congruence system satisfied by the z box referenced by* `Handle`.

```
ppl_Z_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if z box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Z_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if z box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Z_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean, ?Point)
```

Succeeds if and only if z box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Z_Box_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean, ?Point)
```

Succeeds if and only if z box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Z_Box_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the z box referenced by* `Handle`.

```
ppl_Z_Box_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the z box referenced by* `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Z_Box_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the z box referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the z box

These predicates may modify the z box referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the z box by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Z_Box_add_constraint(+Handle, +Constraint)`

Updates the z box referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_Z_Box_add_congruence(+Handle, +Congruence)`

Updates the z box referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Z_Box_add_constraints( +Handle, +Constraint_System)`

Updates the z box referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Z_Box_add_congruences( +Handle, +Congruence_System)`

Updates the z box referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Z_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the z box referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Z_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the z box referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Z_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the z box referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Z_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the z box referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the z box

These predicates enable transformations such as taking the topological closure (which for the domain of z box is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Z_Box_topological_closure_assign(+Handle)`

Assigns to the z box referenced by `Handle` its topological closure.

`ppl_Z_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the z box `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Z_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the z box `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Z_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the z box referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_Z_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the z box referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_Z_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_-
Expr_2, +Coeff)
```

Assigns to z box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_Z_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1,
+Lin_Expr_2, +Coeff)
```

Assigns to z box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_Z_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_-
Symbol, +Lin_Expr, +Coeff)
```

Assigns to z box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`*.*

```
ppl_Z_Box_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_-
Symbol, +Lin_Expr, +Coeff)
```

Assigns to z box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`*.*

```
ppl_Z_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1,
+Relation_Symbol, +Lin_Expr_2)
```

Assigns to z box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`*.*

```
ppl_Z_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1,
+Relation_Symbol, +Lin_Expr_2)
```

Assigns to z box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`*.*

```
ppl_Z_Box_wrap_assign(+Handle, +List_of_PPL_Var, +Width,
+Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)
```

Transforms the z box `P` *referenced by* `Handle` *by wrapping the dimensions given by* `List_of_PPL_-Vars` *while respecting the specified* `Width`*,* `Representation` *and* `Overflow` *behavior of all these variables. The* `Constraint_System` *represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer* `C_unsigned` *and* `Boolean` *allow control of the complexity/precision ratio; higher values for* `C_unsigned` *will lead to possibly greater precision while a true value for* `Boolean` *indicates that the space dimensions should be wrapped individually. See*

Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one z box

These predicates include the binary operators which will assign to the z box referred to by the first argument its combination with the z box referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Z_Box_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the z box `P` referenced by `Handle_1` the intersection of `P` and the z box referenced by `Handle_2`.

`ppl_Z_Box_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the z box `P` referenced by `Handle_1` the upper bound of `P` and the z box referenced by `Handle_2`.

`ppl_Z_Box_difference_assign(+Handle_1, +Handle_2)`

Assigns to the z box `P` referenced by `Handle_1` the difference of `P` and the z box referenced by `Handle_2`.

`ppl_Z_Box_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the z box `P` referenced by `Handle_1` the time elapse of `P` and the z box referenced by `Handle_2`.

`ppl_Z_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the z box `P_1` referenced by `Handle_1` with the z box referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Z_Box_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of z box `P_1` referenced by `Handle_1` and the z box `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

`ppl_Z_Box_linear_partition(+Handle_1, +Handle_2, -Handle_3, -Handle_4)`

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Z_Box domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the z box referred to by the second argument has to be contained in (or equal to) the z box referred to by the first argument.

`ppl_Z_Box_CC76_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the z box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the z box referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_Z_Box_CC76_widening_assign(+Handle_1, +Handle_2)`

Assigns to the z box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the z box referenced by `Handle_2`.

`ppl_Z_Box_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Same as predicate `ppl_Z_Box_CC76_widening_assign_with_tokens/4`

`ppl_Z_Box_widening_assign(+Handle_1, +Handle_2)`

Same as predicate `ppl_Z_Box_CC76_widening_assign/2`

`ppl_Z_Box_limited_CC76_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the z box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the z box referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_Z_Box_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the z box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the z box referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the z box referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Z_Box_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the z box `P` referenced by `Handle_1` the concatenation of `P` and the z box referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the z box referred to in the first argument. These predicates enable the modification of the vector space of the z box referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Z_Box_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the z box `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Z_Box_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` *new dimensions to the space enclosing the* z *box* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

`ppl_Z_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the z *box* `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Z_Box_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the z *box* `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`*.*

`ppl_Z_Box_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` *dimension of the vector space enclosing the* z *box referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

`ppl_Z_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the z *box referenced by* `Handle` *by folding the space dimensions contained in* `List_of_PPL_-Vars` *into dimension* `PPL_Var`*. The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

`ppl_Z_Box_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for the Rational_Box Domain

Here we provide a short description for each of the predicates available for the domain of rational boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for rational boxes

The constructor predicates build a rational box from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Rational_Box_from_space_dimension(+Dimension_Type, +Universe_-or_Empty, -Handle)`

Builds a new rational box $\mathcal{P}$ *with* `Dimension_Type` *dimensions; it is empty or the universe depending on whether* `Atom` *is* `empty` *or* `universe`*, respectively.* `Handle` *is unified with the handle for* $\mathcal{P}$*. Thus the query*

```
?- ppl_new_Rational_Box_from_space_dimension(3, universe, X).
```

creates the rational box defining the 3-dimensional vector space $\mathbb{R}^3$ *with* `X` *bound to a valid handle for accessing it.*

`ppl_new_Rational_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new rational box `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Rational_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new rational box `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Rational_Box_from_generators(+Generator_System, -Handle)`

Builds a new rational box `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new rational boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Rational_Box_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Grid(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new rational box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Uint8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Uint16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Uint32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Uint64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Z_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Rational_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Rational_Box_from_BD_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new rational box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Rational_Box_from_BD_Shape_int16_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new rational box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Rational_Box_from_BD_Shape_int32_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new rational box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Rational_Box_from_BD_Shape_int64_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new rational box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Rational_Box_from_BD_Shape_mpz_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new rational box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Rational_Box_from_BD_Shape_mpq_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new rational box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Rational_Box_from_Octagonal_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new rational box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Rational_Box_from_Octagonal_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new rational box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Rational_Box_from_Octagonal_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new rational box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Octagonal_Shape_int64_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Octagonal_Shape_mpz_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Octagonal_Shape_mpq_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Octagonal_Shape_float_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Rational_Box_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new rational box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Rational_Box domain.

`ppl_delete_Rational_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the rational box

Test Predicates

These predicates test the rational box for different properties and succeed or fail depending on the outcome.

`ppl_Rational_Box_is_empty(+Handle)`

Succeeds if and only if the rational box referenced by `Handle` is empty.

`ppl_Rational_Box_is_universe(+Handle)`

Succeeds if and only if the rational box referenced by `Handle` is the universe.

`ppl_Rational_Box_is_bounded(+Handle)`

Succeeds if and only if the rational box referenced by `Handle` is bounded.

`ppl_Rational_Box_contains_integer_point(+Handle)`

Succeeds if and only if the rational box referenced by `Handle` contains an integer point.

`ppl_Rational_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the rational box referenced by `Handle` is topologically closed.

`ppl_Rational_Box_is_discrete(+Handle)`

Succeeds if and only if the rational box referenced by `Handle` is discrete.

`ppl_Rational_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the rational box referenced by `Handle`.

`ppl_Rational_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the rational box referenced by `Handle`.

`ppl_Rational_Box_contains_Rational_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the rational box referenced by `Handle_2` is included in or equal to the rational box referenced by `Handle_1`.

`ppl_Rational_Box_strictly_contains_Rational_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the rational box referenced by `Handle_2` is included in but not equal to the rational box referenced by `Handle_1`.

`ppl_Rational_Box_is_disjoint_from_Rational_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the rational box referenced by `Handle_2` is disjoint from the rational box referenced by `Handle_1`.

`ppl_Rational_Box_equals_Rational_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the rational box referenced by `Handle_1` is equal to the rational box referenced by `Handle_2`.

`ppl_Rational_Box_OK(+Handle)`

Succeeds only if the rational box referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Rational_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the rational box referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the rational box

These predicates will obtain more detailed information about the rational box unifying some of their arguments with the results.

`ppl_Rational_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the rational box referenced by `Handle`.

`ppl_Rational_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the rational box referenced by `Handle`.

`ppl_Rational_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` *with the list of relations the rational box referenced by* `Handle` *has with* `Constraint`*. The possible relations are listed in the grammar rules above.*

```
ppl_Rational_Box_relation_with_generator(+Handle, +Generator,
?Relation_List)
```

Unifies `Relation_List` *with the list of relations the rational box referenced by* `Handle` *has with* `Generator`*. The possible relations are listed in the grammar rules above.*

```
ppl_Rational_Box_relation_with_congruence(+Handle, +Congruence,
?Relation_List)
```

Unifies `Relation_List` *with the list of relations the rational box referenced by* `Handle` *has with* `Congruence`*. The possible relations are listed in the grammar rules above.*

```
ppl_Rational_Box_get_constraints(+Handle, ?Constraint_System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the constraint system satisfied by the rational box referenced by* `Handle`*.*

```
ppl_Rational_Box_get_congruences(+Handle, ?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the congruence system satisfied by the rational box referenced by* `Handle`*.*

```
ppl_Rational_Box_get_minimized_constraints(+Handle, ?Constraint_-
System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the minimized constraint system satisfied by the rational box referenced by* `Handle`*.*

```
ppl_Rational_Box_get_minimized_congruences(+Handle, ?Congruence_-
System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the minimized congruence system satisfied by the rational box referenced by* `Handle`*.*

```
ppl_Rational_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean)
```

Succeeds if and only if rational box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Rational_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean)
```

Succeeds if and only if rational box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Rational_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if rational box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the*

supremum is also the maximum, `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Rational_Box_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if rational box `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Rational_Box_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the rational box referenced by* `Handle`*.*

```
ppl_Rational_Box_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the rational box referenced by* `Handle`*.*

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Rational_Box_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the rational box referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the rational box

These predicates may modify the rational box referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the rational box by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Rational_Box_add_constraint(+Handle, +Constraint)
```

Updates the rational box referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

```
ppl_Rational_Box_add_congruence(+Handle, +Congruence)
```

Updates the rational box referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

```
ppl_Rational_Box_add_constraints( +Handle, +Constraint_System)
```

Updates the rational box referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Rational_Box_add_congruences( +Handle, +Congruence_System)`

Updates the rational box referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Rational_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the rational box referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Rational_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the rational box referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Rational_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the rational box referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Rational_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the rational box referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the rational box

These predicates enable transformations such as taking the topological closure (which for the domain of rational box is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Rational_Box_topological_closure_assign(+Handle)`

Assigns to the rational box referenced by `Handle` its topological closure.

`ppl_Rational_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the rational box `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Rational_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the rational box `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Rational_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the rational box referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Rational_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the rational box referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Rational_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to rational box `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $Lin_Expr_1/Coeff \leq PPL_Var \leq Lin_Expr_2/Coeff$.

`ppl_Rational_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to rational box `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

`ppl_Rational_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to rational box `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Rational_Box_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to rational box `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Rational_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to rational box `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Rational_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to rational box `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Rational_Box_wrap_assign(+Handle, +List_of_PPL_Var, +Width, +Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)`

Transforms the rational box `P` referenced by `Handle` by wrapping the dimensions given by `List_of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one rational box

These predicates include the binary operators which will assign to the rational box referred to by the first argument its combination with the rational box referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Rational_Box_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the rational box `P` referenced by `Handle_1` the intersection of `P` and the rational box referenced by `Handle_2`.

`ppl_Rational_Box_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the rational box `P` referenced by `Handle_1` the upper bound of `P` and the rational box referenced by `Handle_2`.

`ppl_Rational_Box_difference_assign(+Handle_1, +Handle_2)`

Assigns to the rational box `P` referenced by `Handle_1` the difference of `P` and the rational box referenced by `Handle_2`.

`ppl_Rational_Box_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the rational box `P` referenced by `Handle_1` the time elapse of `P` and the rational box referenced by `Handle_2`.

`ppl_Rational_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the rational box `P_1` referenced by `Handle_1` with the rational box referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Rational_Box_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of rational box `P_1` referenced by `Handle_1` and the rational box `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

`ppl_Rational_Box_linear_partition(+Handle_1, +Handle_2, -Handle_3, -Handle_4)`

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Rational_Box domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the rational box referred to by the second argument has to be contained in (or equal to) the rational box referred to by the first argument.

`ppl_Rational_Box_CC76_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the rational box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the rational box referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_Rational_Box_CC76_widening_assign(+Handle_1, +Handle_2)`

Assigns to the rational box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the rational box referenced by `Handle_2`.

`ppl_Rational_Box_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Same as predicate `ppl_Rational_Box_CC76_widening_assign_with_tokens/4`

`ppl_Rational_Box_widening_assign(+Handle_1, +Handle_2)`

Same as predicate `ppl_Rational_Box_CC76_widening_assign/2`

`ppl_Rational_Box_limited_CC76_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-`

unsigned_2)

Assigns to the rational box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the rational box referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*. The widening with tokens delay technique is applied with* `C_unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Rational_Box_limited_CC76_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the rational box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the rational box referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*.*

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the rational box referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Rational_Box_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the rational box `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the rational box referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the rational box referred to in the first argument. These predicates enable the modification of the vector space of the rational box referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Rational_Box_add_space_dimensions_and_embed(+Handle, +Dimension_-
Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the rational box* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Rational_Box_add_space_dimensions_and_project(+Handle,
+Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the rational box* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

```
ppl_Rational_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the rational box `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Rational_Box_remove_higher_space_dimensions(+Handle, +Dimension_-
Type)
```

Removes the higher dimensions from the vector space enclosing the rational box `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`*.*

```
ppl_Rational_Box_expand_space_dimension(+Handle, +PPL_Var,
+Dimension_Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the rational box referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

```
ppl_Rational_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars,
+PPL_Var)
```

Modifies the rational box referenced by `Handle` *by folding the space dimensions contained in* `List_of_PPL_Vars` *into dimension* `PPL_Var`*. The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_Rational_Box_map_space_dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for the BD_Shape_int8_t Domain

Here we provide a short description for each of the predicates available for the domain of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for bd shapes

The constructor predicates build a bd shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_BD_Shape_int8_t_from_space_dimension(+Dimension_Type,
+Universe_or_Empty, -Handle)
```

Builds a new bd shape $\mathcal{P}$ *with* `Dimension_Type` *dimensions; it is empty or the universe depending on whether* `Atom` *is* `empty` *or* `universe`*, respectively.* `Handle` *is unified with the handle for* $\mathcal{P}$*. Thus the query*

```
?- ppl_new_BD_Shape_int8_t_from_space_dimension(3, universe, X).
```

creates the bd shape defining the 3-dimensional vector space $\mathbb{R}^3$ *with* `X` *bound to a valid handle for accessing it.*

```
ppl_new_BD_Shape_int8_t_from_constraints(+Constraint_System, -Handle)
```

Builds a new bd shape `P` *from* `Constraint_System`*.* `Handle` *is unified with the handle for* `P`*.*

```
ppl_new_BD_Shape_int8_t_from_congruences(+Congruence_System, -Handle)
```

Builds a new bd shape `P` *from* `Congruence_System`*.* `Handle` *is unified with the handle for* `P`*.*

```
ppl_new_BD_Shape_int8_t_from_generators(+Generator_System, -Handle)
```

Builds a new bd shape `P` *from* `Generator_System`*.* `Handle` *is unified with the handle for* `P`*.*

Predicates that build new bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_BD_Shape_int8_t_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Grid(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_Int16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_Int32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_Int64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_Uint8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_Uint16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_Uint32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_Uint64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_Z_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_Rational_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_BD_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_BD_Shape_int16_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_BD_Shape_int32_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_BD_Shape_int64_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_BD_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_BD_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_mpq_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_float_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_double_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int8_t_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the BD_Shape_int8_t domain.

`ppl_delete_BD_Shape_int8_t(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the bd shape

Test Predicates

These predicates test the bd shape for different properties and succeed or fail depending on the outcome.

`ppl_BD_Shape_int8_t_is_empty(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is empty.

`ppl_BD_Shape_int8_t_is_universe(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is the universe.

`ppl_BD_Shape_int8_t_is_bounded(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is bounded.

`ppl_BD_Shape_int8_t_contains_integer_point(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` contains an integer point.

`ppl_BD_Shape_int8_t_is_topologically_closed(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is topologically closed.

`ppl_BD_Shape_int8_t_is_discrete(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is discrete.

`ppl_BD_Shape_int8_t_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the bd shape referenced by `Handle`.

`ppl_BD_Shape_int8_t_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the bd shape referenced by `Handle`.

```
ppl_BD_Shape_int8_t_contains_BD_Shape_int8_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the bd shape referenced by `Handle_2` is included in or equal to the bd shape referenced by `Handle_1`.

```
ppl_BD_Shape_int8_t_strictly_contains_BD_Shape_int8_t(+Handle_1,
+Handle_2)
```

Succeeds if and only if the bd shape referenced by `Handle_2` is included in but not equal to the bd shape referenced by `Handle_1`.

```
ppl_BD_Shape_int8_t_is_disjoint_from_BD_Shape_int8_t(+Handle_1,
+Handle_2)
```

Succeeds if and only if the bd shape referenced by `Handle_2` is disjoint from the bd shape referenced by `Handle_1`.

```
ppl_BD_Shape_int8_t_equals_BD_Shape_int8_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the bd shape referenced by `Handle_1` is equal to the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_int8_t_OK(+Handle)
```

Succeeds only if the bd shape referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

```
ppl_BD_Shape_int8_t_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the bd shape referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the bd shape

These predicates will obtain more detailed information about the bd shape unifying some of their arguments with the results.

```
ppl_BD_Shape_int8_t_space_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` with the dimension of the vector space enclosing the bd shape referenced by `Handle`.

```
ppl_BD_Shape_int8_t_affine_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` with the affine dimension of the bd shape referenced by `Handle`.

```
ppl_BD_Shape_int8_t_relation_with_constraint(+Handle, +Constraint,
?Relation_List)
```

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

```
ppl_BD_Shape_int8_t_relation_with_generator(+Handle, +Generator,
?Relation_List)
```

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

```
ppl_BD_Shape_int8_t_relation_with_congruence(+Handle, +Congruence,
?Relation_List)
```

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_BD_Shape_int8_t_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_int8_t_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_int8_t_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_int8_t_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_int8_t_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_int8_t_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_int8_t_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_int8_t_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_int8_t_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the total memory in bytes occupied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_int8_t_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the bd shape referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_BD_Shape_int8_t_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the bd shape referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the bd shape

These predicates may modify the bd shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the bd shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_BD_Shape_int8_t_add_constraint(+Handle, +Constraint)`

Updates the bd shape referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_BD_Shape_int8_t_add_congruence(+Handle, +Congruence)`

Updates the bd shape referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_BD_Shape_int8_t_add_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_BD_Shape_int8_t_add_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_BD_Shape_int8_t_refine_with_constraint( +Handle, +Constraint)`

Updates the bd shape referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_BD_Shape_int8_t_refine_with_congruence( +Handle, +Congruence)`

Updates the bd shape referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_BD_Shape_int8_t_refine_with_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_BD_Shape_int8_t_refine_with_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the bd shape

These predicates enable transformations such as taking the topological closure (which for the domain of bd shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_BD_Shape_int8_t_topological_closure_assign(+Handle)`

Assigns to the bd shape referenced by `Handle` its topological closure.

`ppl_BD_Shape_int8_t_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the bd shape `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_BD_Shape_int8_t_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the bd shape `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_BD_Shape_int8_t_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the bd shape referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_BD_Shape_int8_t_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the bd shape referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_BD_Shape_int8_t_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_BD_Shape_int8_t_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_BD_Shape_int8_t_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_`-

Symbol.

`ppl_BD_Shape_int8_t_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` ⋈ `Lin_Expr/Coeff`, where ⋈ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_int8_t_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to bd shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_int8_t_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_int8_t_wrap_assign(+Handle, +List_of_PPL_Var, +Width, +Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)`

Transforms the bd shape `P` referenced by `Handle` by wrapping the dimensions given by `List_of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one bd shape

These predicates include the binary operators which will assign to the bd shape referred to by the first argument its combination with the bd shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_BD_Shape_int8_t_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the intersection of `P` and the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_int8_t_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the upper bound of `P` and the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_int8_t_difference_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the difference of `P` and the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_int8_t_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the time elapse of `P` and the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_int8_t_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the bd shape `P_1` referenced by `Handle_1` with the bd shape referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_BD_Shape_int8_t_simplify_using_context_assign(+Handle_1, +Handle_-
2, ?Boolean)
```

Succeeds if and only if the intersection of bd shape `P_1` referenced by `Handle_1` and the bd shape `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

```
ppl_BD_Shape_int8_t_linear_partition(+Handle_1, +Handle_2, -Handle_3,
-Handle_4)
```

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the BD_Shape_int8_t domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the bd shape referred to by the second argument has to be contained in (or equal to) the bd shape referred to by the first argument.

```
ppl_BD_Shape_int8_t_BHMZ05_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_int8_t_H79_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_int8_t_BHMZ05_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_int8_t_H79_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_int8_t_widening_assign_with_tokens(+Handle_1, +Handle_2,
+C_unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_BD_Shape_int8_t_H79_widening_assign_with_tokens/4`

```
ppl_BD_Shape_int8_t_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_BD_Shape_int8_t_H79_widening_assign/2`

```
ppl_BD_Shape_int8_t_limited_BHMZ05_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
```

```
unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_int8_t_limited_H79_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_int8_t_limited_CC76_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_int8_t_limited_BHMZ05_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_BD_Shape_int8_t_limited_H79_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_BD_Shape_int8_t_limited_CC76_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_BD_Shape_int8_t_CC76_narrowing_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-narrowing of `P_1` and the bd shape referenced by `Handle_2`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_BD_Shape_int8_t_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the concatenation of `P` and the bd shape referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument. These predicates enable the modification of the vector space of the bd shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_BD_Shape_int8_t_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_BD_Shape_int8_t_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_BD_Shape_int8_t_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the bd shape `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_BD_Shape_int8_t_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the bd shape `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_BD_Shape_int8_t_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the bd shape referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_BD_Shape_int8_t_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the bd shape referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_BD_Shape_int8_t_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the BD_Shape_int16_t Domain

Here we provide a short description for each of the predicates available for the domain of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for bd shapes

The constructor predicates build a bd shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_BD_Shape_int16_t_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new bd shape $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_BD_Shape_int16_t_from_space_dimension(3, universe, X).
```

creates the bd shape defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_BD_Shape_int16_t_from_constraints(+Constraint_System, -Handle)`

Builds a new bd shape `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_BD_Shape_int16_t_from_congruences(+Congruence_System, -Handle)`

Builds a new bd shape `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_BD_Shape_int16_t_from_generators(+Generator_System, -Handle)`

Builds a new bd shape `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_BD_Shape_int16_t_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_int8_t(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_int16_t(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_int32_t(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_int64_t(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_mpz_class(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_mpq_class(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_C_Polyhedron(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_NNC_Polyhedron(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Grid(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Float_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_BD_Shape_float(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Double_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_BD_Shape_double(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Long_Double_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_BD_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Int8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Int16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Int32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Int64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Uint8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Uint16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Uint32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Uint64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Z_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_Rational_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_BD_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_BD_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_BD_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_BD_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int16_t_from_BD_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for

P_1.

`ppl_new_BD_Shape_int16_t_from_BD_Shape_mpq_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_int8_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_int16_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_int32_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_int64_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_mpz_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_mpq_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_float_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_double_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_BD_Shape_long_double_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int16_t_from_Octagonal_Shape_long_double_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the BD_Shape_int16_t domain.

`ppl_delete_BD_Shape_int16_t(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the bd shape

Test Predicates

These predicates test the bd shape for different properties and succeed or fail depending on the outcome.

`ppl_BD_Shape_int16_t_is_empty(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is empty.

`ppl_BD_Shape_int16_t_is_universe(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is the universe.

`ppl_BD_Shape_int16_t_is_bounded(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is bounded.

`ppl_BD_Shape_int16_t_contains_integer_point(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` contains an integer point.

`ppl_BD_Shape_int16_t_is_topologically_closed(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is topologically closed.

`ppl_BD_Shape_int16_t_is_discrete(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is discrete.

`ppl_BD_Shape_int16_t_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the bd shape referenced by `Handle`.

`ppl_BD_Shape_int16_t_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the bd shape referenced by `Handle`.

`ppl_BD_Shape_int16_t_contains_BD_Shape_int16_t(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is included in or equal to the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_int16_t_strictly_contains_BD_Shape_int16_t(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is included in but not equal to the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_int16_t_is_disjoint_from_BD_Shape_int16_t(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is disjoint from the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_int16_t_equals_BD_Shape_int16_t(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_1` is equal to the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_int16_t_OK(+Handle)
```

Succeeds only if the bd shape referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

```
ppl_BD_Shape_int16_t_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the bd shape referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the bd shape

These predicates will obtain more detailed information about the bd shape unifying some of their arguments with the results.

```
ppl_BD_Shape_int16_t_space_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` with the dimension of the vector space enclosing the bd shape referenced by `Handle`.

```
ppl_BD_Shape_int16_t_affine_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` with the affine dimension of the bd shape referenced by `Handle`.

```
ppl_BD_Shape_int16_t_relation_with_constraint(+Handle, +Constraint,
?Relation_List)
```

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

```
ppl_BD_Shape_int16_t_relation_with_generator(+Handle, +Generator,
?Relation_List)
```

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

```
ppl_BD_Shape_int16_t_relation_with_congruence(+Handle, +Congruence,
?Relation_List)
```

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

```
ppl_BD_Shape_int16_t_get_constraints(+Handle, ?Constraint_System)
```

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the bd shape referenced by `Handle`.

```
ppl_BD_Shape_int16_t_get_congruences(+Handle, ?Congruence_System)
```

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the bd shape referenced by `Handle`.

```
ppl_BD_Shape_int16_t_get_minimized_constraints(+Handle, ?Constraint_-
System)
```

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the bd shape referenced by `Handle`.

```
ppl_BD_Shape_int16_t_get_minimized_congruences(+Handle, ?Congruence_-
System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the minimized congruence system satisfied by the bd shape referenced by* `Handle`.

```
ppl_BD_Shape_int16_t_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean)
```

Succeeds if and only if bd shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_BD_Shape_int16_t_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean)
```

Succeeds if and only if bd shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_BD_Shape_int16_t_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if bd shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_BD_Shape_int16_t_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if bd shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_BD_Shape_int16_t_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the bd shape referenced by* `Handle`.

```
ppl_BD_Shape_int16_t_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the bd shape referenced by* `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_BD_Shape_int16_t_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the bd shape referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the bd shape

These predicates may modify the bd shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the bd shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_BD_Shape_int16_t_add_constraint(+Handle, +Constraint)`

Updates the bd shape referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_BD_Shape_int16_t_add_congruence(+Handle, +Congruence)`

Updates the bd shape referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_BD_Shape_int16_t_add_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_BD_Shape_int16_t_add_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_BD_Shape_int16_t_refine_with_constraint( +Handle, +Constraint)`

Updates the bd shape referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_BD_Shape_int16_t_refine_with_congruence( +Handle, +Congruence)`

Updates the bd shape referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_BD_Shape_int16_t_refine_with_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_BD_Shape_int16_t_refine_with_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the bd shape

These predicates enable transformations such as taking the topological closure (which for the domain of bd shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_BD_Shape_int16_t_topological_closure_assign(+Handle)`

Assigns to the bd shape referenced by `Handle` its topological closure.

`ppl_BD_Shape_int16_t_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the bd shape `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_BD_Shape_int16_t_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the bd shape `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_BD_Shape_int16_t_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the bd shape referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_BD_Shape_int16_t_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the bd shape referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_BD_Shape_int16_t_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_BD_Shape_int16_t_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_BD_Shape_int16_t_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_int16_t_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_int16_t_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to bd shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_int16_t_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by

Relation_Symbol.

```
ppl_BD_Shape_int16_t_wrap_assign(+Handle, +List_of_PPL_Var, +Width,
+Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)
```

Transforms the bd shape `P` *referenced by* `Handle` *by wrapping the dimensions given by* `List_of_PPL_Vars` *while respecting the specified* `Width`, `Representation` *and* `Overflow` *behavior of all these variables. The* `Constraint_System` *represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer* `C_unsigned` *and* `Boolean` *allow control of the complexity/precision ratio; higher values for* `C_unsigned` *will lead to possibly greater precision while a true value for* `Boolean` *indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.*

Predicates whose results depend on more than one bd shape

These predicates include the binary operators which will assign to the bd shape referred to by the first argument its combination with the bd shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_BD_Shape_int16_t_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` *referenced by* `Handle_1` *the intersection of* `P` *and the bd shape referenced by* `Handle_2`.

```
ppl_BD_Shape_int16_t_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the bd shape referenced by* `Handle_2`.

```
ppl_BD_Shape_int16_t_difference_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` *referenced by* `Handle_1` *the difference of* `P` *and the bd shape referenced by* `Handle_2`.

```
ppl_BD_Shape_int16_t_time_elapse_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the bd shape referenced by* `Handle_2`.

```
ppl_BD_Shape_int16_t_upper_bound_assign_if_exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the bd shape `P_1` *referenced by* `Handle_1` *with the bd shape referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_BD_Shape_int16_t_simplify_using_context_assign(+Handle_1,
+Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of bd shape `P_1` *referenced by* `Handle_1` *and the bd shape* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`.

```
ppl_BD_Shape_int16_t_linear_partition(+Handle_1, +Handle_2, -Handle_3,
-Handle_4)
```

`Handle_1` *and* `Handle_2` *are handles for elements* `P_1` *and* `P_2` *in the BD_Shape_int16_t domain. The predicate unifies handle* `Handle_3` *to a reference to the intersection of* `P_1` *and* `P_2` *and* `Handle_4` *to a reference to a pointset powerset of nnc polyhedra* `P_4`*; where* `P_4` *is the linear partition of* `P_1` *with respect to* `P_2`*. This predicate is only provided if the class* `Pointset_Powerset_NNC_Polyhedron` *has been enabled when configuring the library.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the bd shape referred to by the second argument has to be contained in (or equal to) the bd shape referred to by the first argument.

```
ppl_BD_Shape_int16_t_BHMZ05_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_int16_t_H79_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_int16_t_BHMZ05_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_int16_t_H79_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_int16_t_widening_assign_with_tokens(+Handle_1, +Handle_2,
+C_unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_BD_Shape_int16_t_H79_widening_assign_with_tokens/4`

```
ppl_BD_Shape_int16_t_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_BD_Shape_int16_t_H79_widening_assign/2`

```
ppl_BD_Shape_int16_t_limited_BHMZ05_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_int16_t_limited_H79_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_int16_t_limited_CC76_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_int16_t_limited_BHMZ05_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_BD_Shape_int16_t_limited_H79_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_BD_Shape_int16_t_limited_CC76_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_BD_Shape_int16_t_CC76_narrowing_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-narrowing of `P_1` and the bd shape referenced by `Handle_2`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_BD_Shape_int16_t_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` referenced by `Handle_1` the concatenation of `P` and the bd shape referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument. These predicates enable the modification of the vector space of the bd shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_BD_Shape_int16_t_add_space_dimensions_and_embed(+Handle,
+Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_BD_Shape_int16_t_add_space_dimensions_and_project(+Handle,
+Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_BD_Shape_int16_t_remove_space_dimensions(+Handle, +List_of_PPL_-
Vars)
```

Removes from the vector space enclosing the bd shape `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_BD_Shape_int16_t_remove_higher_space_dimensions(+Handle,
+Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the bd shape `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_BD_Shape_int16_t_expand_space_dimension(+Handle, +PPL_Var,
+Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the bd shape referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_BD_Shape_int16_t_fold_space_dimensions(+Handle, +List_of_PPL_Vars,
+PPL_Var)
```

Modifies the bd shape referenced by `Handle` by folding the space dimensions contained in `List_of_-PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_BD_Shape_int16_t_map_space_dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the BD_Shape_int32_t Domain

Here we provide a short description for each of the predicates available for the domain of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for bd shapes

The constructor predicates build a bd shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_BD_Shape_int32_t_from_space_dimension(+Dimension_Type,
+Universe_or_Empty, -Handle)
```

Builds a new bd shape $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_BD_Shape_int32_t_from_space_dimension(3, universe, X).
```

creates the bd shape defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_BD_Shape_int32_t_from_constraints(+Constraint_System, -Handle)`

Builds a new bd shape `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_BD_Shape_int32_t_from_congruences(+Congruence_System, -Handle)`

Builds a new bd shape `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_BD_Shape_int32_t_from_generators(+Generator_System, -Handle)`

Builds a new bd shape `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_BD_Shape_int32_t_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int32_t_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int32_t_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int32_t_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int32_t_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int32_t_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Uint32_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Uint64_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Z_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Rational_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_BD_Shape_int8_t(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_BD_Shape_int16_t(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_BD_Shape_int32_t(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_BD_Shape_int64_t(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_int8_t(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_int16_t(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_int32_t(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_int64_t(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_mpz_class(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_mpq_class(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_C_Polyhedron(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_NNC_Polyhedron(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Grid(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Float_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_BD_Shape_float(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Double_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_BD_Shape_double(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Long_Double_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_BD_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Int8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Int16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Int32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Int64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Uint8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Uint16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Uint32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Uint64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Z_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Rational_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_BD_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_BD_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_BD_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_BD_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_BD_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_BD_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_C_Polyhedron_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_NNC_Polyhedron_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Grid_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_Float_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int32_t_from_BD_Shape_float_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_float_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int32_t_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int32_t_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_double_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int32_t_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int32_t_from_BD_Shape_long_double_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int32_t_from_Octagonal_Shape_long_double_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the BD_Shape_int32_t domain.

`ppl_delete_BD_Shape_int32_t(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the bd shape

Test Predicates

These predicates test the bd shape for different properties and succeed or fail depending on the outcome.

`ppl_BD_Shape_int32_t_is_empty(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is empty.

`ppl_BD_Shape_int32_t_is_universe(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is the universe.

`ppl_BD_Shape_int32_t_is_bounded(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is bounded.

`ppl_BD_Shape_int32_t_contains_integer_point(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` contains an integer point.

`ppl_BD_Shape_int32_t_is_topologically_closed(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is topologically closed.

`ppl_BD_Shape_int32_t_is_discrete(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is discrete.

`ppl_BD_Shape_int32_t_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the bd shape referenced by `Handle`.

`ppl_BD_Shape_int32_t_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the bd shape referenced by `Handle`.

`ppl_BD_Shape_int32_t_contains_BD_Shape_int32_t(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is included in or equal to the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_int32_t_strictly_contains_BD_Shape_int32_t(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is included in but not equal to the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_int32_t_is_disjoint_from_BD_Shape_int32_t(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is disjoint from the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_int32_t_equals_BD_Shape_int32_t(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_1` is equal to the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_int32_t_OK(+Handle)`

Succeeds only if the bd shape referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_BD_Shape_int32_t_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the bd shape referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the bd shape

These predicates will obtain more detailed information about the bd shape unifying some of their arguments with the results.

`ppl_BD_Shape_int32_t_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the bd shape referenced by `Handle`.

`ppl_BD_Shape_int32_t_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the bd shape referenced by `Handle`.

`ppl_BD_Shape_int32_t_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_BD_Shape_int32_t_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_BD_Shape_int32_t_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_BD_Shape_int32_t_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_int32_t_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_int32_t_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_int32_t_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_int32_t_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_int32_t_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_BD_Shape_int32_t_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if bd shape P referenced by Handle is not empty and Lin_Expr is bounded from above in P.

Coeff_1 is unified with the numerator of the supremum value and Coeff_2 with the denominator of the supremum value and Point with a point or closure point where Lin_Expr reaches this value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_BD_Shape_int32_t_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if bd shape P referenced by Handle is not empty and Lin_Expr is bounded from below in P.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value and Point with a point or closure point where Lin_Expr reaches this value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_BD_Shape_int32_t_external_memory_in_bytes(+Handle, ?Number)
```

Unifies Number with the size of the total memory in bytes occupied by the bd shape referenced by Handle.

```
ppl_BD_Shape_int32_t_total_memory_in_bytes(+Handle, ?Number)
```

Unifies Number with the size of the external memory in bytes occupied by the bd shape referenced by Handle.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_BD_Shape_int32_t_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the bd shape referenced by Handle on the standard output.

Space-dimension preserving predicates that may change the bd shape

These predicates may modify the bd shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the bd shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_BD_Shape_int32_t_add_constraint(+Handle, +Constraint)
```

Updates the bd shape referenced by Handle to one obtained by adding Constraint to its constraint system. For a C polyhedron, Constraint must be an equality or a non-strict inequality.

`ppl_BD_Shape_int32_t_add_congruence(+Handle, +Congruence)`

Updates the bd shape referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_BD_Shape_int32_t_add_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_BD_Shape_int32_t_add_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_BD_Shape_int32_t_refine_with_constraint( +Handle, +Constraint)`

Updates the bd shape referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_BD_Shape_int32_t_refine_with_congruence( +Handle, +Congruence)`

Updates the bd shape referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_BD_Shape_int32_t_refine_with_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_BD_Shape_int32_t_refine_with_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the bd shape

These predicates enable transformations such as taking the topological closure (which for the domain of bd shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_BD_Shape_int32_t_topological_closure_assign(+Handle)`

Assigns to the bd shape referenced by `Handle` its topological closure.

`ppl_BD_Shape_int32_t_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the bd shape `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_BD_Shape_int32_t_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the bd shape `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_BD_Shape_int32_t_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the bd shape referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_BD_Shape_int32_t_affine_preimage(+Handle, +PPL_Var, +Lin_Expr,
+Coeff)
```

Transforms the bd shape referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_BD_Shape_int32_t_bounded_affine_image(+Handle, +PPL_Var, +Lin_-
Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_BD_Shape_int32_t_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_-
Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_BD_Shape_int32_t_generalized_affine_image(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_BD_Shape_int32_t_generalized_affine_preimage(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_BD_Shape_int32_t_generalized_affine_image_lhs_rhs(+Handle, +Lin_-
Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_BD_Shape_int32_t_generalized_affine_preimage_lhs_rhs(+Handle,
+Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_BD_Shape_int32_t_wrap_assign(+Handle, +List_of_PPL_Var, +Width,
+Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)
```

Transforms the bd shape `P` *referenced by* `Handle` *by wrapping the dimensions given by* `List_of_PPL_Vars` *while respecting the specified* `Width`, `Representation` *and* `Overflow` *behavior of all these variables. The* `Constraint_System` *represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer* `C_unsigned` *and* `Boolean` *allow control of the complexity/precision ratio; higher values for* `C_unsigned` *will lead to possibly greater precision while a true value for* `Boolean` *indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.*

Predicates whose results depend on more than one bd shape

These predicates include the binary operators which will assign to the bd shape referred to by the first argument its combination with the bd shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_BD_Shape_int32_t_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the intersection of `P` and the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_int32_t_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the upper bound of `P` and the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_int32_t_difference_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the difference of `P` and the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_int32_t_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the time elapse of `P` and the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_int32_t_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the bd shape `P_1` referenced by `Handle_1` with the bd shape referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_BD_Shape_int32_t_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of bd shape `P_1` referenced by `Handle_1` and the bd shape `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

`ppl_BD_Shape_int32_t_linear_partition(+Handle_1, +Handle_2, -Handle_3, -Handle_4)`

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the BD_Shape_int32_t domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the bd shape referred to by the second argument has to be contained in (or equal to) the bd shape referred to by the first argument.

`ppl_BD_Shape_int32_t_BHMZ05_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1`

tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_int32_t_H79_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_int32_t_BHMZ05_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_int32_t_H79_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_int32_t_widening_assign_with_tokens(+Handle_1, +Handle_2,
+C_unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_BD_Shape_int32_t_H79_widening_assign_with_tokens/4`

```
ppl_BD_Shape_int32_t_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_BD_Shape_int32_t_H79_widening_assign/2`

```
ppl_BD_Shape_int32_t_limited_BHMZ05_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_int32_t_limited_H79_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_int32_t_limited_CC76_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_int32_t_limited_BHMZ05_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

`ppl_BD_Shape_int32_t_limited_H79_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

`ppl_BD_Shape_int32_t_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

`ppl_BD_Shape_int32_t_CC76_narrowing_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-narrowing of `P_1` and the bd shape referenced by `Handle_2`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_BD_Shape_int32_t_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the concatenation of `P` and the bd shape referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument. These predicates enable the modification of the vector space of the bd shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_BD_Shape_int32_t_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_BD_Shape_int32_t_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_BD_Shape_int32_t_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the bd shape `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_BD_Shape_int32_t_remove_higher_space_dimensions(+Handle,
+Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the bd shape `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`*.*

```
ppl_BD_Shape_int32_t_expand_space_dimension(+Handle, +PPL_Var,
+Dimension_Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the bd shape referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

```
ppl_BD_Shape_int32_t_fold_space_dimensions(+Handle, +List_of_PPL_Vars,
+PPL_Var)
```

Modifies the bd shape referenced by `Handle` *by folding the space dimensions contained in* `List_of_PPL_Vars` *into dimension* `PPL_Var`*. The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_BD_Shape_int32_t_map_space_dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for the BD_Shape_int64_t Domain

Here we provide a short description for each of the predicates available for the domain of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for bd shapes

The constructor predicates build a bd shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_BD_Shape_int64_t_from_space_dimension(+Dimension_Type,
+Universe_or_Empty, -Handle)
```

Builds a new bd shape $\mathcal{P}$ *with* `Dimension_Type` *dimensions; it is empty or the universe depending on whether* `Atom` *is* `empty` *or* `universe`*, respectively.* `Handle` *is unified with the handle for* $\mathcal{P}$*. Thus the query*

```
?- ppl_new_BD_Shape_int64_t_from_space_dimension(3, universe, X).
```

creates the bd shape defining the 3-dimensional vector space $\mathbb{R}^3$ *with* `X` *bound to a valid handle for accessing it.*

```
ppl_new_BD_Shape_int64_t_from_constraints(+Constraint_System, -Handle)
```

Builds a new bd shape `P` *from* `Constraint_System`. `Handle` *is unified with the handle for* `P`.

```
ppl_new_BD_Shape_int64_t_from_congruences(+Congruence_System, -Handle)
```

Builds a new bd shape `P` *from* `Congruence_System`. `Handle` *is unified with the handle for* `P`.

```
ppl_new_BD_Shape_int64_t_from_generators(+Generator_System, -Handle)
```

Builds a new bd shape `P` *from* `Generator_System`. `Handle` *is unified with the handle for* `P`.

Predicates that build new bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

```
ppl_new_BD_Shape_int64_t_from_Int8_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` *from the int8_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Int16_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` *from the int16_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Int32_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` *from the int32_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Int64_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` *from the int64_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Uint8_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` *from the uint8_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Uint16_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` *from the uint16_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Uint32_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` *from the uint32_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Uint64_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` *from the uint64_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

`ppl_new_BD_Shape_int64_t_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_mpq_class(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_C_Polyhedron(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_NNC_Polyhedron(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Grid(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Float_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_BD_Shape_float(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Double_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_BD_Shape_double(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Long_Double_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_BD_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Uint8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Uint16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Uint32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Uint64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Z_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Rational_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_BD_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_BD_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_BD_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_BD_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_BD_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_C_Polyhedron_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_NNC_Polyhedron_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Grid_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Float_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_BD_Shape_float_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_float_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Double_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_BD_Shape_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Long_Double_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_BD_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_int64_t_from_Octagonal_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the BD_Shape_int64_t domain.

```
ppl_delete_BD_Shape_int64_t(+Handle)
```

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the bd shape

Test Predicates

These predicates test the bd shape for different properties and succeed or fail depending on the outcome.

```
ppl_BD_Shape_int64_t_is_empty(+Handle)
```

Succeeds if and only if the bd shape referenced by `Handle` is empty.

```
ppl_BD_Shape_int64_t_is_universe(+Handle)
```

Succeeds if and only if the bd shape referenced by `Handle` is the universe.

```
ppl_BD_Shape_int64_t_is_bounded(+Handle)
```

Succeeds if and only if the bd shape referenced by `Handle` is bounded.

```
ppl_BD_Shape_int64_t_contains_integer_point(+Handle)
```

Succeeds if and only if the bd shape referenced by `Handle` contains an integer point.

```
ppl_BD_Shape_int64_t_is_topologically_closed(+Handle)
```

Succeeds if and only if the bd shape referenced by `Handle` is topologically closed.

`ppl_BD_Shape_int64_t_is_discrete(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is discrete.

`ppl_BD_Shape_int64_t_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the bd shape referenced by `Handle`.

`ppl_BD_Shape_int64_t_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the bd shape referenced by `Handle`.

`ppl_BD_Shape_int64_t_contains_BD_Shape_int64_t(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is included in or equal to the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_int64_t_strictly_contains_BD_Shape_int64_t(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is included in but not equal to the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_int64_t_is_disjoint_from_BD_Shape_int64_t(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is disjoint from the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_int64_t_equals_BD_Shape_int64_t(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_1` is equal to the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_int64_t_OK(+Handle)`

Succeeds only if the bd shape referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_BD_Shape_int64_t_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the bd shape referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the bd shape

These predicates will obtain more detailed information about the bd shape unifying some of their arguments with the results.

`ppl_BD_Shape_int64_t_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the bd shape referenced by `Handle`.

`ppl_BD_Shape_int64_t_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the bd shape referenced by `Handle`.

`ppl_BD_Shape_int64_t_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_BD_Shape_int64_t_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_BD_Shape_int64_t_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_BD_Shape_int64_t_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_int64_t_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_int64_t_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_int64_t_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_int64_t_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_int64_t_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_int64_t_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_int64_t_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_int64_t_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the total memory in bytes occupied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_int64_t_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the bd shape referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_BD_Shape_int64_t_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the bd shape referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the bd shape

These predicates may modify the bd shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the bd shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_BD_Shape_int64_t_add_constraint(+Handle, +Constraint)`

Updates the bd shape referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_BD_Shape_int64_t_add_congruence(+Handle, +Congruence)`

Updates the bd shape referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_BD_Shape_int64_t_add_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_BD_Shape_int64_t_add_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_BD_Shape_int64_t_refine_with_constraint( +Handle, +Constraint)`

Updates the bd shape referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_BD_Shape_int64_t_refine_with_congruence( +Handle, +Congruence)`

Updates the bd shape referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_BD_Shape_int64_t_refine_with_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_BD_Shape_int64_t_refine_with_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the bd shape

These predicates enable transformations such as taking the topological closure (which for the domain of bd shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_BD_Shape_int64_t_topological_closure_assign(+Handle)`

Assigns to the bd shape referenced by `Handle` its topological closure.

`ppl_BD_Shape_int64_t_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the bd shape `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_BD_Shape_int64_t_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the bd shape `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_BD_Shape_int64_t_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the bd shape referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_BD_Shape_int64_t_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the bd shape referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_BD_Shape_int64_t_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

`ppl_BD_Shape_int64_t_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

`ppl_BD_Shape_int64_t_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_int64_t_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_int64_t_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to bd shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_int64_t_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_int64_t_wrap_assign(+Handle, +List_of_PPL_Var, +Width, +Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)`

Transforms the bd shape `P` referenced by `Handle` by wrapping the dimensions given by `List_of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one bd shape

These predicates include the binary operators which will assign to the bd shape referred to by the first argument its combination with the bd shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_BD_Shape_int64_t_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the intersection of `P` and the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_int64_t_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the upper bound of `P` and the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_int64_t_difference_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the difference of `P` and the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_int64_t_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the time elapse of `P` and the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_int64_t_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the bd shape `P_1` referenced by `Handle_1` with the bd shape referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_BD_Shape_int64_t_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of bd shape `P_1` referenced by `Handle_1` and the bd shape `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

`ppl_BD_Shape_int64_t_linear_partition(+Handle_1, +Handle_2, -Handle_3, -Handle_4)`

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the BD_Shape_int64_t domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the bd shape referred to by the second argument has to be contained in (or equal to) the bd shape referred to by the first argument.

`ppl_BD_Shape_int64_t_BHMZ05_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_BD_Shape_int64_t_H79_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_BD_Shape_int64_t_BHMZ05_widening_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_int64_t_H79_widening_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_int64_t_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Same as predicate `ppl_BD_Shape_int64_t_H79_widening_assign_with_tokens/4`

`ppl_BD_Shape_int64_t_widening_assign(+Handle_1, +Handle_2)`

Same as predicate `ppl_BD_Shape_int64_t_H79_widening_assign/2`

`ppl_BD_Shape_int64_t_limited_BHMZ05_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_BD_Shape_int64_t_limited_H79_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_BD_Shape_int64_t_limited_CC76_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_BD_Shape_int64_t_limited_BHMZ05_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

`ppl_BD_Shape_int64_t_limited_H79_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

`ppl_BD_Shape_int64_t_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

`ppl_BD_Shape_int64_t_CC76_narrowing_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-narrowing of `P_1` and the bd shape referenced by `Handle_2`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_BD_Shape_int64_t_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the concatenation of `P` and the bd shape referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument. These predicates enable the modification of the vector space of the bd shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_BD_Shape_int64_t_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_BD_Shape_int64_t_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_BD_Shape_int64_t_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the bd shape `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_BD_Shape_int64_t_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the bd shape `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_BD_Shape_int64_t_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the bd shape referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_BD_Shape_int64_t_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the bd shape referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_BD_Shape_int64_t_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the BD_Shape_mpz_class Domain

Here we provide a short description for each of the predicates available for the domain of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for bd shapes

The constructor predicates build a bd shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_BD_Shape_mpz_class_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new bd shape $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_BD_Shape_mpz_class_from_space_dimension(3, universe, X).
```

creates the bd shape defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_BD_Shape_mpz_class_from_constraints(+Constraint_System, -Handle)`

Builds a new bd shape `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_BD_Shape_mpz_class_from_congruences(+Congruence_System, -Handle)`

Builds a new bd shape `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_BD_Shape_mpz_class_from_generators(+Generator_System, -Handle)`

Builds a new bd shape `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions

- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_BD_Shape_mpz_class_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Grid(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Float_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_BD_Shape_float(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Double_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_BD_Shape_double(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Long_Double_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_BD_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Int8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Int16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Int32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Int64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Uint8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Uint16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Uint32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Uint64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Z_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Rational_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_BD_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_BD_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_BD_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_BD_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_BD_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_BD_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for

P_1.

`ppl_new_BD_Shape_mpz_class_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpz_class_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the BD_Shape_mpz_class domain.

`ppl_delete_BD_Shape_mpz_class(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the bd shape

Test Predicates

These predicates test the bd shape for different properties and succeed or fail depending on the outcome.

`ppl_BD_Shape_mpz_class_is_empty(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is empty.

`ppl_BD_Shape_mpz_class_is_universe(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is the universe.

`ppl_BD_Shape_mpz_class_is_bounded(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is bounded.

`ppl_BD_Shape_mpz_class_contains_integer_point(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` contains an integer point.

`ppl_BD_Shape_mpz_class_is_topologically_closed(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is topologically closed.

`ppl_BD_Shape_mpz_class_is_discrete(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is discrete.

`ppl_BD_Shape_mpz_class_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the bd shape referenced by `Handle`.

`ppl_BD_Shape_mpz_class_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the bd shape referenced by `Handle`.

`ppl_BD_Shape_mpz_class_contains_BD_Shape_mpz_class(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is included in or equal to the bd shape referenced by `Handle_1`.

```
ppl_BD_Shape_mpz_class_strictly_contains_BD_Shape_mpz_class(+Handle_1,
+Handle_2)
```

Succeeds if and only if the bd shape referenced by `Handle_2` is included in but not equal to the bd shape referenced by `Handle_1`.

```
ppl_BD_Shape_mpz_class_is_disjoint_from_BD_Shape_mpz_class(+Handle_1,
+Handle_2)
```

Succeeds if and only if the bd shape referenced by `Handle_2` is disjoint from the bd shape referenced by `Handle_1`.

```
ppl_BD_Shape_mpz_class_equals_BD_Shape_mpz_class(+Handle_1, +Handle_2)
```

Succeeds if and only if the bd shape referenced by `Handle_1` is equal to the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_mpz_class_OK(+Handle)
```

Succeeds only if the bd shape referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

```
ppl_BD_Shape_mpz_class_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the bd shape referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the bd shape

These predicates will obtain more detailed information about the bd shape unifying some of their arguments with the results.

```
ppl_BD_Shape_mpz_class_space_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` with the dimension of the vector space enclosing the bd shape referenced by `Handle`.

```
ppl_BD_Shape_mpz_class_affine_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` with the affine dimension of the bd shape referenced by `Handle`.

```
ppl_BD_Shape_mpz_class_relation_with_constraint(+Handle, +Constraint,
?Relation_List)
```

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

```
ppl_BD_Shape_mpz_class_relation_with_generator(+Handle, +Generator,
?Relation_List)
```

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

```
ppl_BD_Shape_mpz_class_relation_with_congruence(+Handle, +Congruence,
?Relation_List)
```

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

```
ppl_BD_Shape_mpz_class_get_constraints(+Handle, ?Constraint_System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the constraint system satisfied by the bd shape referenced by* `Handle`.

```
ppl_BD_Shape_mpz_class_get_congruences(+Handle, ?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the congruence system satisfied by the bd shape referenced by* `Handle`.

```
ppl_BD_Shape_mpz_class_get_minimized_constraints(+Handle,
?Constraint_System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the minimized constraint system satisfied by the bd shape referenced by* `Handle`.

```
ppl_BD_Shape_mpz_class_get_minimized_congruences(+Handle,
?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the minimized congruence system satisfied by the bd shape referenced by* `Handle`.

```
ppl_BD_Shape_mpz_class_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_-
2, ?Boolean)
```

Succeeds if and only if bd shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_BD_Shape_mpz_class_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_-
2, ?Boolean)
```

Succeeds if and only if bd shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_BD_Shape_mpz_class_maximize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if bd shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_BD_Shape_mpz_class_minimize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if bd shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_BD_Shape_mpz_class_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the bd shape referenced by* `Handle`.

`ppl_BD_Shape_mpz_class_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the external memory in bytes occupied by the bd shape referenced by* `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_BD_Shape_mpz_class_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the bd shape referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the bd shape

These predicates may modify the bd shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the bd shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_BD_Shape_mpz_class_add_constraint(+Handle, +Constraint)`

Updates the bd shape referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_BD_Shape_mpz_class_add_congruence(+Handle, +Congruence)`

Updates the bd shape referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_BD_Shape_mpz_class_add_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`. *For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_BD_Shape_mpz_class_add_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`. *For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_BD_Shape_mpz_class_refine_with_constraint( +Handle, +Constraint)`

Updates the bd shape referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`.

`ppl_BD_Shape_mpz_class_refine_with_congruence( +Handle, +Congruence)`

Updates the bd shape referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`.

`ppl_BD_Shape_mpz_class_refine_with_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`.

```
ppl_BD_Shape_mpz_class_refine_with_congruences( +Handle, +Congruence_-
System)
```

Updates the bd shape referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`.

Predicates that transform the bd shape

These predicates enable transformations such as taking the topological closure (which for the domain of bd shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_BD_Shape_mpz_class_topological_closure_assign(+Handle)
```

Assigns to the bd shape referenced by `Handle` *its topological closure.*

```
ppl_BD_Shape_mpz_class_unconstrain_space_dimension(+Handle, +PPL_Var)
```

Modifies the bd shape `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`.

```
ppl_BD_Shape_mpz_class_unconstrain_space_dimensions(+Handle, +List_-
of_PPL_Var)
```

Modifies the bd shape `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_BD_Shape_mpz_class_affine_image(+Handle, +PPL_Var, +Lin_Expr,
+Coeff)
```

Transforms the bd shape referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_BD_Shape_mpz_class_affine_preimage(+Handle, +PPL_Var, +Lin_Expr,
+Coeff)
```

Transforms the bd shape referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_BD_Shape_mpz_class_bounded_affine_image(+Handle, +PPL_Var, +Lin_-
Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_BD_Shape_mpz_class_bounded_affine_preimage(+Handle, +PPL_Var,
+Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_BD_Shape_mpz_class_generalized_affine_image(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`.

`ppl_BD_Shape_mpz_class_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` ⋈ `Lin_Expr/Coeff`, where ⋈ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_mpz_class_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to bd shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_mpz_class_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_mpz_class_wrap_assign(+Handle, +List_of_PPL_Var, +Width, +Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)`

Transforms the bd shape `P` referenced by `Handle` by wrapping the dimensions given by `List_of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one bd shape

These predicates include the binary operators which will assign to the bd shape referred to by the first argument its combination with the bd shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_BD_Shape_mpz_class_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the intersection of `P` and the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_mpz_class_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the upper bound of `P` and the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_mpz_class_difference_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the difference of `P` and the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_mpz_class_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the time elapse of `P` and the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_mpz_class_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the bd shape `P_1` referenced by `Handle_1` with the bd shape referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_BD_Shape_mpz_class_simplify_using_context_assign(+Handle_1,
+Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of bd shape `P_1` referenced by `Handle_1` and the bd shape `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

```
ppl_BD_Shape_mpz_class_linear_partition(+Handle_1, +Handle_2,
-Handle_3, -Handle_4)
```

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the BD_Shape_mpz_class domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the bd shape referred to by the second argument has to be contained in (or equal to) the bd shape referred to by the first argument.

```
ppl_BD_Shape_mpz_class_BHMZ05_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_mpz_class_H79_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_mpz_class_BHMZ05_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_mpz_class_H79_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_mpz_class_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_BD_Shape_mpz_class_H79_widening_assign_with_tokens/4`

```
ppl_BD_Shape_mpz_class_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_BD_Shape_mpz_class_H79_widening_assign/2`

```
ppl_BD_Shape_mpz_class_limited_BHMZ05_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
```

unsigned_2)

Assigns to the bd shape P_1 referenced by Handle_1 the BHMZ05-widening of P_1 with the bd shape referenced by Handle_2 intersected with the constraints in Constraint_System that are satisfied by all the points of P_1. The widening with tokens delay technique is applied with C_unsigned_1 tokens; C_unsigned_2 is unified with the number of tokens remaining at the end of the operation.

ppl_BD_Shape_mpz_class_limited_H79_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)

Assigns to the bd shape P_1 referenced by Handle_1 the H79-widening of P_1 with the bd shape referenced by Handle_2 intersected with the constraints in Constraint_System that are satisfied by all the points of P_1. The widening with tokens delay technique is applied with C_unsigned_1 tokens; C_unsigned_2 is unified with the number of tokens remaining at the end of the operation.

ppl_BD_Shape_mpz_class_limited_CC76_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)

Assigns to the bd shape P_1 referenced by Handle_1 the CC76-widening of P_1 with the bd shape referenced by Handle_2 intersected with the constraints in Constraint_System that are satisfied by all the points of P_1. The widening with tokens delay technique is applied with C_unsigned_1 tokens; C_unsigned_2 is unified with the number of tokens remaining at the end of the operation.

ppl_BD_Shape_mpz_class_limited_BHMZ05_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)

Assigns to the bd shape P_1 referenced by Handle_1 the BHMZ05-widening of P_1 with the bd shape referenced by Handle_2 intersected with the constraints in Constraint_System that are satisfied by all the points of P_1.

ppl_BD_Shape_mpz_class_limited_H79_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)

Assigns to the bd shape P_1 referenced by Handle_1 the H79-widening of P_1 with the bd shape referenced by Handle_2 intersected with the constraints in Constraint_System that are satisfied by all the points of P_1.

ppl_BD_Shape_mpz_class_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)

Assigns to the bd shape P_1 referenced by Handle_1 the CC76-widening of P_1 with the bd shape referenced by Handle_2 intersected with the constraints in Constraint_System that are satisfied by all the points of P_1.

ppl_BD_Shape_mpz_class_CC76_narrowing_assign(+Handle_1, +Handle_2)

Assigns to the bd shape P_1 referenced by Handle_1 the CC76-narrowing of P_1 and the bd shape referenced by Handle_2.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_BD_Shape_mpz_class_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the concatenation of `P` and the bd shape referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument. These predicates enable the modification of the vector space of the bd shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_BD_Shape_mpz_class_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_BD_Shape_mpz_class_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_BD_Shape_mpz_class_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the bd shape `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_BD_Shape_mpz_class_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the bd shape `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_BD_Shape_mpz_class_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the bd shape referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_BD_Shape_mpz_class_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the bd shape referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_BD_Shape_mpz_class_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the BD_Shape_mpq_class Domain

Here we provide a short description for each of the predicates available for the domain of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for bd shapes

The constructor predicates build a bd shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_BD_Shape_mpq_class_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new bd shape $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_BD_Shape_mpq_class_from_space_dimension(3, universe, X).
```

creates the bd shape defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_BD_Shape_mpq_class_from_constraints(+Constraint_System, -Handle)`

Builds a new bd shape `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_BD_Shape_mpq_class_from_congruences(+Congruence_System, -Handle)`

Builds a new bd shape `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_BD_Shape_mpq_class_from_generators(+Generator_System, -Handle)`

Builds a new bd shape `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_BD_Shape_mpq_class_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Grid(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpq_class_from_BD_Shape_float(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpq_class_from_Double_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpq_class_from_BD_Shape_double(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpq_class_from_Long_Double_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpq_class_from_BD_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpq_class_from_Int8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpq_class_from_Int16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_mpq_class_from_Int32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Uint8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Uint16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Uint32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Uint64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Z_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Rational_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_BD_Shape_int8_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_BD_Shape_int16_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_BD_Shape_int32_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_BD_Shape_int64_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_BD_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_NNC_Polyhedron_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_BD_Shape_float_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_float_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_BD_Shape_double_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_double_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Long_Double_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_BD_Shape_long_double_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_mpq_class_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the BD_Shape_mpq_class domain.

`ppl_delete_BD_Shape_mpq_class(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the bd shape

Test Predicates

These predicates test the bd shape for different properties and succeed or fail depending on the outcome.

`ppl_BD_Shape_mpq_class_is_empty(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is empty.

`ppl_BD_Shape_mpq_class_is_universe(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is the universe.

`ppl_BD_Shape_mpq_class_is_bounded(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is bounded.

`ppl_BD_Shape_mpq_class_contains_integer_point(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` contains an integer point.

`ppl_BD_Shape_mpq_class_is_topologically_closed(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is topologically closed.

`ppl_BD_Shape_mpq_class_is_discrete(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is discrete.

`ppl_BD_Shape_mpq_class_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the bd shape referenced by `Handle`.

`ppl_BD_Shape_mpq_class_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the bd shape referenced by `Handle`.

`ppl_BD_Shape_mpq_class_contains_BD_Shape_mpq_class(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is included in or equal to the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_mpq_class_strictly_contains_BD_Shape_mpq_class(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` *is included in but not equal to the bd shape referenced by* `Handle_1`.

```
ppl_BD_Shape_mpq_class_is_disjoint_from_BD_Shape_mpq_class(+Handle_1,
+Handle_2)
```

Succeeds if and only if the bd shape referenced by `Handle_2` *is disjoint from the bd shape referenced by* `Handle_1`.

```
ppl_BD_Shape_mpq_class_equals_BD_Shape_mpq_class(+Handle_1, +Handle_2)
```

Succeeds if and only if the bd shape referenced by `Handle_1` *is equal to the bd shape referenced by* `Handle_2`.

```
ppl_BD_Shape_mpq_class_OK(+Handle)
```

Succeeds only if the bd shape referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_BD_Shape_mpq_class_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the bd shape referenced by `Handle` *constrains the dimension* `PPL_Var`.

Predicates that return information about the bd shape

These predicates will obtain more detailed information about the bd shape unifying some of their arguments with the results.

```
ppl_BD_Shape_mpq_class_space_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the bd shape referenced by* `Handle`.

```
ppl_BD_Shape_mpq_class_affine_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` *with the affine dimension of the bd shape referenced by* `Handle`.

```
ppl_BD_Shape_mpq_class_relation_with_constraint(+Handle, +Constraint,
?Relation_List)
```

Unifies `Relation_List` *with the list of relations the bd shape referenced by* `Handle` *has with* `Constraint`. *The possible relations are listed in the grammar rules above.*

```
ppl_BD_Shape_mpq_class_relation_with_generator(+Handle, +Generator,
?Relation_List)
```

Unifies `Relation_List` *with the list of relations the bd shape referenced by* `Handle` *has with* `Generator`. *The possible relations are listed in the grammar rules above.*

```
ppl_BD_Shape_mpq_class_relation_with_congruence(+Handle, +Congruence,
?Relation_List)
```

Unifies `Relation_List` *with the list of relations the bd shape referenced by* `Handle` *has with* `Congruence`. *The possible relations are listed in the grammar rules above.*

```
ppl_BD_Shape_mpq_class_get_constraints(+Handle, ?Constraint_System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the constraint system satisfied by the bd shape referenced by* `Handle`.

```
ppl_BD_Shape_mpq_class_get_congruences(+Handle, ?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the congruence system satisfied by the bd shape referenced by* `Handle`.

```
ppl_BD_Shape_mpq_class_get_minimized_constraints(+Handle,
?Constraint_System)
```

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the bd shape referenced by `Handle`.

```
ppl_BD_Shape_mpq_class_get_minimized_congruences(+Handle,
?Congruence_System)
```

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the bd shape referenced by `Handle`.

```
ppl_BD_Shape_mpq_class_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_-
2, ?Boolean)
```

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_BD_Shape_mpq_class_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_-
2, ?Boolean)
```

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_BD_Shape_mpq_class_maximize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_BD_Shape_mpq_class_minimize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_BD_Shape_mpq_class_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the total memory in bytes occupied by the bd shape referenced by `Handle`.

```
ppl_BD_Shape_mpq_class_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the external memory in bytes occupied by the bd shape referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_BD_Shape_mpq_class_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the bd shape referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the bd shape

These predicates may modify the bd shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the bd shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_BD_Shape_mpq_class_add_constraint(+Handle, +Constraint)`

Updates the bd shape referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_BD_Shape_mpq_class_add_congruence(+Handle, +Congruence)`

Updates the bd shape referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_BD_Shape_mpq_class_add_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_BD_Shape_mpq_class_add_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_BD_Shape_mpq_class_refine_with_constraint( +Handle, +Constraint)`

Updates the bd shape referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_BD_Shape_mpq_class_refine_with_congruence( +Handle, +Congruence)`

Updates the bd shape referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_BD_Shape_mpq_class_refine_with_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_BD_Shape_mpq_class_refine_with_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the bd shape

These predicates enable transformations such as taking the topological closure (which for the domain of bd shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_BD_Shape_mpq_class_topological_closure_assign(+Handle)`

Assigns to the bd shape referenced by `Handle` its topological closure.

`ppl_BD_Shape_mpq_class_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the bd shape `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_BD_Shape_mpq_class_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the bd shape `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_BD_Shape_mpq_class_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the bd shape referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_BD_Shape_mpq_class_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the bd shape referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_BD_Shape_mpq_class_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_BD_Shape_mpq_class_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_BD_Shape_mpq_class_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_mpq_class_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_mpq_class_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to bd shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_BD_Shape_mpq_class_generalized_affine_preimage_lhs_rhs(+Handle,
+Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_BD_Shape_mpq_class_wrap_assign(+Handle, +List_of_PPL_Var, +Width,
+Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)
```

Transforms the bd shape `P` referenced by `Handle` by wrapping the dimensions given by `List_of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one bd shape

These predicates include the binary operators which will assign to the bd shape referred to by the first argument its combination with the bd shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_BD_Shape_mpq_class_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` referenced by `Handle_1` the intersection of `P` and the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_mpq_class_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` referenced by `Handle_1` the upper bound of `P` and the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_mpq_class_difference_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` referenced by `Handle_1` the difference of `P` and the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_mpq_class_time_elapse_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` referenced by `Handle_1` the time elapse of `P` and the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_mpq_class_upper_bound_assign_if_exact(+Handle_1,
+Handle_2)
```

Succeeds if the least upper bound of the bd shape `P_1` referenced by `Handle_1` with the bd shape referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_BD_Shape_mpq_class_simplify_using_context_assign(+Handle_1,
+Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of bd shape `P_1` referenced by `Handle_1` and the bd shape `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

```
ppl_BD_Shape_mpq_class_linear_partition(+Handle_1, +Handle_2,
-Handle_3, -Handle_4)
```

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the BD_Shape_mpq_class domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the bd shape referred to by the second argument has to be contained in (or equal to) the bd shape referred to by the first argument.

```
ppl_BD_Shape_mpq_class_BHMZ05_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_mpq_class_H79_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_mpq_class_BHMZ05_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_mpq_class_H79_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_mpq_class_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_BD_Shape_mpq_class_H79_widening_assign_with_tokens/4`

```
ppl_BD_Shape_mpq_class_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_BD_Shape_mpq_class_H79_widening_assign/2`

```
ppl_BD_Shape_mpq_class_limited_BHMZ05_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_mpq_class_limited_H79_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
```

```
unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_mpq_class_limited_CC76_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_mpq_class_limited_BHMZ05_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_BD_Shape_mpq_class_limited_H79_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_BD_Shape_mpq_class_limited_CC76_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_BD_Shape_mpq_class_CC76_narrowing_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-narrowing of `P_1` and the bd shape referenced by `Handle_2`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_BD_Shape_mpq_class_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` referenced by `Handle_1` the concatenation of `P` and the bd shape referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument. These predicates enable the modification of the vector space of the bd shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_BD_Shape_mpq_class_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_BD_Shape_mpq_class_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_BD_Shape_mpq_class_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the bd shape `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_BD_Shape_mpq_class_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the bd shape `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_BD_Shape_mpq_class_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the bd shape referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_BD_Shape_mpq_class_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the bd shape referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_BD_Shape_mpq_class_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Octagonal_Shape_int8_t Domain

Here we provide a short description for each of the predicates available for the domain of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for octagonal shapes

The constructor predicates build a octagonal shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Octagonal_Shape_int8_t_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new octagonal shape $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Octagonal_Shape_int8_t_from_space_dimension(3, universe, X).
```

creates the octagonal shape defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Octagonal_Shape_int8_t_from_constraints(+Constraint_System, -Handle)`

Builds a new octagonal shape `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Octagonal_Shape_int8_t_from_congruences(+Congruence_System, -Handle)`

Builds a new octagonal shape `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Octagonal_Shape_int8_t_from_generators(+Generator_System, -Handle)`

Builds a new octagonal shape `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Octagonal_Shape_int8_t_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_mpz_class(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_mpq_class(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_int8_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_int16_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_int32_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_int64_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_mpz_-
class(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_-1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_mpq_-
class(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_-1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_C_Polyhedron(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_NNC_Polyhedron(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Grid(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Float_Box(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Double_Box(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Long_Double_Box(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_long_-
double(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Int8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Int16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Int32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Int64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Uint8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Uint16_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Uint32_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Uint64_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Z_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_Rational_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_mpq_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_C_Polyhedron_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_NNC_Polyhedron_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_float_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_float_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Double_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_double_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_double_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the

handle for `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` *from the long_double_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` *from the bd_shape_long_double referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`.

`ppl_new_Octagonal_Shape_int8_t_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` *from the octagonal_shape_long_double referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`.

Destructor predicate

Below is the destructor predicate for the Octagonal_Shape_int8_t domain.

`ppl_delete_Octagonal_Shape_int8_t(+Handle)`

Invalidates the handle referenced by `Handle`*: this makes sure the corresponding resources will eventually be released.*

Predicates that do not change the octagonal shape

Test Predicates

These predicates test the octagonal shape for different properties and succeed or fail depending on the outcome.

`ppl_Octagonal_Shape_int8_t_is_empty(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is empty.*

`ppl_Octagonal_Shape_int8_t_is_universe(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is the universe.*

`ppl_Octagonal_Shape_int8_t_is_bounded(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is bounded.*

`ppl_Octagonal_Shape_int8_t_contains_integer_point(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *contains an integer point.*

`ppl_Octagonal_Shape_int8_t_is_topologically_closed(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is topologically closed.*

`ppl_Octagonal_Shape_int8_t_is_discrete(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is discrete.*

`ppl_Octagonal_Shape_int8_t_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_int8_t_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_int8_t_contains_Octagonal_Shape_int8_t(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` is included in or equal to the octagonal shape referenced by `Handle_1`.

`ppl_Octagonal_Shape_int8_t_strictly_contains_Octagonal_Shape_int8_t(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` is included in but not equal to the octagonal shape referenced by `Handle_1`.

`ppl_Octagonal_Shape_int8_t_is_disjoint_from_Octagonal_Shape_int8_t(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` is disjoint from the octagonal shape referenced by `Handle_1`.

`ppl_Octagonal_Shape_int8_t_equals_Octagonal_Shape_int8_t(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_1` is equal to the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_int8_t_OK(+Handle)`

Succeeds only if the octagonal shape referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Octagonal_Shape_int8_t_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the octagonal shape referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the octagonal shape

These predicates will obtain more detailed information about the octagonal shape unifying some of their arguments with the results.

`ppl_Octagonal_Shape_int8_t_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_int8_t_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_int8_t_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the octagonal shape referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Octagonal_Shape_int8_t_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` *with the list of relations the octagonal shape referenced by* `Handle` *has with* `Generator`*. The possible relations are listed in the grammar rules above.*

```
ppl_Octagonal_Shape_int8_t_relation_with_congruence(+Handle,
+Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the octagonal shape referenced by* `Handle` *has with* `Congruence`*. The possible relations are listed in the grammar rules above.*

```
ppl_Octagonal_Shape_int8_t_get_constraints(+Handle, ?Constraint_-
System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the constraint system satisfied by the octagonal shape referenced by* `Handle`*.*

```
ppl_Octagonal_Shape_int8_t_get_congruences(+Handle, ?Congruence_-
System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the congruence system satisfied by the octagonal shape referenced by* `Handle`*.*

```
ppl_Octagonal_Shape_int8_t_get_minimized_constraints(+Handle,
?Constraint_System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the minimized constraint system satisfied by the octagonal shape referenced by* `Handle`*.*

```
ppl_Octagonal_Shape_int8_t_get_minimized_congruences(+Handle,
?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the minimized congruence system satisfied by the octagonal shape referenced by* `Handle`*.*

```
ppl_Octagonal_Shape_int8_t_maximize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if octagonal shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Octagonal_Shape_int8_t_minimize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if octagonal shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Octagonal_Shape_int8_t_maximize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if octagonal shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

`ppl_Octagonal_Shape_int8_t_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Octagonal_Shape_int8_t_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the total memory in bytes occupied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_int8_t_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the octagonal shape referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Octagonal_Shape_int8_t_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the octagonal shape referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the octagonal shape

These predicates may modify the octagonal shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the octagonal shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Octagonal_Shape_int8_t_add_constraint(+Handle, +Constraint)`

Updates the octagonal shape referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_Octagonal_Shape_int8_t_add_congruence(+Handle, +Congruence)`

Updates the octagonal shape referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Octagonal_Shape_int8_t_add_constraints( +Handle, +Constraint_System)`

Updates the octagonal shape referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Octagonal_Shape_int8_t_add_congruences( +Handle, +Congruence_System)`

Updates the octagonal shape referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Octagonal_Shape_int8_t_refine_with_constraint( +Handle, +Constraint)`

Updates the octagonal shape referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Octagonal_Shape_int8_t_refine_with_congruence( +Handle, +Congruence)`

Updates the octagonal shape referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Octagonal_Shape_int8_t_refine_with_constraints( +Handle, +Constraint_System)`

Updates the octagonal shape referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Octagonal_Shape_int8_t_refine_with_congruences( +Handle, +Congruence_System)`

Updates the octagonal shape referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the octagonal shape

These predicates enable transformations such as taking the topological closure (which for the domain of octagonal shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Octagonal_Shape_int8_t_topological_closure_assign(+Handle)`

Assigns to the octagonal shape referenced by `Handle` its topological closure.

`ppl_Octagonal_Shape_int8_t_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the octagonal shape `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Octagonal_Shape_int8_t_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the octagonal shape `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Octagonal_Shape_int8_t_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the octagonal shape referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Octagonal_Shape_int8_t_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the octagonal shape referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Octagonal_Shape_int8_t_bounded_affine_image(+Handle, +PPL_Var,
+Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Octagonal_Shape_int8_t_bounded_affine_preimage(+Handle, +PPL_Var,
+Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Octagonal_Shape_int8_t_generalized_affine_image(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Octagonal_Shape_int8_t_generalized_affine_preimage(+Handle, +PPL_
Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Octagonal_Shape_int8_t_generalized_affine_image_lhs_rhs(+Handle,
+Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Octagonal_Shape_int8_t_generalized_affine_preimage_lhs_
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Octagonal_Shape_int8_t_wrap_assign(+Handle, +List_of_PPL_Var,
+Width, +Representation, +Overflow, +Constraint_System, +C_unsigned,
+Boolean)
```

Transforms the octagonal shape `P` *referenced by* `Handle` *by wrapping the dimensions given by* `List_of_PPL_Vars` *while respecting the specified* `Width`, `Representation` *and* `Overflow` *behavior of all these variables. The* `Constraint_System` *represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer* `C_unsigned` *and* `Boolean` *allow control of the complexity/precision ratio; higher values for* `C_unsigned` *will lead to possibly greater precision while a true value for* `Boolean` *indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.*

Predicates whose results depend on more than one octagonal shape

These predicates include the binary operators which will assign to the octagonal shape referred to by the first argument its combination with the octagonal shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral*

Difference; and a linear partitioning operator described below.

`ppl_Octagonal_Shape_int8_t_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the intersection of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_int8_t_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the upper bound of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_int8_t_difference_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the difference of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_int8_t_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the time elapse of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_int8_t_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the octagonal shape `P_1` referenced by `Handle_1` with the octagonal shape referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Octagonal_Shape_int8_t_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of octagonal shape `P_1` referenced by `Handle_1` and the octagonal shape `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

`ppl_Octagonal_Shape_int8_t_linear_partition(+Handle_1, +Handle_2, -Handle_3, -Handle_4)`

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Octagonal_Shape_int8_t domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the octagonal shape referred to by the second argument has to be contained in (or equal to) the octagonal shape referred to by the first argument.

`ppl_Octagonal_Shape_int8_t_BHMZ05_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_Octagonal_Shape_int8_t_BHMZ05_widening_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_int8_t_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Same as predicate `ppl_Octagonal_Shape_int8_t_H79_widening_assign_with_tokens/4`

`ppl_Octagonal_Shape_int8_t_widening_assign(+Handle_1, +Handle_2)`

Same as predicate `ppl_Octagonal_Shape_int8_t_H79_widening_assign/2`

`ppl_Octagonal_Shape_int8_t_limited_BHMZ05_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_Octagonal_Shape_int8_t_limited_CC76_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_Octagonal_Shape_int8_t_limited_BHMZ05_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

`ppl_Octagonal_Shape_int8_t_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

`ppl_Octagonal_Shape_int8_t_CC76_narrowing_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the CC76-narrowing of `P_1` and the octagonal shape referenced by `Handle_2`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Octagonal_Shape_int8_t_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the concatenation of `P` and the octagonal shape referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Octagonal_Shape_int8_t_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the octagonal shape `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Octagonal_Shape_int8_t_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the octagonal shape `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Octagonal_Shape_int8_t_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the octagonal shape `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Octagonal_Shape_int8_t_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the octagonal shape `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Octagonal_Shape_int8_t_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the octagonal shape referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Octagonal_Shape_int8_t_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the octagonal shape referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Octagonal_Shape_int8_t_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial

function.

Predicates for the Octagonal_Shape_int16_t Domain

Here we provide a short description for each of the predicates available for the domain of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for octagonal shapes

The constructor predicates build a octagonal shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Octagonal_Shape_int16_t_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new octagonal shape $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Octagonal_Shape_int16_t_from_space_dimension(3, universe, X).
```

creates the octagonal shape defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Octagonal_Shape_int16_t_from_constraints(+Constraint_System, -Handle)`

Builds a new octagonal shape `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Octagonal_Shape_int16_t_from_congruences(+Congruence_System, -Handle)`

Builds a new octagonal shape `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Octagonal_Shape_int16_t_from_generators(+Generator_System, -Handle)`

Builds a new octagonal shape `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Octagonal_Shape_int16_t_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_NNC_Polyhedron(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Grid(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Float_Box(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Double_Box(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Long_Double_Box(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_long_-
double(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Int8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Int16_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Int32_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Int64_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Uint8_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Uint16_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Uint32_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Uint64_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Z_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_Rational_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_mpz_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_mpq_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_C_Polyhedron_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_NNC_Polyhedron_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Float_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_float_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_float_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Double_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_double_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for

P_1.

`ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int16_t_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Octagonal_Shape_int16_t domain.

`ppl_delete_Octagonal_Shape_int16_t(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the octagonal shape

Test Predicates

These predicates test the octagonal shape for different properties and succeed or fail depending on the outcome.

`ppl_Octagonal_Shape_int16_t_is_empty(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is empty.

`ppl_Octagonal_Shape_int16_t_is_universe(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is the universe.

`ppl_Octagonal_Shape_int16_t_is_bounded(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is bounded.

`ppl_Octagonal_Shape_int16_t_contains_integer_point(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` contains an integer point.

`ppl_Octagonal_Shape_int16_t_is_topologically_closed(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is topologically closed.

`ppl_Octagonal_Shape_int16_t_is_discrete(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is discrete.

`ppl_Octagonal_Shape_int16_t_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_int16_t_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_int16_t_contains_Octagonal_Shape_int16_t(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` is included in or equal to the octagonal shape referenced by `Handle_1`.

`ppl_Octagonal_Shape_int16_t_strictly_contains_Octagonal_Shape_int16_t(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` is included in but not equal to the octagonal shape referenced by `Handle_1`.

`ppl_Octagonal_Shape_int16_t_is_disjoint_from_Octagonal_Shape_int16_t(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` is disjoint from the octagonal shape referenced by `Handle_1`.

`ppl_Octagonal_Shape_int16_t_equals_Octagonal_Shape_int16_t(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_1` is equal to the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_int16_t_OK(+Handle)`

Succeeds only if the octagonal shape referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Octagonal_Shape_int16_t_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the octagonal shape referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the octagonal shape

These predicates will obtain more detailed information about the octagonal shape unifying some of their arguments with the results.

`ppl_Octagonal_Shape_int16_t_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_int16_t_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_int16_t_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` *with the list of relations the octagonal shape referenced by* `Handle` *has with* `Constraint`*. The possible relations are listed in the grammar rules above.*

```
ppl_Octagonal_Shape_int16_t_relation_with_generator(+Handle,
+Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the octagonal shape referenced by* `Handle` *has with* `Generator`*. The possible relations are listed in the grammar rules above.*

```
ppl_Octagonal_Shape_int16_t_relation_with_congruence(+Handle,
+Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the octagonal shape referenced by* `Handle` *has with* `Congruence`*. The possible relations are listed in the grammar rules above.*

```
ppl_Octagonal_Shape_int16_t_get_constraints(+Handle, ?Constraint_-
System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the constraint system satisfied by the octagonal shape referenced by* `Handle`*.*

```
ppl_Octagonal_Shape_int16_t_get_congruences(+Handle, ?Congruence_-
System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the congruence system satisfied by the octagonal shape referenced by* `Handle`*.*

```
ppl_Octagonal_Shape_int16_t_get_minimized_constraints(+Handle,
?Constraint_System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the minimized constraint system satisfied by the octagonal shape referenced by* `Handle`*.*

```
ppl_Octagonal_Shape_int16_t_get_minimized_congruences(+Handle,
?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the minimized congruence system satisfied by the octagonal shape referenced by* `Handle`*.*

```
ppl_Octagonal_Shape_int16_t_maximize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if octagonal shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Octagonal_Shape_int16_t_minimize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if octagonal shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Octagonal_Shape_int16_t_maximize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if octagonal shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Octagonal_Shape_int16_t_minimize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Octagonal_Shape_int16_t_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the total memory in bytes occupied by the octagonal shape referenced by `Handle`.

```
ppl_Octagonal_Shape_int16_t_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the external memory in bytes occupied by the octagonal shape referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Octagonal_Shape_int16_t_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the octagonal shape referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the octagonal shape

These predicates may modify the octagonal shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the octagonal shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Octagonal_Shape_int16_t_add_constraint(+Handle, +Constraint)
```

Updates the octagonal shape referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

```
ppl_Octagonal_Shape_int16_t_add_congruence(+Handle, +Congruence)
```

Updates the octagonal shape referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

```
ppl_Octagonal_Shape_int16_t_add_constraints( +Handle, +Constraint_-
System)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

```
ppl_Octagonal_Shape_int16_t_add_congruences( +Handle, +Congruence_-
System)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

```
ppl_Octagonal_Shape_int16_t_refine_with_constraint( +Handle,
+Constraint)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

```
ppl_Octagonal_Shape_int16_t_refine_with_congruence( +Handle,
+Congruence)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

```
ppl_Octagonal_Shape_int16_t_refine_with_constraints( +Handle,
+Constraint_System)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

```
ppl_Octagonal_Shape_int16_t_refine_with_congruences( +Handle,
+Congruence_System)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the octagonal shape

These predicates enable transformations such as taking the topological closure (which for the domain of octagonal shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Octagonal_Shape_int16_t_topological_closure_assign(+Handle)
```

Assigns to the octagonal shape referenced by `Handle` *its topological closure.*

```
ppl_Octagonal_Shape_int16_t_unconstrain_space_dimension(+Handle,
+PPL_Var)
```

Modifies the octagonal shape `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

```
ppl_Octagonal_Shape_int16_t_unconstrain_space_dimensions(+Handle,
+List_of_PPL_Var)
```

Modifies the octagonal shape `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Octagonal_Shape_int16_t_affine_image(+Handle, +PPL_Var, +Lin_Expr,
+Coeff)
```

Transforms the octagonal shape referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Octagonal_Shape_int16_t_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the octagonal shape referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Octagonal_Shape_int16_t_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Octagonal_Shape_int16_t_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Octagonal_Shape_int16_t_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Octagonal_Shape_int16_t_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Octagonal_Shape_int16_t_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Octagonal_Shape_int16_t_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Octagonal_Shape_int16_t_wrap_assign(+Handle, +List_of_PPL_Var, +Width, +Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)
```

Transforms the octagonal shape `P` *referenced by* `Handle` *by wrapping the dimensions given by* `List_of_PPL_Vars` *while respecting the specified* `Width`, `Representation` *and* `Overflow` *behavior of all these variables. The* `Constraint_System` *represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer* `C_unsigned` *and* `Boolean` *allow control of the complexity/precision ratio; higher values for* `C_unsigned` *will lead to possibly greater precision while a true value for* `Boolean` *indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.*

Predicates whose results depend on more than one octagonal shape

These predicates include the binary operators which will assign to the octagonal shape referred to by the first argument its combination with the octagonal shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Octagonal_Shape_int16_t_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the intersection of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_int16_t_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the upper bound of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_int16_t_difference_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the difference of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_int16_t_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the time elapse of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_int16_t_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the octagonal shape `P_1` referenced by `Handle_1` with the octagonal shape referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Octagonal_Shape_int16_t_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of octagonal shape `P_1` referenced by `Handle_1` and the octagonal shape `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

`ppl_Octagonal_Shape_int16_t_linear_partition(+Handle_1, +Handle_2, -Handle_3, -Handle_4)`

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Octagonal_Shape_int16_t domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the octagonal shape referred to by the second argument has to be contained in (or equal to) the octagonal shape referred to by the first argument.

`ppl_Octagonal_Shape_int16_t_BHMZ05_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_int16_t_BHMZ05_widening_assign(+Handle_1,
+Handle_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_int16_t_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_Octagonal_Shape_int16_t_H79_widening_assign_with_tokens/4`

```
ppl_Octagonal_Shape_int16_t_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_Octagonal_Shape_int16_t_H79_widening_assign/2`

```
ppl_Octagonal_Shape_int16_t_limited_BHMZ05_extrapolation_assign_with_
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_
unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_int16_t_limited_CC76_extrapolation_assign_with_
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_
unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_int16_t_limited_BHMZ05_extrapolation_
assign(+Handle_1, +Handle_2, +Constraint_System)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_Octagonal_Shape_int16_t_limited_CC76_extrapolation_
assign(+Handle_1, +Handle_2, +Constraint_System)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_Octagonal_Shape_int16_t_CC76_narrowing_assign(+Handle_1, +Handle_
2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the CC76-narrowing of `P_1` and the octagonal shape referenced by `Handle_2`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Octagonal_Shape_int16_t_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the concatenation of `P` and the octagonal shape referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Octagonal_Shape_int16_t_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the octagonal shape `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Octagonal_Shape_int16_t_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the octagonal shape `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Octagonal_Shape_int16_t_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the octagonal shape `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Octagonal_Shape_int16_t_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the octagonal shape `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Octagonal_Shape_int16_t_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the octagonal shape referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Octagonal_Shape_int16_t_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the octagonal shape referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars`

is a waste but an innocuous one.

`ppl_Octagonal_Shape_int16_t_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for the Octagonal_Shape_int32_t Domain

Here we provide a short description for each of the predicates available for the domain of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for octagonal shapes

The constructor predicates build a octagonal shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Octagonal_Shape_int32_t_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new octagonal shape $\mathcal{P}$ *with* `Dimension_Type` *dimensions; it is empty or the universe depending on whether* `Atom` *is* `empty` *or* `universe`*, respectively.* `Handle` *is unified with the handle for* $\mathcal{P}$*. Thus the query*

```
?- ppl_new_Octagonal_Shape_int32_t_from_space_dimension(3, universe, X).
```

creates the octagonal shape defining the 3-dimensional vector space $\mathbb{R}^3$ *with* `X` *bound to a valid handle for accessing it.*

`ppl_new_Octagonal_Shape_int32_t_from_constraints(+Constraint_System, -Handle)`

Builds a new octagonal shape `P` *from* `Constraint_System`*.* `Handle` *is unified with the handle for* `P`*.*

`ppl_new_Octagonal_Shape_int32_t_from_congruences(+Congruence_System, -Handle)`

Builds a new octagonal shape `P` *from* `Congruence_System`*.* `Handle` *is unified with the handle for* `P`*.*

`ppl_new_Octagonal_Shape_int32_t_from_generators(+Generator_System, -Handle)`

Builds a new octagonal shape `P` *from* `Generator_System`*.* `Handle` *is unified with the handle for* `P`*.*

Predicates that build new octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions

- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Octagonal_Shape_int32_t_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_int32_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_int64_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_mpz_class(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_mpq_class(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_int8_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_int16_t(+Handle_-
1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_int32_t(+Handle_-
1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_int64_t(+Handle_-
1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_mpz_-
class(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_-1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_mpq_-
class(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_-1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_C_Polyhedron(+Handle_1, -Handle_-
2)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_NNC_Polyhedron(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_Grid(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_Float_Box(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_Double_Box(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_Long_Double_Box(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_long_-
double(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Uint8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Uint16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Uint32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Uint64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Z_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Rational_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for

P_1.

`ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the

handle for `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` *from the octagonal_shape_mpz_class referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` *from the octagonal_shape_mpq_class referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` *from the polyhedron referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` *from the polyhedron referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` *from the grid referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` *from the float_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` *from the bd_shape_float referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` *from the octagonal_shape_float referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` *from the double_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`.

`ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_Long_Double_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_BD_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int32_t_from_Octagonal_Shape_long_double_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Octagonal_Shape_int32_t domain.

```
ppl_delete_Octagonal_Shape_int32_t(+Handle)
```

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the octagonal shape

Test Predicates

These predicates test the octagonal shape for different properties and succeed or fail depending on the outcome.

```
ppl_Octagonal_Shape_int32_t_is_empty(+Handle)
```

Succeeds if and only if the octagonal shape referenced by `Handle` is empty.

```
ppl_Octagonal_Shape_int32_t_is_universe(+Handle)
```

Succeeds if and only if the octagonal shape referenced by `Handle` is the universe.

```
ppl_Octagonal_Shape_int32_t_is_bounded(+Handle)
```

Succeeds if and only if the octagonal shape referenced by `Handle` is bounded.

```
ppl_Octagonal_Shape_int32_t_contains_integer_point(+Handle)
```

Succeeds if and only if the octagonal shape referenced by `Handle` *contains an integer point.*

`ppl_Octagonal_Shape_int32_t_is_topologically_closed(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is topologically closed.*

`ppl_Octagonal_Shape_int32_t_is_discrete(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is discrete.*

`ppl_Octagonal_Shape_int32_t_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from above in the octagonal shape referenced by* `Handle`*.*

`ppl_Octagonal_Shape_int32_t_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from below in the octagonal shape referenced by* `Handle`*.*

`ppl_Octagonal_Shape_int32_t_contains_Octagonal_Shape_int32_t(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` *is included in or equal to the octagonal shape referenced by* `Handle_1`*.*

`ppl_Octagonal_Shape_int32_t_strictly_contains_Octagonal_Shape_int32_t(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` *is included in but not equal to the octagonal shape referenced by* `Handle_1`*.*

`ppl_Octagonal_Shape_int32_t_is_disjoint_from_Octagonal_Shape_int32_t(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` *is disjoint from the octagonal shape referenced by* `Handle_1`*.*

`ppl_Octagonal_Shape_int32_t_equals_Octagonal_Shape_int32_t(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_1` *is equal to the octagonal shape referenced by* `Handle_2`*.*

`ppl_Octagonal_Shape_int32_t_OK(+Handle)`

Succeeds only if the octagonal shape referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

`ppl_Octagonal_Shape_int32_t_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the octagonal shape referenced by `Handle` *constrains the dimension* `PPL_Var`*.*

Predicates that return information about the octagonal shape

These predicates will obtain more detailed information about the octagonal shape unifying some of their arguments with the results.

`ppl_Octagonal_Shape_int32_t_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the octagonal shape referenced by* `Handle`*.*

`ppl_Octagonal_Shape_int32_t_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` *with the affine dimension of the octagonal shape referenced by* `Handle`*.*

```
ppl_Octagonal_Shape_int32_t_relation_with_constraint(+Handle,
+Constraint, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the octagonal shape referenced by* `Handle` *has with* `Constraint`*. The possible relations are listed in the grammar rules above.*

```
ppl_Octagonal_Shape_int32_t_relation_with_generator(+Handle,
+Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the octagonal shape referenced by* `Handle` *has with* `Generator`*. The possible relations are listed in the grammar rules above.*

```
ppl_Octagonal_Shape_int32_t_relation_with_congruence(+Handle,
+Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the octagonal shape referenced by* `Handle` *has with* `Congruence`*. The possible relations are listed in the grammar rules above.*

```
ppl_Octagonal_Shape_int32_t_get_constraints(+Handle, ?Constraint_-
System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the constraint system satisfied by the octagonal shape referenced by* `Handle`*.*

```
ppl_Octagonal_Shape_int32_t_get_congruences(+Handle, ?Congruence_-
System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the congruence system satisfied by the octagonal shape referenced by* `Handle`*.*

```
ppl_Octagonal_Shape_int32_t_get_minimized_constraints(+Handle,
?Constraint_System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the minimized constraint system satisfied by the octagonal shape referenced by* `Handle`*.*

```
ppl_Octagonal_Shape_int32_t_get_minimized_congruences(+Handle,
?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the minimized congruence system satisfied by the octagonal shape referenced by* `Handle`*.*

```
ppl_Octagonal_Shape_int32_t_maximize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if octagonal shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Octagonal_Shape_int32_t_minimize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if octagonal shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Octagonal_Shape_int32_t_maximize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if octagonal shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Octagonal_Shape_int32_t_minimize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if octagonal shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Octagonal_Shape_int32_t_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the octagonal shape referenced by* `Handle`*.*

```
ppl_Octagonal_Shape_int32_t_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the octagonal shape referenced by* `Handle`*.*

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Octagonal_Shape_int32_t_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the octagonal shape referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the octagonal shape

These predicates may modify the octagonal shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the octagonal shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Octagonal_Shape_int32_t_add_constraint(+Handle, +Constraint)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

```
ppl_Octagonal_Shape_int32_t_add_congruence(+Handle, +Congruence)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Octagonal_Shape_int32_t_add_constraints( +Handle, +Constraint_System)`

Updates the octagonal shape referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Octagonal_Shape_int32_t_add_congruences( +Handle, +Congruence_System)`

Updates the octagonal shape referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Octagonal_Shape_int32_t_refine_with_constraint( +Handle, +Constraint)`

Updates the octagonal shape referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Octagonal_Shape_int32_t_refine_with_congruence( +Handle, +Congruence)`

Updates the octagonal shape referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Octagonal_Shape_int32_t_refine_with_constraints( +Handle, +Constraint_System)`

Updates the octagonal shape referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Octagonal_Shape_int32_t_refine_with_congruences( +Handle, +Congruence_System)`

Updates the octagonal shape referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the octagonal shape

These predicates enable transformations such as taking the topological closure (which for the domain of octagonal shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Octagonal_Shape_int32_t_topological_closure_assign(+Handle)`

Assigns to the octagonal shape referenced by `Handle` its topological closure.

`ppl_Octagonal_Shape_int32_t_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the octagonal shape `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Octagonal_Shape_int32_t_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the octagonal shape `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Octagonal_Shape_int32_t_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the octagonal shape referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Octagonal_Shape_int32_t_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the octagonal shape referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Octagonal_Shape_int32_t_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $Lin_Expr_1/Coeff \leq PPL_Var \leq Lin_Expr_2/Coeff$.

`ppl_Octagonal_Shape_int32_t_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $Lin_Expr_1/Coeff \leq PPL_Var \leq Lin_Expr_2/Coeff$.

`ppl_Octagonal_Shape_int32_t_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $PPL_Var \bowtie Lin_Expr/Coeff$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_int32_t_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $PPL_Var \bowtie Lin_Expr/Coeff$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_int32_t_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $Lin_Expr_1 \bowtie Lin_Expr_2$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_int32_t_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $Lin_Expr_1 \bowtie Lin_Expr_2$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_int32_t_wrap_assign(+Handle, +List_of_PPL_Var, +Width, +Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)`

Transforms the octagonal shape `P` referenced by `Handle` by wrapping the dimensions given by `List_of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped

individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one octagonal shape

These predicates include the binary operators which will assign to the octagonal shape referred to by the first argument its combination with the octagonal shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Octagonal_Shape_int32_t_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the intersection of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_int32_t_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the upper bound of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_int32_t_difference_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the difference of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_int32_t_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the time elapse of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_int32_t_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the octagonal shape `P_1` referenced by `Handle_1` with the octagonal shape referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Octagonal_Shape_int32_t_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of octagonal shape `P_1` referenced by `Handle_1` and the octagonal shape `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

`ppl_Octagonal_Shape_int32_t_linear_partition(+Handle_1, +Handle_2, -Handle_3, -Handle_4)`

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Octagonal_Shape_int32_t domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the octagonal shape referred to by the second argument has to be contained in (or equal to) the octagonal shape referred to by the first argument.

```
ppl_Octagonal_Shape_int32_t_BHMZ05_widening_assign_with_-
tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_-unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_int32_t_BHMZ05_widening_assign(+Handle_1,
+Handle_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_int32_t_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_Octagonal_Shape_int32_t_H79_widening_assign_with_-tokens/4`

```
ppl_Octagonal_Shape_int32_t_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_Octagonal_Shape_int32_t_H79_widening_assign/2`

```
ppl_Octagonal_Shape_int32_t_limited_BHMZ05_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_-unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_int32_t_limited_CC76_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_-unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_int32_t_limited_BHMZ05_extrapolation_-
assign(+Handle_1, +Handle_2, +Constraint_System)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_Octagonal_Shape_int32_t_limited_CC76_extrapolation_-
assign(+Handle_1, +Handle_2, +Constraint_System)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_Octagonal_Shape_int32_t_CC76_narrowing_assign(+Handle_1, +Handle_-
2)
```

Assigns to the octagonal shape `P_1` *referenced by* `Handle_1` *the CC76-narrowing of* `P_1` *and the octagonal shape referenced by* `Handle_2`*.*

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Octagonal_Shape_int32_t_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the octagonal shape referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Octagonal_Shape_int32_t_add_space_dimensions_and_embed(+Handle,
+Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the octagonal shape* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Octagonal_Shape_int32_t_add_space_dimensions_and_project(+Handle,
+Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the octagonal shape* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

```
ppl_Octagonal_Shape_int32_t_remove_space_dimensions(+Handle, +List_-
of_PPL_Vars)
```

Removes from the vector space enclosing the octagonal shape `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Octagonal_Shape_int32_t_remove_higher_space_dimensions(+Handle,
+Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the octagonal shape `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`*.*

```
ppl_Octagonal_Shape_int32_t_expand_space_dimension(+Handle, +PPL_Var,
+Dimension_Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the octagonal shape referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

`ppl_Octagonal_Shape_int32_t_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the octagonal shape referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Octagonal_Shape_int32_t_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Octagonal_Shape_int64_t Domain

Here we provide a short description for each of the predicates available for the domain of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for octagonal shapes

The constructor predicates build a octagonal shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Octagonal_Shape_int64_t_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new octagonal shape $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Octagonal_Shape_int64_t_from_space_dimension(3, universe, X).
```

creates the octagonal shape defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Octagonal_Shape_int64_t_from_constraints(+Constraint_System, -Handle)`

Builds a new octagonal shape `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Octagonal_Shape_int64_t_from_congruences(+Congruence_System, -Handle)`

Builds a new octagonal shape `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Octagonal_Shape_int64_t_from_generators(+Generator_System, -Handle)`

Builds a new octagonal shape `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Octagonal_Shape_int64_t_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Grid(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_long_-
double(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Int8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Int16_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Int32_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Int64_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Uint8_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Uint16_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Uint32_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Uint64_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Z_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Rational_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Grid_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_int64_t_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Octagonal_Shape_int64_t domain.

`ppl_delete_Octagonal_Shape_int64_t(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the octagonal shape

Test Predicates

These predicates test the octagonal shape for different properties and succeed or fail depending on the outcome.

`ppl_Octagonal_Shape_int64_t_is_empty(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is empty.

`ppl_Octagonal_Shape_int64_t_is_universe(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is the universe.

`ppl_Octagonal_Shape_int64_t_is_bounded(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is bounded.

`ppl_Octagonal_Shape_int64_t_contains_integer_point(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` contains an integer point.

`ppl_Octagonal_Shape_int64_t_is_topologically_closed(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is topologically closed.

`ppl_Octagonal_Shape_int64_t_is_discrete(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is discrete.

`ppl_Octagonal_Shape_int64_t_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_int64_t_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_int64_t_contains_Octagonal_Shape_int64_t(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` is included in or equal to the octagonal shape referenced by `Handle_1`.

`ppl_Octagonal_Shape_int64_t_strictly_contains_Octagonal_Shape_int64_t(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` is included in but not equal to the octagonal shape referenced by `Handle_1`.

`ppl_Octagonal_Shape_int64_t_is_disjoint_from_Octagonal_Shape_int64_t(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` is disjoint from the octagonal shape referenced by `Handle_1`.

`ppl_Octagonal_Shape_int64_t_equals_Octagonal_Shape_int64_t(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_1` is equal to the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_int64_t_OK(+Handle)`

Succeeds only if the octagonal shape referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Octagonal_Shape_int64_t_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the octagonal shape referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the octagonal shape

These predicates will obtain more detailed information about the octagonal shape unifying some of their arguments with the results.

`ppl_Octagonal_Shape_int64_t_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the octagonal shape referenced by* `Handle`.

```
ppl_Octagonal_Shape_int64_t_affine_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` *with the affine dimension of the octagonal shape referenced by* `Handle`.

```
ppl_Octagonal_Shape_int64_t_relation_with_constraint(+Handle,
+Constraint, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the octagonal shape referenced by* `Handle` *has with* `Constraint`. *The possible relations are listed in the grammar rules above.*

```
ppl_Octagonal_Shape_int64_t_relation_with_generator(+Handle,
+Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the octagonal shape referenced by* `Handle` *has with* `Generator`. *The possible relations are listed in the grammar rules above.*

```
ppl_Octagonal_Shape_int64_t_relation_with_congruence(+Handle,
+Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the octagonal shape referenced by* `Handle` *has with* `Congruence`. *The possible relations are listed in the grammar rules above.*

```
ppl_Octagonal_Shape_int64_t_get_constraints(+Handle, ?Constraint_-
System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the constraint system satisfied by the octagonal shape referenced by* `Handle`.

```
ppl_Octagonal_Shape_int64_t_get_congruences(+Handle, ?Congruence_-
System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the congruence system satisfied by the octagonal shape referenced by* `Handle`.

```
ppl_Octagonal_Shape_int64_t_get_minimized_constraints(+Handle,
?Constraint_System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the minimized constraint system satisfied by the octagonal shape referenced by* `Handle`.

```
ppl_Octagonal_Shape_int64_t_get_minimized_congruences(+Handle,
?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the minimized congruence system satisfied by the octagonal shape referenced by* `Handle`.

```
ppl_Octagonal_Shape_int64_t_maximize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if octagonal shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Octagonal_Shape_int64_t_minimize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if octagonal shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Octagonal_Shape_int64_t_maximize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if octagonal shape P referenced by Handle is not empty and Lin_Expr is bounded from above in P.

Coeff_1 is unified with the numerator of the supremum value and Coeff_2 with the denominator of the supremum value and Point with a point or closure point where Lin_Expr reaches this value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Octagonal_Shape_int64_t_minimize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if octagonal shape P referenced by Handle is not empty and Lin_Expr is bounded from below in P.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value and Point with a point or closure point where Lin_Expr reaches this value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Octagonal_Shape_int64_t_external_memory_in_bytes(+Handle, ?Number)
```

Unifies Number with the size of the total memory in bytes occupied by the octagonal shape referenced by Handle.

```
ppl_Octagonal_Shape_int64_t_total_memory_in_bytes(+Handle, ?Number)
```

Unifies Number with the size of the external memory in bytes occupied by the octagonal shape referenced by Handle.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Octagonal_Shape_int64_t_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the octagonal shape referenced by Handle on the standard output.

Space-dimension preserving predicates that may change the octagonal shape

These predicates may modify the octagonal shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the octagonal shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Octagonal_Shape_int64_t_add_constraint(+Handle, +Constraint)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

```
ppl_Octagonal_Shape_int64_t_add_congruence(+Handle, +Congruence)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

```
ppl_Octagonal_Shape_int64_t_add_constraints( +Handle, +Constraint_-
System)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

```
ppl_Octagonal_Shape_int64_t_add_congruences( +Handle, +Congruence_-
System)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

```
ppl_Octagonal_Shape_int64_t_refine_with_constraint( +Handle,
+Constraint)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

```
ppl_Octagonal_Shape_int64_t_refine_with_congruence( +Handle,
+Congruence)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

```
ppl_Octagonal_Shape_int64_t_refine_with_constraints( +Handle,
+Constraint_System)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

```
ppl_Octagonal_Shape_int64_t_refine_with_congruences( +Handle,
+Congruence_System)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the octagonal shape

These predicates enable transformations such as taking the topological closure (which for the domain of octagonal shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Octagonal_Shape_int64_t_topological_closure_assign(+Handle)
```

Assigns to the octagonal shape referenced by `Handle` *its topological closure.*

```
ppl_Octagonal_Shape_int64_t_unconstrain_space_dimension(+Handle,
+PPL_Var)
```

Modifies the octagonal shape `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

`ppl_Octagonal_Shape_int64_t_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the octagonal shape `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Octagonal_Shape_int64_t_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the octagonal shape referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Octagonal_Shape_int64_t_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the octagonal shape referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Octagonal_Shape_int64_t_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Octagonal_Shape_int64_t_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Octagonal_Shape_int64_t_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_int64_t_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_int64_t_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_int64_t_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_int64_t_wrap_assign(+Handle, +List_of_PPL_Var, +Width, +Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)`

Transforms the octagonal shape `P` *referenced by* `Handle` *by wrapping the dimensions given by* `List_of_PPL_Vars` *while respecting the specified* `Width`, `Representation` *and* `Overflow` *behavior of all these variables. The* `Constraint_System` *represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer* `C_unsigned` *and* `Boolean` *allow control of the complexity/precision ratio; higher values for* `C_unsigned` *will lead to possibly greater precision while a true value for* `Boolean` *indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.*

Predicates whose results depend on more than one octagonal shape

These predicates include the binary operators which will assign to the octagonal shape referred to by the first argument its combination with the octagonal shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Octagonal_Shape_int64_t_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P` *referenced by* `Handle_1` *the intersection of* `P` *and the octagonal shape referenced by* `Handle_2`.

```
ppl_Octagonal_Shape_int64_t_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the octagonal shape referenced by* `Handle_2`.

```
ppl_Octagonal_Shape_int64_t_difference_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P` *referenced by* `Handle_1` *the difference of* `P` *and the octagonal shape referenced by* `Handle_2`.

```
ppl_Octagonal_Shape_int64_t_time_elapse_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the octagonal shape referenced by* `Handle_2`.

```
ppl_Octagonal_Shape_int64_t_upper_bound_assign_if_exact(+Handle_1,
+Handle_2)
```

Succeeds if the least upper bound of the octagonal shape `P_1` *referenced by* `Handle_1` *with the octagonal shape referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Octagonal_Shape_int64_t_simplify_using_context_assign(+Handle_1,
+Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of octagonal shape `P_1` *referenced by* `Handle_1` *and the octagonal shape* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`.

```
ppl_Octagonal_Shape_int64_t_linear_partition(+Handle_1, +Handle_2,
-Handle_3, -Handle_4)
```

`Handle_1` *and* `Handle_2` *are handles for elements* `P_1` *and* `P_2` *in the Octagonal_Shape_int64_t domain. The predicate unifies handle* `Handle_3` *to a reference to the intersection of* `P_1` *and* `P_2` *and* `Handle_4` *to a reference to a pointset powerset of nnc polyhedra* `P_4`*; where* `P_4` *is the linear partition of* `P_1` *with respect to* `P_2`*. This predicate is only provided if the class* `Pointset_Powerset_NNC_Polyhedron` *has been enabled when configuring the library.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the octagonal shape referred to by the second argument has to be contained in (or equal to) the octagonal shape referred to by the first argument.

```
ppl_Octagonal_Shape_int64_t_BHMZ05_widening_assign_with_-
tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_-unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_int64_t_BHMZ05_widening_assign(+Handle_1,
+Handle_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_int64_t_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_Octagonal_Shape_int64_t_H79_widening_assign_with_-tokens/4`

```
ppl_Octagonal_Shape_int64_t_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_Octagonal_Shape_int64_t_H79_widening_assign/2`

```
ppl_Octagonal_Shape_int64_t_limited_BHMZ05_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_-unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_int64_t_limited_CC76_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_-unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_int64_t_limited_BHMZ05_extrapolation_-
assign(+Handle_1, +Handle_2, +Constraint_System)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_Octagonal_Shape_int64_t_limited_CC76_extrapolation_-
assign(+Handle_1, +Handle_2, +Constraint_System)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_Octagonal_Shape_int64_t_CC76_narrowing_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the CC76-narrowing of `P_1` and the octagonal shape referenced by `Handle_2`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Octagonal_Shape_int64_t_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P` referenced by `Handle_1` the concatenation of `P` and the octagonal shape referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Octagonal_Shape_int64_t_add_space_dimensions_and_embed(+Handle,
+Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the octagonal shape `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Octagonal_Shape_int64_t_add_space_dimensions_and_project(+Handle,
+Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the octagonal shape `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Octagonal_Shape_int64_t_remove_space_dimensions(+Handle, +List_
of_PPL_Vars)
```

Removes from the vector space enclosing the octagonal shape `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Octagonal_Shape_int64_t_remove_higher_space_dimensions(+Handle,
+Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the octagonal shape `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Octagonal_Shape_int64_t_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the octagonal shape referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Octagonal_Shape_int64_t_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the octagonal shape referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Octagonal_Shape_int64_t_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Octagonal_Shape_mpz_class Domain

Here we provide a short description for each of the predicates available for the domain of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for octagonal shapes

The constructor predicates build a octagonal shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Octagonal_Shape_mpz_class_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new octagonal shape $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Octagonal_Shape_mpz_class_from_space_dimension(3, universe, X).
```

creates the octagonal shape defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Octagonal_Shape_mpz_class_from_constraints(+Constraint_System, -Handle)`

Builds a new octagonal shape `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Octagonal_Shape_mpz_class_from_congruences(+Congruence_System, -Handle)`

Builds a new octagonal shape `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Octagonal_Shape_mpz_class_from_generators(+Generator_System, -Handle)`

Builds a new octagonal shape `P` *from* `Generator_System`*.* `Handle` *is unified with the handle for* `P`*.*

Predicates that build new octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Octagonal_Shape_mpz_class_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` *from the int8_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Octagonal_Shape_mpz_class_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` *from the int16_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Octagonal_Shape_mpz_class_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` *from the int32_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Octagonal_Shape_mpz_class_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` *from the int64_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Octagonal_Shape_mpz_class_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` *from the uint8_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Octagonal_Shape_mpz_class_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` *from the uint16_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Octagonal_Shape_mpz_class_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` *from the uint32_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Octagonal_Shape_mpz_class_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` *from the uint64_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Octagonal_Shape_mpz_class_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Rational_Box(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_int8_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_int16_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_int32_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_int64_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_mpz_class(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_mpq_class(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_int8_-
t(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_int16_-
t(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_int32_-
t(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_int64_-
t(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_mpz_-
class(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_-1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_mpq_-
class(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_-1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_C_Polyhedron(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_NNC_Polyhedron(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Grid(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Float_Box(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_float(+Handle_-
1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Double_Box(+Handle_1, -Handle_-
2)
```

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_-
double(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Long_Double_Box(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_long_-
double(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Int8_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Int16_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Int32_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Int64_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Uint8_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Uint16_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Uint32_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Uint64_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Z_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Rational_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_mpz_class_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_mpz_class_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_mpz_class_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_mpz_class_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_float_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Double_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Long_Double_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_BD_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpz_class_from_Octagonal_Shape_long_double_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Octagonal_Shape_mpz_class domain.

```
ppl_delete_Octagonal_Shape_mpz_class(+Handle)
```

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the octagonal shape

Test Predicates

These predicates test the octagonal shape for different properties and succeed or fail depending on the outcome.

`ppl_Octagonal_Shape_mpz_class_is_empty(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is empty.

`ppl_Octagonal_Shape_mpz_class_is_universe(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is the universe.

`ppl_Octagonal_Shape_mpz_class_is_bounded(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is bounded.

`ppl_Octagonal_Shape_mpz_class_contains_integer_point(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` contains an integer point.

`ppl_Octagonal_Shape_mpz_class_is_topologically_closed(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is topologically closed.

`ppl_Octagonal_Shape_mpz_class_is_discrete(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is discrete.

`ppl_Octagonal_Shape_mpz_class_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_mpz_class_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_mpz_class_contains_Octagonal_Shape_mpz_class(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` is included in or equal to the octagonal shape referenced by `Handle_1`.

`ppl_Octagonal_Shape_mpz_class_strictly_contains_Octagonal_Shape_mpz_class(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` is included in but not equal to the octagonal shape referenced by `Handle_1`.

`ppl_Octagonal_Shape_mpz_class_is_disjoint_from_Octagonal_Shape_mpz_class(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` is disjoint from the octagonal shape referenced by `Handle_1`.

`ppl_Octagonal_Shape_mpz_class_equals_Octagonal_Shape_mpz_class(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_1` is equal to the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_mpz_class_OK(+Handle)`

Succeeds only if the octagonal shape referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Octagonal_Shape_mpz_class_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the octagonal shape referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the octagonal shape

These predicates will obtain more detailed information about the octagonal shape unifying some of their arguments with the results.

`ppl_Octagonal_Shape_mpz_class_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_mpz_class_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_mpz_class_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the octagonal shape referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Octagonal_Shape_mpz_class_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the octagonal shape referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Octagonal_Shape_mpz_class_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the octagonal shape referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Octagonal_Shape_mpz_class_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_mpz_class_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_mpz_class_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_mpz_class_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_mpz_class_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Octagonal_Shape_mpz_class_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Octagonal_Shape_mpz_class_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Octagonal_Shape_mpz_class_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Octagonal_Shape_mpz_class_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the total memory in bytes occupied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_mpz_class_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the octagonal shape referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Octagonal_Shape_mpz_class_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the octagonal shape referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the octagonal shape

These predicates may modify the octagonal shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the octagonal shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Octagonal_Shape_mpz_class_add_constraint(+Handle, +Constraint)`

Updates the octagonal shape referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Octagonal_Shape_mpz_class_add_congruence(+Handle, +Congruence)`

Updates the octagonal shape referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Octagonal_Shape_mpz_class_add_constraints( +Handle, +Constraint_System)`

Updates the octagonal shape referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Octagonal_Shape_mpz_class_add_congruences( +Handle, +Congruence_System)`

Updates the octagonal shape referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Octagonal_Shape_mpz_class_refine_with_constraint( +Handle, +Constraint)`

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Octagonal_Shape_mpz_class_refine_with_congruence( +Handle, +Congruence)`

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Octagonal_Shape_mpz_class_refine_with_constraints( +Handle, +Constraint_System)`

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Octagonal_Shape_mpz_class_refine_with_congruences( +Handle, +Congruence_System)`

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the octagonal shape

These predicates enable transformations such as taking the topological closure (which for the domain of octagonal shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Octagonal_Shape_mpz_class_topological_closure_assign(+Handle)`

Assigns to the octagonal shape referenced by `Handle` *its topological closure.*

`ppl_Octagonal_Shape_mpz_class_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the octagonal shape `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

`ppl_Octagonal_Shape_mpz_class_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the octagonal shape `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Octagonal_Shape_mpz_class_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the octagonal shape referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

`ppl_Octagonal_Shape_mpz_class_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the octagonal shape referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

`ppl_Octagonal_Shape_mpz_class_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Octagonal_Shape_mpz_class_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Octagonal_Shape_mpz_class_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Octagonal_Shape_mpz_class_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Octagonal_Shape_mpz_class_generalized_affine_image_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to octagonal shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Octagonal_Shape_mpz_class_generalized_affine_preimage_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to octagonal shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Octagonal_Shape_mpz_class_wrap_assign(+Handle, +List_of_PPL_Var,
+Width, +Representation, +Overflow, +Constraint_System, +C_unsigned,
+Boolean)
```

Transforms the octagonal shape `P` referenced by `Handle` by wrapping the dimensions given by `List_-of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one octagonal shape

These predicates include the binary operators which will assign to the octagonal shape referred to by the first argument its combination with the octagonal shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Octagonal_Shape_mpz_class_intersection_assign(+Handle_1, +Handle_-
2)
```

Assigns to the octagonal shape `P` referenced by `Handle_1` the intersection of `P` and the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_mpz_class_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P` referenced by `Handle_1` the upper bound of `P` and the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_mpz_class_difference_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P` referenced by `Handle_1` the difference of `P` and the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_mpz_class_time_elapse_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P` referenced by `Handle_1` the time elapse of `P` and the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_mpz_class_upper_bound_assign_if_exact(+Handle_1,
+Handle_2)
```

Succeeds if the least upper bound of the octagonal shape `P_1` referenced by `Handle_1` with the octagonal shape referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_Octagonal_Shape_mpz_class_simplify_using_context_assign(+Handle_1,
+Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of octagonal shape `P_1` referenced by `Handle_1` and the octagonal shape `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

```
ppl_Octagonal_Shape_mpz_class_linear_partition(+Handle_1, +Handle_2,
-Handle_3, -Handle_4)
```

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Octagonal_Shape_mpz_class domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the octagonal shape referred to by the second argument has to be contained in (or equal to) the octagonal shape referred to by the first argument.

```
ppl_Octagonal_Shape_mpz_class_BHMZ05_widening_assign_with_
tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_mpz_class_BHMZ05_widening_assign(+Handle_1,
+Handle_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_mpz_class_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_Octagonal_Shape_mpz_class_H79_widening_assign_with_tokens/4`

```
ppl_Octagonal_Shape_mpz_class_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_Octagonal_Shape_mpz_class_H79_widening_assign/2`

```
ppl_Octagonal_Shape_mpz_class_limited_BHMZ05_extrapolation_assign_
with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1,
?C_unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_mpz_class_limited_CC76_extrapolation_assign_with_
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_
```

unsigned_2)

Assigns to the octagonal shape `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the octagonal shape referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*. The widening with tokens delay technique is applied with* `C_unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Octagonal_Shape_mpz_class_limited_BHMZ05_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)
```

Assigns to the octagonal shape `P_1` *referenced by* `Handle_1` *the BHMZ05-widening of* `P_1` *with the octagonal shape referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*.*

```
ppl_Octagonal_Shape_mpz_class_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)
```

Assigns to the octagonal shape `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the octagonal shape referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*.*

```
ppl_Octagonal_Shape_mpz_class_CC76_narrowing_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P_1` *referenced by* `Handle_1` *the CC76-narrowing of* `P_1` *and the octagonal shape referenced by* `Handle_2`*.*

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Octagonal_Shape_mpz_class_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the octagonal shape referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Octagonal_Shape_mpz_class_add_space_dimensions_and_embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the octagonal shape* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

`ppl_Octagonal_Shape_mpz_class_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the octagonal shape `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Octagonal_Shape_mpz_class_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the octagonal shape `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Octagonal_Shape_mpz_class_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the octagonal shape `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Octagonal_Shape_mpz_class_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the octagonal shape referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Octagonal_Shape_mpz_class_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the octagonal shape referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Octagonal_Shape_mpz_class_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Octagonal_Shape_mpq_class Domain

Here we provide a short description for each of the predicates available for the domain of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for octagonal shapes

The constructor predicates build a octagonal shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Octagonal_Shape_mpq_class_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new octagonal shape $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Octagonal_Shape_mpq_class_from_space_dimension(3, universe, X).
```

creates the octagonal shape defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Octagonal_Shape_mpq_class_from_constraints(+Constraint_System, -Handle)`

Builds a new octagonal shape `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Octagonal_Shape_mpq_class_from_congruences(+Congruence_System, -Handle)`

Builds a new octagonal shape `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Octagonal_Shape_mpq_class_from_generators(+Generator_System, -Handle)`

Builds a new octagonal shape `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Octagonal_Shape_mpq_class_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_mpq_class_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_mpq_class_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_mpq_class_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_mpq_class_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_mpq_class_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Uint32_Box(+Handle_1, -Handle_-
2)
```

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Uint64_Box(+Handle_1, -Handle_-
2)
```

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Z_Box(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Rational_Box(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_int8_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_int16_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_int32_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_int64_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_mpz_class(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_mpq_class(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_int8_-
t(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_int16_-
t(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_int32_-
t(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_int64_-
t(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_mpz_-
class(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_-1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_mpq_-
class(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_-1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_C_Polyhedron(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_NNC_Polyhedron(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Grid(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Float_Box(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_float(+Handle_-
1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Double_Box(+Handle_1, -Handle_-
2)
```

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_-
double(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Long_Double_Box(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_long_-
double(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Int8_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Int16_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Int32_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Int64_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Uint8_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Uint16_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Uint32_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Uint64_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Z_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Rational_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_mpz_class_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_mpq_class_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_C_Polyhedron_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_NNC_Polyhedron_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Grid_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Float_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_float_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_float_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Double_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_Long_Double_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_mpq_class_from_BD_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_mpq_class_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Octagonal_Shape_mpq_class domain.

`ppl_delete_Octagonal_Shape_mpq_class(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the octagonal shape

Test Predicates

These predicates test the octagonal shape for different properties and succeed or fail depending on the outcome.

`ppl_Octagonal_Shape_mpq_class_is_empty(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is empty.

`ppl_Octagonal_Shape_mpq_class_is_universe(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is the universe.

`ppl_Octagonal_Shape_mpq_class_is_bounded(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is bounded.

`ppl_Octagonal_Shape_mpq_class_contains_integer_point(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` contains an integer point.

`ppl_Octagonal_Shape_mpq_class_is_topologically_closed(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is topologically closed.

`ppl_Octagonal_Shape_mpq_class_is_discrete(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` is discrete.

`ppl_Octagonal_Shape_mpq_class_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_mpq_class_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_mpq_class_contains_Octagonal_Shape_mpq_class(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` is included in or equal to the octagonal shape referenced by `Handle_1`.

`ppl_Octagonal_Shape_mpq_class_strictly_contains_Octagonal_Shape_mpq_class(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` is included in but not equal to the octagonal shape referenced by `Handle_1`.

```
ppl_Octagonal_Shape_mpq_class_is_disjoint_from_Octagonal_Shape_mpq_-
class(+Handle_1, +Handle_2)
```

Succeeds if and only if the octagonal shape referenced by `Handle_2` is disjoint from the octagonal shape referenced by `Handle_1`.

```
ppl_Octagonal_Shape_mpq_class_equals_Octagonal_Shape_mpq_-
class(+Handle_1, +Handle_2)
```

Succeeds if and only if the octagonal shape referenced by `Handle_1` is equal to the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_mpq_class_OK(+Handle)
```

Succeeds only if the octagonal shape referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

```
ppl_Octagonal_Shape_mpq_class_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the octagonal shape referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the octagonal shape

These predicates will obtain more detailed information about the octagonal shape unifying some of their arguments with the results.

```
ppl_Octagonal_Shape_mpq_class_space_dimension(+Handle, ?Dimension_-
Type)
```

Unifies `Dimension_Type` with the dimension of the vector space enclosing the octagonal shape referenced by `Handle`.

```
ppl_Octagonal_Shape_mpq_class_affine_dimension(+Handle, ?Dimension_-
Type)
```

Unifies `Dimension_Type` with the affine dimension of the octagonal shape referenced by `Handle`.

```
ppl_Octagonal_Shape_mpq_class_relation_with_constraint(+Handle,
+Constraint, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the octagonal shape referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

```
ppl_Octagonal_Shape_mpq_class_relation_with_generator(+Handle,
+Generator, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the octagonal shape referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

```
ppl_Octagonal_Shape_mpq_class_relation_with_congruence(+Handle,
+Congruence, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the octagonal shape referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

```
ppl_Octagonal_Shape_mpq_class_get_constraints(+Handle, ?Constraint_-
System)
```

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_mpq_class_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_mpq_class_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_mpq_class_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_mpq_class_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Octagonal_Shape_mpq_class_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Octagonal_Shape_mpq_class_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Octagonal_Shape_mpq_class_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Octagonal_Shape_mpq_class_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the total memory in bytes occupied by the octagonal shape referenced by* `Handle`*.*

```
ppl_Octagonal_Shape_mpq_class_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the octagonal shape referenced by* `Handle`*.*

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Octagonal_Shape_mpq_class_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the octagonal shape referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the octagonal shape

These predicates may modify the octagonal shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the octagonal shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Octagonal_Shape_mpq_class_add_constraint(+Handle, +Constraint)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

```
ppl_Octagonal_Shape_mpq_class_add_congruence(+Handle, +Congruence)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

```
ppl_Octagonal_Shape_mpq_class_add_constraints( +Handle, +Constraint_-
System)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

```
ppl_Octagonal_Shape_mpq_class_add_congruences( +Handle, +Congruence_-
System)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

```
ppl_Octagonal_Shape_mpq_class_refine_with_constraint( +Handle,
+Constraint)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

```
ppl_Octagonal_Shape_mpq_class_refine_with_congruence( +Handle,
+Congruence)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`.

```
ppl_Octagonal_Shape_mpq_class_refine_with_constraints( +Handle,
+Constraint_System)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`.

```
ppl_Octagonal_Shape_mpq_class_refine_with_congruences( +Handle,
+Congruence_System)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`.

Predicates that transform the octagonal shape

These predicates enable transformations such as taking the topological closure (which for the domain of octagonal shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Octagonal_Shape_mpq_class_topological_closure_assign(+Handle)
```

Assigns to the octagonal shape referenced by `Handle` *its topological closure.*

```
ppl_Octagonal_Shape_mpq_class_unconstrain_space_dimension(+Handle,
+PPL_Var)
```

Modifies the octagonal shape `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`.

```
ppl_Octagonal_Shape_mpq_class_unconstrain_space_dimensions(+Handle,
+List_of_PPL_Var)
```

Modifies the octagonal shape `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Octagonal_Shape_mpq_class_affine_image(+Handle, +PPL_Var, +Lin_-
Expr, +Coeff)
```

Transforms the octagonal shape referenced by `Handle` *assigning the affine expression for* `Lin_-Expr/Coeff` *to* `PPL_Var`.

```
ppl_Octagonal_Shape_mpq_class_affine_preimage(+Handle, +PPL_Var,
+Lin_Expr, +Coeff)
```

Transforms the octagonal shape referenced by `Handle` *substituting the affine expression for* `Lin_-Expr/Coeff` *to* `PPL_Var`.

```
ppl_Octagonal_Shape_mpq_class_bounded_affine_image(+Handle, +PPL_Var,
+Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Octagonal_Shape_mpq_class_bounded_affine_preimage(+Handle, +PPL_-
Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Octagonal_Shape_mpq_class_generalized_affine_image(+Handle, +PPL_-
Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to octagonal shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `PPL_Var` ⋈ `Lin_Expr/Coeff`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Octagonal_Shape_mpq_class_generalized_affine_preimage(+Handle,
+PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to octagonal shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` ⋈ `Lin_Expr/Coeff`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Octagonal_Shape_mpq_class_generalized_affine_image_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to octagonal shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Octagonal_Shape_mpq_class_generalized_affine_preimage_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to octagonal shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Octagonal_Shape_mpq_class_wrap_assign(+Handle, +List_of_PPL_Var,
+Width, +Representation, +Overflow, +Constraint_System, +C_unsigned,
+Boolean)
```

Transforms the octagonal shape `P` referenced by `Handle` by wrapping the dimensions given by `List_-of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one octagonal shape

These predicates include the binary operators which will assign to the octagonal shape referred to by the first argument its combination with the octagonal shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Octagonal_Shape_mpq_class_intersection_assign(+Handle_1, +Handle_-
2)
```

Assigns to the octagonal shape `P` referenced by `Handle_1` the intersection of `P` and the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_mpq_class_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P` referenced by `Handle_1` the upper bound of `P` and the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_mpq_class_difference_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P` referenced by `Handle_1` the difference of `P` and the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_mpq_class_time_elapse_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P` referenced by `Handle_1` the time elapse of `P` and the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_mpq_class_upper_bound_assign_if_exact(+Handle_1,
+Handle_2)
```

Succeeds if the least upper bound of the octagonal shape `P_1` referenced by `Handle_1` with the octagonal shape referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_Octagonal_Shape_mpq_class_simplify_using_context_assign(+Handle_1,
+Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of octagonal shape `P_1` referenced by `Handle_1` and the octagonal shape `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

```
ppl_Octagonal_Shape_mpq_class_linear_partition(+Handle_1, +Handle_2,
-Handle_3, -Handle_4)
```

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Octagonal_Shape_mpq_class domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the octagonal shape referred to by the second argument has to be contained in (or equal to) the octagonal shape referred to by the first argument.

```
ppl_Octagonal_Shape_mpq_class_BHMZ05_widening_assign_with_
tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_mpq_class_BHMZ05_widening_assign(+Handle_1,
+Handle_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_mpq_class_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_Octagonal_Shape_mpq_class_H79_widening_assign_with_tokens/4`

```
ppl_Octagonal_Shape_mpq_class_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_Octagonal_Shape_mpq_class_H79_widening_assign/2`

```
ppl_Octagonal_Shape_mpq_class_limited_BHMZ05_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the octagonal shape `P_1` *referenced by* `Handle_1` *the BHMZ05-widening of* `P_1` *with the octagonal shape referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*. The widening with tokens delay technique is applied with* `C_unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Octagonal_Shape_mpq_class_limited_CC76_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the octagonal shape `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the octagonal shape referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*. The widening with tokens delay technique is applied with* `C_unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Octagonal_Shape_mpq_class_limited_BHMZ05_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)
```

Assigns to the octagonal shape `P_1` *referenced by* `Handle_1` *the BHMZ05-widening of* `P_1` *with the octagonal shape referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*.*

```
ppl_Octagonal_Shape_mpq_class_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)
```

Assigns to the octagonal shape `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the octagonal shape referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*.*

```
ppl_Octagonal_Shape_mpq_class_CC76_narrowing_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P_1` *referenced by* `Handle_1` *the CC76-narrowing of* `P_1` *and the octagonal shape referenced by* `Handle_2`*.*

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Octagonal_Shape_mpq_class_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the octagonal shape referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Octagonal_Shape_mpq_class_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the octagonal shape `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Octagonal_Shape_mpq_class_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the octagonal shape `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Octagonal_Shape_mpq_class_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the octagonal shape `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Octagonal_Shape_mpq_class_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the octagonal shape `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Octagonal_Shape_mpq_class_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the octagonal shape referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Octagonal_Shape_mpq_class_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the octagonal shape referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Octagonal_Shape_mpq_class_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Polyhedron Domain

Here we provide a short description for each of the predicates available for the domain of polyhedra.

Constructor, copy, conversion and destructor predicates

Constructor predicates for polyhedra

The constructor predicates build a polyhedron from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs. For example, a C Polyhedron can be built from a list of non-strict inequality or equality constraints or a list of equality congruences or a list of generators that contains no closure points.

`ppl_new_C_Polyhedron_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new C polyhedron $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_C_Polyhedron_from_space_dimension(3, universe, X).
```

creates the C polyhedron defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_NNC_Polyhedron_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new NNC polyhedron $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_NNC_Polyhedron_from_space_dimension(3, universe, X).
```

creates the NNC polyhedron defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_C_Polyhedron_from_constraints(+Constraint_System, -Handle)`

Builds a new C polyhedron `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_NNC_Polyhedron_from_constraints(+Constraint_System, -Handle)`

Builds a new NNC polyhedron `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_C_Polyhedron_from_congruences(+Congruence_System, -Handle)`

Builds a new C polyhedron `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_NNC_Polyhedron_from_congruences(+Congruence_System, -Handle)`

Builds a new NNC polyhedron `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_C_Polyhedron_from_generators(+Generator_System, -Handle)`

Builds a new C polyhedron `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

`ppl_new_NNC_Polyhedron_from_generators(+Generator_System, -Handle)`

Builds a new NNC polyhedron `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new polyhedra by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class (e.g., ppl_new_Grid_from_C_Polyhedron, ppl_new_C_Polyhedron_from_BD_Shape_mpq_class, etc.).

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_C_Polyhedron_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Grid(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Grid(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new NNC polyhedron `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new C polyhedron `P_1` *from the octagonal_shape_double referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_NNC_Polyhedron_from_Octagonal_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new NNC polyhedron `P_1` *from the octagonal_shape_double referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_C_Polyhedron_from_Long_Double_Box(+Handle_1, -Handle_2)
```

Builds a new C polyhedron `P_1` *from the long_double_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_NNC_Polyhedron_from_Long_Double_Box(+Handle_1, -Handle_2)
```

Builds a new NNC polyhedron `P_1` *from the long_double_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_C_Polyhedron_from_BD_Shape_long_double(+Handle_1, -Handle_2)
```

Builds a new C polyhedron `P_1` *from the bd_shape_long_double referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_NNC_Polyhedron_from_BD_Shape_long_double(+Handle_1, -Handle_2)
```

Builds a new NNC polyhedron `P_1` *from the bd_shape_long_double referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_C_Polyhedron_from_Octagonal_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new C polyhedron `P_1` *from the octagonal_shape_long_double referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_NNC_Polyhedron_from_Octagonal_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new NNC polyhedron `P_1` *from the octagonal_shape_long_double referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_C_Polyhedron_from_Int8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new C polyhedron `P_1` *from the int8_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`; `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_NNC_Polyhedron_from_Int8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` *from the int8_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`; `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_C_Polyhedron_from_Int16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new C polyhedron `P_1` *from the int16_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`; `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_NNC_Polyhedron_from_Int16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` *from the int16_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`; `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_C_Polyhedron_from_Int32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Int32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_Int64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Int64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_Uint8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Uint8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_Uint16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Uint16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_Uint32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Uint32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_Uint64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Uint64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_Z_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new C polyhedron `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Z_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_Rational_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Rational_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_BD_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_BD_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_BD_Shape_int16_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_BD_Shape_int16_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_BD_Shape_int32_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_BD_Shape_int32_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_BD_Shape_int64_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_BD_Shape_int64_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_BD_Shape_mpz_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_BD_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_BD_Shape_mpq_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_BD_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_Octagonal_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Octagonal_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_Grid_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Grid_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new NNC polyhedron `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new C polyhedron `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new NNC polyhedron `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new C polyhedron `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new NNC polyhedron `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new C polyhedron `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new NNC polyhedron `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new C polyhedron `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_NNC_Polyhedron_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new NNC polyhedron `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_C_Polyhedron_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new C polyhedron `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Long_Double_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_BD_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_BD_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_C_Polyhedron_from_Octagonal_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new C polyhedron `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_NNC_Polyhedron_from_Octagonal_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new NNC polyhedron `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Polyhedron domain.

```
ppl_delete_Polyhedron(+Handle)
```

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the polyhedron

Test Predicates

These predicates test the polyhedron for different properties and succeed or fail depending on the outcome.

```
ppl_Polyhedron_is_empty(+Handle)
```

Succeeds if and only if the polyhedron referenced by `Handle` is empty.

```
ppl_Polyhedron_is_universe(+Handle)
```

Succeeds if and only if the polyhedron referenced by `Handle` is the universe.

`ppl_Polyhedron_is_bounded(+Handle)`

Succeeds if and only if the polyhedron referenced by `Handle` is bounded.

`ppl_Polyhedron_contains_integer_point(+Handle)`

Succeeds if and only if the polyhedron referenced by `Handle` contains an integer point.

`ppl_Polyhedron_is_topologically_closed(+Handle)`

Succeeds if and only if the polyhedron referenced by `Handle` is topologically closed.

`ppl_Polyhedron_is_discrete(+Handle)`

Succeeds if and only if the polyhedron referenced by `Handle` is discrete.

`ppl_Polyhedron_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the polyhedron referenced by `Handle`.

`ppl_Polyhedron_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the polyhedron referenced by `Handle`.

`ppl_Polyhedron_contains_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the polyhedron referenced by `Handle_2` is included in or equal to the polyhedron referenced by `Handle_1`.

`ppl_Polyhedron_strictly_contains_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the polyhedron referenced by `Handle_2` is included in but not equal to the polyhedron referenced by `Handle_1`.

`ppl_Polyhedron_is_disjoint_from_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the polyhedron referenced by `Handle_2` is disjoint from the polyhedron referenced by `Handle_1`.

`ppl_Polyhedron_equals_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the polyhedron referenced by `Handle_1` is equal to the polyhedron referenced by `Handle_2`.

`ppl_Polyhedron_OK(+Handle)`

Succeeds only if the polyhedron referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Polyhedron_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the polyhedron referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the polyhedron

These predicates will obtain more detailed information about the polyhedron unifying some of their arguments with the results.

`ppl_Polyhedron_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the polyhedron referenced by `Handle`.

`ppl_Polyhedron_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the polyhedron referenced by `Handle`.

`ppl_Polyhedron_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the polyhedron referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Polyhedron_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the polyhedron referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Polyhedron_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the polyhedron referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Polyhedron_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the polyhedron referenced by `Handle`.

`ppl_Polyhedron_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the polyhedron referenced by `Handle`.

`ppl_Polyhedron_get_generators(+Handle, ?Generator_System)`

Unifies `Generator_System` with the generators (in the form of a list) in the generator system for the polyhedron referenced by `Handle`.

`ppl_Polyhedron_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the polyhedron referenced by `Handle`.

`ppl_Polyhedron_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system for the polyhedron referenced by `Handle`.

`ppl_Polyhedron_get_minimized_generators(+Handle, ?Generator_System)`

Unifies `Generator_System` with the generators (in the form of a list) in the minimized generator system satisfied by the polyhedron referenced by `Handle`.

`ppl_Polyhedron_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if polyhedron `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Polyhedron_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if polyhedron `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise,

unified with the atom `false`.

```
ppl_Polyhedron_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if polyhedron `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Polyhedron_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if polyhedron `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Polyhedron_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the polyhedron referenced by* `Handle`.

```
ppl_Polyhedron_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the polyhedron referenced by* `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Polyhedron_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the polyhedron referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the polyhedron

These predicates may modify the polyhedron referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the polyhedron by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Polyhedron_add_constraint(+Handle, +Constraint)
```

Updates the polyhedron referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Polyhedron_add_congruence(+Handle, +Congruence)`

Updates the polyhedron referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Polyhedron_add_generator(+Handle, +Generator)`

Updates the polyhedron referenced by `Handle` to one obtained by adding `Generator` to its generator system. For a C polyhedron, `Generator` must be a line, ray or point.

`ppl_Polyhedron_add_constraints( +Handle, +Constraint_System)`

Updates the polyhedron referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Polyhedron_add_congruences( +Handle, +Congruence_System)`

Updates the polyhedron referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Polyhedron_add_generators( +Handle, +Generator_System)`

Updates the polyhedron referenced by `Handle` to one obtained by adding to its generator system the generators in `Generator_System`. For a C polyhedron, `Generators` must be a list of lines, rays and points.

`ppl_Polyhedron_refine_with_constraint( +Handle, +Constraint)`

Updates the polyhedron referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Polyhedron_refine_with_congruence( +Handle, +Congruence)`

Updates the polyhedron referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Polyhedron_refine_with_constraints( +Handle, +Constraint_System)`

Updates the polyhedron referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Polyhedron_refine_with_congruences( +Handle, +Congruence_System)`

Updates the polyhedron referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the polyhedron

These predicates enable transformations such as taking the topological closure (which for the domain of polyhedron is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Polyhedron_topological_closure_assign(+Handle)`

Assigns to the polyhedron referenced by `Handle` its topological closure.

`ppl_Polyhedron_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the polyhedron `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Polyhedron_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the polyhedron `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Polyhedron_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the polyhedron referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_Polyhedron_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the polyhedron referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_Polyhedron_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1,
+Lin_Expr_2, +Coeff)
```

Assigns to polyhedron `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_Polyhedron_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1,
+Lin_Expr_2, +Coeff)
```

Assigns to polyhedron `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_Polyhedron_generalized_affine_image(+Handle, +PPL_Var, +Relation_
Symbol, +Lin_Expr, +Coeff)
```

Assigns to polyhedron `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Polyhedron_generalized_affine_preimage(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to polyhedron `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Polyhedron_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1,
+Relation_Symbol, +Lin_Expr_2)
```

Assigns to polyhedron `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Polyhedron_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_
Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to polyhedron `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Polyhedron_wrap_assign(+Handle, +List_of_PPL_Var, +Width,
+Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)
```

Transforms the polyhedron `P` *referenced by* `Handle` *by wrapping the dimensions given by* `List_of_PPL_Vars` *while respecting the specified* `Width`*,* `Representation` *and* `Overflow` *behavior of all these variables. The* `Constraint_System` *represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer* `C_unsigned` *and* `Boolean` *allow control of the complexity/precision ratio; higher values for* `C_unsigned` *will lead to possibly greater precision while a true value for* `Boolean` *indicates that the space dimensions should be wrapped individ-*

ually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one polyhedron

These predicates include the binary operators which will assign to the polyhedron referred to by the first argument its combination with the polyhedron referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Polyhedron_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the polyhedron `P` referenced by `Handle_1` the intersection of `P` and the polyhedron referenced by `Handle_2`.

`ppl_Polyhedron_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the polyhedron `P` referenced by `Handle_1` the upper bound of `P` and the polyhedron referenced by `Handle_2`.

`ppl_Polyhedron_difference_assign(+Handle_1, +Handle_2)`

Assigns to the polyhedron `P` referenced by `Handle_1` the difference of `P` and the polyhedron referenced by `Handle_2`.

`ppl_Polyhedron_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the polyhedron `P` referenced by `Handle_1` the time elapse of `P` and the polyhedron referenced by `Handle_2`.

`ppl_Polyhedron_poly_hull(+Handle_1, +Handle_2)`

Assigns to the polyhedron `P` referenced by `Handle_1` the poly-hull of `P` and the polyhedron referenced by `Handle_2`.

`ppl_Polyhedron_poly_difference(+Handle_1, +Handle_2)`

Assigns to the polyhedron `P` referenced by `Handle_1` the poly-difference of `P` and the polyhedron referenced by `Handle_2`.

`ppl_Polyhedron_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the polyhedron `P_1` referenced by `Handle_1` with the polyhedron referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Polyhedron_poly_hull_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the polyhedron `P_1` referenced by `Handle_1` with the polyhedron referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Polyhedron_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of polyhedron `P_1` referenced by `Handle_1` and the polyhedron `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

`ppl_Polyhedron_linear_partition(+Handle_1, +Handle_2, -Handle_3, -Handle_4)`

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Polyhedron domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron`

has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the polyhedron referred to by the second argument has to be contained in (or equal to) the polyhedron referred to by the first argument.

`ppl_Polyhedron_BHRZ03_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the polyhedron `P_1` referenced by `Handle_1` the BHRZ03-widening of `P_1` with the polyhedron referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_Polyhedron_H79_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the polyhedron `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the polyhedron referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_Polyhedron_BHRZ03_widening_assign(+Handle_1, +Handle_2)`

Assigns to the polyhedron `P_1` referenced by `Handle_1` the BHRZ03-widening of `P_1` with the polyhedron referenced by `Handle_2`.

`ppl_Polyhedron_H79_widening_assign(+Handle_1, +Handle_2)`

Assigns to the polyhedron `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the polyhedron referenced by `Handle_2`.

`ppl_Polyhedron_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Same as predicate `ppl_Polyhedron_H79_widening_assign_with_tokens/4`

`ppl_Polyhedron_widening_assign(+Handle_1, +Handle_2)`

Same as predicate `ppl_Polyhedron_H79_widening_assign/2`

`ppl_Polyhedron_limited_BHRZ03_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the polyhedron `P_1` referenced by `Handle_1` the BHRZ03-widening of `P_1` with the polyhedron referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_Polyhedron_bounded_BHRZ03_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the polyhedron `P_1` referenced by `Handle_1` the BHRZ03-widening of `P_1` with the polyhedron referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`, further intersected with the smallest box containing `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the

number of tokens remaining at the end of the operation.

```
ppl_Polyhedron_limited_H79_extrapolation_assign_with_tokens(+Handle_1,
+Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the polyhedron `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the polyhedron referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Polyhedron_bounded_H79_extrapolation_assign_with_tokens(+Handle_1,
+Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the polyhedron `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the polyhedron referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`, further intersected with the smallest box containing `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Polyhedron_limited_BHRZ03_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the polyhedron `P_1` referenced by `Handle_1` the BHRZ03-widening of `P_1` with the polyhedron referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_Polyhedron_bounded_BHRZ03_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the polyhedron `P_1` referenced by `Handle_1` the BHRZ03-widening of `P_1` with the polyhedron referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`, further intersected with the smallest box containing `P_1`.

```
ppl_Polyhedron_limited_H79_extrapolation_assign(+Handle_1, +Handle_2,
+Constraint_System)
```

Assigns to the polyhedron `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the polyhedron referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_Polyhedron_bounded_H79_extrapolation_assign(+Handle_1, +Handle_2,
+Constraint_System)
```

Assigns to the polyhedron `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the polyhedron referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`, further intersected with the smallest box containing `P_1`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the polyhedron referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Polyhedron_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the polyhedron `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the polyhedron referenced by* `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the polyhedron referred to in the first argument. These predicates enable the modification of the vector space of the polyhedron referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Polyhedron_add_space_dimensions_and_embed(+Handle, +Dimension_-
Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the polyhedron* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Polyhedron_add_space_dimensions_and_project(+Handle, +Dimension_-
Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the polyhedron* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

```
ppl_Polyhedron_remove_space_dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the polyhedron `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Polyhedron_remove_higher_space_dimensions(+Handle, +Dimension_-
Type)
```

Removes the higher dimensions from the vector space enclosing the polyhedron `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`.

```
ppl_Polyhedron_expand_space_dimension(+Handle, +PPL_Var, +Dimension_-
Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the polyhedron referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

```
ppl_Polyhedron_fold_space_dimensions(+Handle, +List_of_PPL_Vars,
+PPL_Var)
```

Modifies the polyhedron referenced by `Handle` *by folding the space dimensions contained in* `List_of_-PPL_Vars` *into dimension* `PPL_Var`. *The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_Polyhedron_map_space_dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, *which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for the Grid Domain

Here we provide a short description for each of the predicates available for the domain of grids.

Constructor, copy, conversion and destructor predicates

Constructor predicates for grids

The constructor predicates build a grid from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Grid_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new grid $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Grid_from_space_dimension(3, universe, X).
```

creates the grid defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Grid_from_constraints(+Constraint_System, -Handle)`

Builds a new grid `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Grid_from_congruences(+Congruence_System, -Handle)`

Builds a new grid `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Grid_from_grid_generators(+Grid_Generator_System, -Handle)`

Builds a new grid `P` from `Grid_Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new grids by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Grid_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Grid(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new grid `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Uint8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Uint16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Uint32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Uint64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Z_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Rational_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_BD_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_BD_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_BD_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_BD_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_BD_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Grid_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new grid `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Grid_from_BD_Shape_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new grid `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Grid_from_Octagonal_Shape_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new grid `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Grid_from_Long_Double_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new grid `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Grid_from_BD_Shape_long_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new grid `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Grid_from_Octagonal_Shape_long_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new grid `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Grid domain.

```
ppl_delete_Grid(+Handle)
```

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the grid

Test Predicates

These predicates test the grid for different properties and succeed or fail depending on the outcome.

```
ppl_Grid_is_empty(+Handle)
```

Succeeds if and only if the grid referenced by `Handle` is empty.

```
ppl_Grid_is_universe(+Handle)
```

Succeeds if and only if the grid referenced by `Handle` is the universe.

```
ppl_Grid_is_bounded(+Handle)
```

Succeeds if and only if the grid referenced by `Handle` is bounded.

```
ppl_Grid_contains_integer_point(+Handle)
```

Succeeds if and only if the grid referenced by `Handle` *contains an integer point.*

`ppl_Grid_is_topologically_closed(+Handle)`

Succeeds if and only if the grid referenced by `Handle` *is topologically closed.*

`ppl_Grid_is_discrete(+Handle)`

Succeeds if and only if the grid referenced by `Handle` *is discrete.*

`ppl_Grid_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from above in the grid referenced by* `Handle`*.*

`ppl_Grid_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from below in the grid referenced by* `Handle`*.*

`ppl_Grid_contains_Grid(+Handle_1, +Handle_2)`

Succeeds if and only if the grid referenced by `Handle_2` *is included in or equal to the grid referenced by* `Handle_1`*.*

`ppl_Grid_strictly_contains_Grid(+Handle_1, +Handle_2)`

Succeeds if and only if the grid referenced by `Handle_2` *is included in but not equal to the grid referenced by* `Handle_1`*.*

`ppl_Grid_is_disjoint_from_Grid(+Handle_1, +Handle_2)`

Succeeds if and only if the grid referenced by `Handle_2` *is disjoint from the grid referenced by* `Handle_1`*.*

`ppl_Grid_equals_Grid(+Handle_1, +Handle_2)`

Succeeds if and only if the grid referenced by `Handle_1` *is equal to the grid referenced by* `Handle_2`*.*

`ppl_Grid_OK(+Handle)`

Succeeds only if the grid referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

`ppl_Grid_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the grid referenced by `Handle` *constrains the dimension* `PPL_Var`*.*

Predicates that return information about the grid

These predicates will obtain more detailed information about the grid unifying some of their arguments with the results.

`ppl_Grid_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the grid referenced by* `Handle`*.*

`ppl_Grid_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` *with the affine dimension of the grid referenced by* `Handle`*.*

`ppl_Grid_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` *with the list of relations the grid referenced by* `Handle` *has with* `Constraint`*. The possible relations are listed in the grammar rules above.*

`ppl_Grid_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` *with the list of relations the grid referenced by* `Handle` *has with* `Generator`. *The possible relations are listed in the grammar rules above.*

```
ppl_Grid_relation_with_congruence(+Handle, +Congruence, ?Relation_-
List)
```

Unifies `Relation_List` *with the list of relations the grid referenced by* `Handle` *has with* `Congruence`. *The possible relations are listed in the grammar rules above.*

```
ppl_Grid_relation_with_grid_generator(+Handle, +Grid_Generator,
?Relation_List)
```

Unifies `Relation_List` *with the list of relations the grid referenced by* `Handle` *has with* `Grid_-Generator`. *The possible relations are listed in the grammar rules above.*

```
ppl_Grid_get_constraints(+Handle, ?Constraint_System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the constraint system satisfied by the grid referenced by* `Handle`.

```
ppl_Grid_get_congruences(+Handle, ?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the congruence system satisfied by the grid referenced by* `Handle`.

```
ppl_Grid_get_grid_generators(+Handle, ?Grid_Generator_System)
```

Unifies `Grid_Generator_System` *with the grid_generators (in the form of a list) in the grid_generator system for the grid referenced by* `Handle`.

```
ppl_Grid_get_minimized_constraints(+Handle, ?Constraint_System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the minimized constraint system satisfied by the grid referenced by* `Handle`.

```
ppl_Grid_get_minimized_congruences(+Handle, ?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the minimized congruence system satisfied by the grid referenced by* `Handle`.

```
ppl_Grid_get_minimized_grid_generators(+Handle, ?Grid_Generator_-
System)
```

Unifies `Grid_Generator_System` *with the grid_generators (in the form of a list) in the minimized grid_generator system for the grid referenced by* `Handle`.

```
ppl_Grid_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if grid `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Grid_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if grid `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Grid_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean, ?Point)
```

Succeeds if and only if grid `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Grid_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean, ?Point)
```

Succeeds if and only if grid `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Grid_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the grid referenced by* `Handle`*.*

```
ppl_Grid_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the grid referenced by* `Handle`*.*

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Grid_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the grid referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the grid

These predicates may modify the grid referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the grid by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Grid_add_constraint(+Handle, +Constraint)
```

Updates the grid referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

```
ppl_Grid_add_congruence(+Handle, +Congruence)
```

Updates the grid referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

```
ppl_Grid_add_grid_generator(+Handle, +Grid_Generator)
```

Updates the grid referenced by `Handle` *to one obtained by adding* `Grid_Generator` *to its grid_-generator system. For a C polyhedron,* `Grid_Generator` *must be a line, ray or point.*

`ppl_Grid_add_constraints( +Handle, +Constraint_System)`

Updates the grid referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Grid_add_congruences( +Handle, +Congruence_System)`

Updates the grid referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Grid_add_grid_generators( +Handle, +Grid_Generator_System)`

Updates the grid referenced by `Handle` *to one obtained by adding to its grid_generator system the grid_-generators in* `Grid_Generator_System`*. For a C polyhedron,* `Grid_Generators` *must be a list of lines, rays and points.*

`ppl_Grid_refine_with_constraint( +Handle, +Constraint)`

Updates the grid referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Grid_refine_with_congruence( +Handle, +Congruence)`

Updates the grid referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Grid_refine_with_constraints( +Handle, +Constraint_System)`

Updates the grid referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Grid_refine_with_congruences( +Handle, +Congruence_System)`

Updates the grid referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the grid

These predicates enable transformations such as taking the topological closure (which for the domain of grid is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Grid_topological_closure_assign(+Handle)`

Assigns to the grid referenced by `Handle` *its topological closure.*

`ppl_Grid_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the grid `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

`ppl_Grid_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the grid `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Grid_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the grid referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Grid_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the grid referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Grid_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_-
Expr_2, +Coeff)
```

Assigns to grid `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Grid_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1,
+Lin_Expr_2, +Coeff)
```

Assigns to grid `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Grid_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol,
+Lin_Expr, +Coeff)
```

Assigns to grid `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`.

```
ppl_Grid_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_-
Symbol, +Lin_Expr, +Coeff)
```

Assigns to grid `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`.

```
ppl_Grid_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1,
+Relation_Symbol, +Lin_Expr_2)
```

Assigns to grid `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`.

```
ppl_Grid_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1,
+Relation_Symbol, +Lin_Expr_2)
```

Assigns to grid `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`.

```
ppl_Grid_generalized_affine_image_with_congruence(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff_1, +Coeff_2)
```

Assigns to grid `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \equiv_{\texttt{f}} \texttt{Lin_Expr_2/Coeff_1}$, *where* `Coeff_2` *is bound to the modulus* `f`.

```
ppl_Grid_generalized_affine_preimage_with_congruence(+Handle, +PPL_-
Var, +Relation_Symbol, +Lin_Expr, +Coeff_1, +Coeff_2)
```

Assigns to grid `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \equiv_{\texttt{f}} \texttt{Lin_Expr_2/Coeff_1}$, *where* `Coeff_2` *is bound to the modulus* `f`.

`ppl_Grid_generalized_affine_image_lhs_rhs_with_congruence(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2, +Coeff)`

Assigns to grid `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` $\equiv_f$ `Lin_Expr_2`. where `Coeff` is bound to the modulus `f`.

`ppl_Grid_generalized_affine_preimage_lhs_rhs_with_congruence(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2, +Coeff)`

Assigns to grid `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` $\equiv_f$ `Lin_Expr_2`. where `Coeff` is bound to the modulus `f`.

`ppl_Grid_wrap_assign(+Handle, +List_of_PPL_Var, +Width, +Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)`

Transforms the grid `P` referenced by `Handle` by wrapping the dimensions given by `List_of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one grid

These predicates include the binary operators which will assign to the grid referred to by the first argument its combination with the grid referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Grid_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the grid `P` referenced by `Handle_1` the intersection of `P` and the grid referenced by `Handle_2`.

`ppl_Grid_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the grid `P` referenced by `Handle_1` the upper bound of `P` and the grid referenced by `Handle_2`.

`ppl_Grid_difference_assign(+Handle_1, +Handle_2)`

Assigns to the grid `P` referenced by `Handle_1` the difference of `P` and the grid referenced by `Handle_2`.

`ppl_Grid_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the grid `P` referenced by `Handle_1` the time elapse of `P` and the grid referenced by `Handle_2`.

`ppl_Grid_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the grid `P_1` referenced by `Handle_1` with the grid referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Grid_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of grid `P_1` referenced by `Handle_1` and the grid `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

`ppl_Grid_approximate_partition(+Handle_1, +Handle_2, ?Boolean, -Handle_3, -Handle_4)`

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Grid domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of grids `P_4` where:

if there is a finite linear partition of `P_1` *wrt* `P_2` `Boolean` *is unified with the atom* `true` *and* `P_4` *is the linear partition;*

otherwise `Boolean` *is unified with the atom* `false` *and* `P_4` *is set to the singleton set that contains* `P_2`*.*

This predicate is only provided if the class `Pointset_Powerset_Grid` *has been enabled when configuring the library.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the grid referred to by the second argument has to be contained in (or equal to) the grid referred to by the first argument.

```
ppl_Grid_congruence_widening_assign_with_tokens(+Handle_1, +Handle_2,
+C_unsigned_1, ?C_unsigned_2)
```

Assigns to the grid `P_1` *referenced by* `Handle_1` *the congruence-widening of* `P_1` *with the grid referenced by* `Handle_2`*. The widening with tokens delay technique is applied with* `C_unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Grid_generator_widening_assign_with_tokens(+Handle_1, +Handle_2,
+C_unsigned_1, ?C_unsigned_2)
```

Assigns to the grid `P_1` *referenced by* `Handle_1` *the generator-widening of* `P_1` *with the grid referenced by* `Handle_2`*. The widening with tokens delay technique is applied with* `C_unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Grid_congruence_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the grid `P_1` *referenced by* `Handle_1` *the congruence-widening of* `P_1` *with the grid referenced by* `Handle_2`*.*

```
ppl_Grid_generator_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the grid `P_1` *referenced by* `Handle_1` *the generator-widening of* `P_1` *with the grid referenced by* `Handle_2`*.*

```
ppl_Grid_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_
unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_Grid_congruence_widening_assign_with_tokens/4`

```
ppl_Grid_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_Grid_congruence_widening_assign/2`

```
ppl_Grid_limited_congruence_extrapolation_assign_with_tokens(+Handle_
1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the grid `P_1` *referenced by* `Handle_1` *the congruence-widening of* `P_1` *with the grid referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*. The widening with tokens delay technique is applied with* `C_unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Grid_limited_generator_extrapolation_assign_with_tokens(+Handle_1,
+Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the grid `P_1` *referenced by* `Handle_1` *the generator-widening of* `P_1` *with the grid referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the*

points of `P_1`*. The widening with tokens delay technique is applied with* `C_unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Grid_limited_congruence_extrapolation_assign(+Handle_1, +Handle_2,
+Constraint_System)
```

Assigns to the grid `P_1` *referenced by* `Handle_1` *the congruence-widening of* `P_1` *with the grid referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*.*

```
ppl_Grid_limited_generator_extrapolation_assign(+Handle_1, +Handle_2,
+Constraint_System)
```

Assigns to the grid `P_1` *referenced by* `Handle_1` *the generator-widening of* `P_1` *with the grid referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*.*

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the grid referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Grid_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the grid `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the grid referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the grid referred to in the first argument. These predicates enable the modification of the vector space of the grid referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Grid_add_space_dimensions_and_embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the grid* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Grid_add_space_dimensions_and_project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the grid* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

```
ppl_Grid_remove_space_dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the grid `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Grid_remove_higher_space_dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the grid `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`*.*

```
ppl_Grid_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the grid referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

```
ppl_Grid_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)
```

Modifies the grid referenced by `Handle` *by folding the space dimensions contained in* `List_of_PPL_Vars` *into dimension* `PPL_Var`*. The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_Grid_map_space_dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for the Pointset_Powerset_Int8_Box Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of int8 boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of int8 boxes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Pointset_Powerset_Int8_Box_from_space_dimension(+Dimension_-
Type, +Universe_or_Empty, -Handle)
```

Builds a new pointset powerset $\mathcal{P}$ *with* `Dimension_Type` *dimensions; it is empty or the universe depending on whether* `Atom` *is* `empty` *or* `universe`*, respectively.* `Handle` *is unified with the handle for* $\mathcal{P}$*. Thus the query*

```
?- ppl_new_Pointset_Powerset_Int8_Box_from_space_dimension(3, universe, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ *with* `X` *bound to a valid handle for accessing it.*

```
ppl_new_Pointset_Powerset_Int8_Box_from_constraints(+Constraint_-
System, -Handle)
```

Builds a new pointset powerset `P` *from* `Constraint_System`*.* `Handle` *is unified with the handle for* `P`*.*

```
ppl_new_Pointset_Powerset_Int8_Box_from_congruences(+Congruence_-
System, -Handle)
```

Builds a new pointset powerset `P` *from* `Congruence_System`*.* `Handle` *is unified with the handle for* `P`*.*

Predicates that build new pointset powersets of int8 boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Int8_Box_from_Pointset_Powerset_Int8_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Int8_Box_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Int8_Box_from_Pointset_Powerset_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Int8_Box_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Int8_Box domain.

`ppl_delete_Pointset_Powerset_Int8_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Int8_Box_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_Int8_Box_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_Int8_Box_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_Int8_Box_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_Int8_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_Int8_Box_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_Int8_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Int8_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Int8_Box_contains_Pointset_Powerset_Int8_-`
`Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Int8_Box_strictly_contains_Pointset_Powerset_-`
`Int8_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Int8_Box_is_disjoint_from_Pointset_Powerset_-`
`Int8_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Int8_Box_geometrically_covers_Pointset_-`
`Powerset_Int8_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Int8_Box_geometrically_equals_Pointset_-`
`Powerset_Int8_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Int8_Box_equals_Pointset_Powerset_Int8_-`
`Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Int8_Box_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Pointset_Powerset_Int8_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

`ppl_Pointset_Powerset_Int8_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Int8_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Int8_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Int8_Box_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Int8_Box_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Int8_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_Int8_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_Int8_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_Int8_Box_minimize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_Int8_Box_external_memory_in_bytes(+Handle,
?Number)
```

Unifies `Number` with the size of the total memory in bytes occupied by the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_Int8_Box_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the external memory in bytes occupied by the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_Int8_Box_size(+Handle, ?Number)
```

Unifies `Number` with the size of the memory in bytes occupied by the pointset powerset referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_Int8_Box_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Pointset_Powerset_Int8_Box_add_constraint(+Handle, +Constraint)
```

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

```
ppl_Pointset_Powerset_Int8_Box_add_congruence(+Handle, +Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Pointset_Powerset_Int8_Box_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Pointset_Powerset_Int8_Box_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Pointset_Powerset_Int8_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Pointset_Powerset_Int8_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Pointset_Powerset_Int8_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Pointset_Powerset_Int8_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_Int8_Box_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

`ppl_Pointset_Powerset_Int8_Box_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

`ppl_Pointset_Powerset_Int8_Box_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

`ppl_Pointset_Powerset_Int8_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`.

```
ppl_Pointset_Powerset_Int8_Box_unconstrain_space_dimensions(+Handle,
+List_of_PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Int8_Box_affine_image(+Handle, +PPL_Var, +Lin_-
Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_Int8_Box_affine_preimage(+Handle, +PPL_Var,
+Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_Int8_Box_bounded_affine_image(+Handle, +PPL_Var,
+Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_Int8_Box_bounded_affine_preimage(+Handle, +PPL_-
Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_Int8_Box_generalized_affine_image(+Handle,
+PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Int8_Box_generalized_affine_preimage(+Handle,
+PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Int8_Box_generalized_affine_image_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Int8_Box_generalized_affine_preimage_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Pointset_Powerset_Int8_Box_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the intersection of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Int8_Box_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the upper bound of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Int8_Box_difference_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the difference of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Int8_Box_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the time elapse of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Int8_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the pointset powerset `P_1` referenced by `Handle_1` with the pointset powerset referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Pointset_Powerset_Int8_Box_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of pointset powerset `P_1` referenced by `Handle_1` and the pointset powerset `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Pointset_Powerset_Int8_Box_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Pointset_Powerset_Int8_Box_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Pointset_Powerset_Int8_Box_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Pointset_Powerset_Int8_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Int8_Box_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Pointset_Powerset_Int8_Box_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Pointset_Powerset_Int8_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Int8_Box_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

`ppl_new_Pointset_Powerset_Int8_Box_iterator_from_iterator(+Iterator_1, -Iterator_2)`

Builds a new iterator `it` *from the iterator referenced by* `Iterator_1`*.* `Iterator_2` *is unified with the handle for* `it`*.*

`ppl_Pointset_Powerset_Int8_Box_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` *with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

`ppl_Pointset_Powerset_Int8_Box_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` *with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

`ppl_Pointset_Powerset_Int8_Box_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` *is equal to the iterator referenced by* `Iterator_2`*.*

`ppl_Pointset_Powerset_Int8_Box_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` *so that it "points" to the next disjunct.*

`ppl_Pointset_Powerset_Int8_Box_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` *so that it "points" to the previous disjunct.*

`ppl_Pointset_Powerset_Int8_Box_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` *a reference to the disjunct referred to by* `Iterator_1`*.*

`ppl_delete_Pointset_Powerset_Int8_Box_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`*: this makes sure the corresponding resources will eventually be released.*

`ppl_Pointset_Powerset_Int8_Box_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` *a disjunct referred to by* `Handle_2`*.*

`ppl_Pointset_Powerset_Int8_Box_drop_disjunct(+Handle, +Iterator)`

If `it` *is the iterator referred to by* `Iterator`*, drops from the pointset powerset referenced by* `Handle` *the disjunct pointed to by* `it` *and assigns to* `it` *an iterator to the next disjunct.*

`ppl_Pointset_Powerset_Int8_Box_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Int16_Box Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of int16 boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of int16 boxes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_Int16_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Int16_Box_from_space_dimension(3, universe, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_Int16_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_Int16_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of int16 boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Int16_Box_from_Pointset_Powerset_Int16_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Int16_Box_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Int16_Box_from_Pointset_Powerset_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Int16_Box_from_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Int16_Box domain.

`ppl_delete_Pointset_Powerset_Int16_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Int16_Box_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_Int16_Box_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_Int16_Box_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_Int16_Box_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_Int16_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_Int16_Box_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is discrete.*

```
ppl_Pointset_Powerset_Int16_Box_bounds_from_above(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from above in the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Int16_Box_bounds_from_below(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from below in the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Int16_Box_contains_Pointset_Powerset_Int16_-
Box(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in or equal to the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_Int16_Box_strictly_contains_Pointset_Powerset_-
Int16_Box(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in but not equal to the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_Int16_Box_is_disjoint_from_Pointset_Powerset_-
Int16_Box(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is disjoint from the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_Int16_Box_geometrically_covers_Pointset_-
Powerset_Int16_Box(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically covers the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_Int16_Box_geometrically_equals_Pointset_-
Powerset_Int16_Box(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically equals the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_Int16_Box_equals_Pointset_Powerset_Int16_-
Box(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_1` *is equal to the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Int16_Box_OK(+Handle)
```

Succeeds only if the pointset powerset referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_Pointset_Powerset_Int16_Box_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *constrains the dimension* `PPL_Var`*.*

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_Int16_Box_space_dimension(+Handle, ?Dimension_-
Type)
```

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_Int16_Box_affine_dimension(+Handle, ?Dimension_-
Type)
```

Unifies `Dimension_Type` *with the affine dimension of the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_Int16_Box_relation_with_constraint(+Handle,
+Constraint, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Constraint`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Int16_Box_relation_with_generator(+Handle,
+Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Generator`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Int16_Box_relation_with_congruence(+Handle,
+Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Congruence`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Int16_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_Int16_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_Int16_Box_maximize_with_point(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_Int16_Box_minimize_with_point(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_Int16_Box_external_memory_in_bytes(+Handle,
?Number)
```

Unifies `Number` with the size of the total memory in bytes occupied by the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_Int16_Box_total_memory_in_bytes(+Handle,
?Number)
```

Unifies `Number` with the size of the external memory in bytes occupied by the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_Int16_Box_size(+Handle, ?Number)
```

Unifies `Number` with the size of the memory in bytes occupied by the pointset powerset referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_Int16_Box_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Pointset_Powerset_Int16_Box_add_constraint(+Handle, +Constraint)
```

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

```
ppl_Pointset_Powerset_Int16_Box_add_congruence(+Handle, +Congruence)
```

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

```
ppl_Pointset_Powerset_Int16_Box_add_constraints( +Handle,
+Constraint_System)
```

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Pointset_Powerset_Int16_Box_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Pointset_Powerset_Int16_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Pointset_Powerset_Int16_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Pointset_Powerset_Int16_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Pointset_Powerset_Int16_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_Int16_Box_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its topological closure.

`ppl_Pointset_Powerset_Int16_Box_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its pairwise reduction.

`ppl_Pointset_Powerset_Int16_Box_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its omega reduction.

`ppl_Pointset_Powerset_Int16_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Pointset_Powerset_Int16_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Int16_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Int16_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Int16_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_Int16_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_Int16_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Int16_Box_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Int16_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Int16_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Pointset_Powerset_Int16_Box_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Int16_Box_upper_bound_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Int16_Box_difference_assign(+Handle_1, +Handle_-
2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the difference of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Int16_Box_time_elapse_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Int16_Box_upper_bound_assign_if_exact(+Handle_1,
+Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` *referenced by* `Handle_1` *with the pointset powerset referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Pointset_Powerset_Int16_Box_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_Int16_Box_concatenate_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_Int16_Box_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Pointset_Powerset_Int16_Box_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Pointset_Powerset_Int16_Box_remove_space_dimensions(+Handle,
+List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Int16_Box_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Pointset_Powerset_Int16_Box_expand_space_dimension(+Handle, +PPL_-
Var, +Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Pointset_Powerset_Int16_Box_fold_space_dimensions(+Handle, +List_-
of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Int16_Box_map_space_dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

`ppl_new_Pointset_Powerset_Int16_Box_iterator_from_iterator(+Iterator_1, -Iterator_2)`

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

`ppl_Pointset_Powerset_Int16_Box_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Int16_Box_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Int16_Box_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_Int16_Box_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_Int16_Box_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_Int16_Box_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_Int16_Box_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_Int16_Box_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_Int16_Box_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Int16_Box_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Int32_Box Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of int32 boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of int32 boxes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_Int32_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Int32_Box_from_space_dimension(3, universe, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_Int32_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_Int32_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of int32 boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Int32_Box_from_Pointset_Powerset_Int32_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Int32_Box_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` *from the int32_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Pointset_Powerset_Int32_Box_from_Pointset_Powerset_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` *from the pointset_powerset_int32_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Pointset_Powerset_Int32_Box_from_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` *from the int32_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Int32_Box domain.

`ppl_delete_Pointset_Powerset_Int32_Box(+Handle)`

Invalidates the handle referenced by `Handle`*: this makes sure the corresponding resources will eventually be released.*

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Int32_Box_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is empty.*

`ppl_Pointset_Powerset_Int32_Box_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is the universe.*

`ppl_Pointset_Powerset_Int32_Box_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is bounded.*

`ppl_Pointset_Powerset_Int32_Box_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *contains an integer point.*

`ppl_Pointset_Powerset_Int32_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is topologically closed.*

`ppl_Pointset_Powerset_Int32_Box_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is discrete.*

`ppl_Pointset_Powerset_Int32_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from above in the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_Int32_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from below in the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_Int32_Box_contains_Pointset_Powerset_Int32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Int32_Box_strictly_contains_Pointset_Powerset_Int32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Int32_Box_is_disjoint_from_Pointset_Powerset_Int32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Int32_Box_geometrically_covers_Pointset_Powerset_Int32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Int32_Box_geometrically_equals_Pointset_Powerset_Int32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Int32_Box_equals_Pointset_Powerset_Int32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Int32_Box_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Pointset_Powerset_Int32_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

`ppl_Pointset_Powerset_Int32_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Int32_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Int32_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Constraint`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Int32_Box_relation_with_generator(+Handle,
+Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Generator`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Int32_Box_relation_with_congruence(+Handle,
+Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Congruence`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Int32_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Int32_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Int32_Box_maximize_with_point(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Int32_Box_minimize_with_point(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Int32_Box_external_memory_in_bytes(+Handle,
?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_Int32_Box_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Int32_Box_size(+Handle, ?Number)`

Unifies `Number` with the size of the memory in bytes occupied by the pointset powerset referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Pointset_Powerset_Int32_Box_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_Int32_Box_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_Pointset_Powerset_Int32_Box_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Pointset_Powerset_Int32_Box_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Pointset_Powerset_Int32_Box_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Pointset_Powerset_Int32_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`.

```
ppl_Pointset_Powerset_Int32_Box_refine_with_congruence( +Handle,
+Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`.

```
ppl_Pointset_Powerset_Int32_Box_refine_with_constraints( +Handle,
+Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`.

```
ppl_Pointset_Powerset_Int32_Box_refine_with_congruences( +Handle,
+Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Pointset_Powerset_Int32_Box_topological_closure_assign(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

```
ppl_Pointset_Powerset_Int32_Box_pairwise_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

```
ppl_Pointset_Powerset_Int32_Box_omega_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

```
ppl_Pointset_Powerset_Int32_Box_unconstrain_space_dimension(+Handle,
+PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`.

```
ppl_Pointset_Powerset_Int32_Box_unconstrain_space_dimensions(+Handle,
+List_of_PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Int32_Box_affine_image(+Handle, +PPL_Var, +Lin_-
Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_-Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_Int32_Box_affine_preimage(+Handle, +PPL_Var,
+Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_Int32_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_Int32_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_Int32_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Int32_Box_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Int32_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Int32_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_Int32_Box_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`.

```
ppl_Pointset_Powerset_Int32_Box_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`.

`ppl_Pointset_Powerset_Int32_Box_difference_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the difference of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Int32_Box_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the time elapse of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Int32_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the pointset powerset `P_1` referenced by `Handle_1` with the pointset powerset referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Pointset_Powerset_Int32_Box_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of pointset powerset `P_1` referenced by `Handle_1` and the pointset powerset `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Pointset_Powerset_Int32_Box_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections

Adding New Dimensions to the Vector Space, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_Int32_Box_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Pointset_Powerset_Int32_Box_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Pointset_Powerset_Int32_Box_remove_space_dimensions(+Handle,
+List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Int32_Box_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Pointset_Powerset_Int32_Box_expand_space_dimension(+Handle, +PPL_-
Var, +Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Pointset_Powerset_Int32_Box_fold_space_dimensions(+Handle, +List_-
of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Int32_Box_map_space_dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_Int32_Box_iterator_from_iterator(+Iterator_-
1, -Iterator_2)
```

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

```
ppl_Pointset_Powerset_Int32_Box_begin_iterator(+Handle, -Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

`ppl_Pointset_Powerset_Int32_Box_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` *with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

`ppl_Pointset_Powerset_Int32_Box_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` *is equal to the iterator referenced by* `Iterator_2`*.*

`ppl_Pointset_Powerset_Int32_Box_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` *so that it "points" to the next disjunct.*

`ppl_Pointset_Powerset_Int32_Box_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` *so that it "points" to the previous disjunct.*

`ppl_Pointset_Powerset_Int32_Box_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` *a reference to the disjunct referred to by* `Iterator_1`*.*

`ppl_delete_Pointset_Powerset_Int32_Box_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`*: this makes sure the corresponding resources will eventually be released.*

`ppl_Pointset_Powerset_Int32_Box_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` *a disjunct referred to by* `Handle_2`*.*

`ppl_Pointset_Powerset_Int32_Box_drop_disjunct(+Handle, +Iterator)`

If `it` *is the iterator referred to by* `Iterator`*, drops from the pointset powerset referenced by* `Handle` *the disjunct pointed to by* `it` *and assigns to* `it` *an iterator to the next disjunct.*

`ppl_Pointset_Powerset_Int32_Box_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` *and* `it_2` *are the iterators referred to by* `Iterator_1` *and* `Iterator_2`*, respectively, drops from the pointset powerset referenced by* `Handle` *all the disjuncts from* `it_1` *to* `it_2` *(excluded).*

Predicates for the Pointset_Powerset_Int64_Box Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of int64 boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of int64 boxes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.

- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_Int64_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Int64_Box_from_space_dimension(3, universe, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_Int64_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_Int64_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of int64 boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Int64_Box_from_Pointset_Powerset_Int64_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Int64_Box_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Int64_Box_from_Pointset_Powerset_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Int64_Box_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Int64_Box domain.

`ppl_delete_Pointset_Powerset_Int64_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Int64_Box_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_Int64_Box_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_Int64_Box_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_Int64_Box_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_Int64_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_Int64_Box_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_Int64_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Int64_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Int64_Box_contains_Pointset_Powerset_Int64_-Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Int64_Box_strictly_contains_Pointset_Powerset_-Int64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Int64_Box_is_disjoint_from_Pointset_Powerset_Int64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Int64_Box_geometrically_covers_Pointset_Powerset_Int64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Int64_Box_geometrically_equals_Pointset_Powerset_Int64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Int64_Box_equals_Pointset_Powerset_Int64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Int64_Box_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Pointset_Powerset_Int64_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

`ppl_Pointset_Powerset_Int64_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Int64_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Int64_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Int64_Box_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Int64_Box_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Congruence`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Int64_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Int64_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Int64_Box_maximize_with_point(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Int64_Box_minimize_with_point(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Int64_Box_external_memory_in_bytes(+Handle,
?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Int64_Box_total_memory_in_bytes(+Handle,
?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Int64_Box_size(+Handle, ?Number)
```

Unifies `Number` *with the size of the memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Pointset_Powerset_Int64_Box_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_Int64_Box_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Pointset_Powerset_Int64_Box_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Pointset_Powerset_Int64_Box_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Pointset_Powerset_Int64_Box_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Pointset_Powerset_Int64_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Pointset_Powerset_Int64_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Pointset_Powerset_Int64_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

```
ppl_Pointset_Powerset_Int64_Box_refine_with_congruences( +Handle,
+Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Pointset_Powerset_Int64_Box_topological_closure_assign(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

```
ppl_Pointset_Powerset_Int64_Box_pairwise_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

```
ppl_Pointset_Powerset_Int64_Box_omega_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

```
ppl_Pointset_Powerset_Int64_Box_unconstrain_space_dimension(+Handle,
+PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

```
ppl_Pointset_Powerset_Int64_Box_unconstrain_space_dimensions(+Handle,
+List_of_PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Int64_Box_affine_image(+Handle, +PPL_Var, +Lin_-
Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_-Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_Pointset_Powerset_Int64_Box_affine_preimage(+Handle, +PPL_Var,
+Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_-Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_Pointset_Powerset_Int64_Box_bounded_affine_image(+Handle, +PPL_-
Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_Pointset_Powerset_Int64_Box_bounded_affine_preimage(+Handle,
+PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

```
ppl_Pointset_Powerset_Int64_Box_generalized_affine_image(+Handle,
+PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Int64_Box_generalized_affine_preimage(+Handle,
+PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Int64_Box_generalized_affine_image_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Int64_Box_generalized_affine_preimage_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_Int64_Box_intersection_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`.

```
ppl_Pointset_Powerset_Int64_Box_upper_bound_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`.

```
ppl_Pointset_Powerset_Int64_Box_difference_assign(+Handle_1, +Handle_-
2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the difference of* `P` *and the pointset powerset referenced by* `Handle_2`.

```
ppl_Pointset_Powerset_Int64_Box_time_elapse_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the pointset powerset referenced by* `Handle_2`.

`ppl_Pointset_Powerset_Int64_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the pointset powerset `P_1` referenced by `Handle_1` with the pointset powerset referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Pointset_Powerset_Int64_Box_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of pointset powerset `P_1` referenced by `Handle_1` and the pointset powerset `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Pointset_Powerset_Int64_Box_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Pointset_Powerset_Int64_Box_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Pointset_Powerset_Int64_Box_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Pointset_Powerset_Int64_Box_remove_space_dimensions(+Handle,
+List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Int64_Box_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Pointset_Powerset_Int64_Box_expand_space_dimension(+Handle, +PPL_-
Var, +Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Pointset_Powerset_Int64_Box_fold_space_dimensions(+Handle, +List_-
of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Int64_Box_map_space_dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_Int64_Box_iterator_from_iterator(+Iterator_-
1, -Iterator_2)
```

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

```
ppl_Pointset_Powerset_Int64_Box_begin_iterator(+Handle, -Iterator)
```

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

```
ppl_Pointset_Powerset_Int64_Box_end_iterator(+Handle, -Iterator)
```

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

```
ppl_Pointset_Powerset_Int64_Box_iterator_equals_iterator(+Iterator_1,
+Iterator_2)
```

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_Int64_Box_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_Int64_Box_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_Int64_Box_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_Int64_Box_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_Int64_Box_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_Int64_Box_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Int64_Box_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Uint8_Box Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of uint8 boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of uint8 boxes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_Uint8_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Uint8_Box_from_space_dimension(3, universe, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

```
ppl_new_Pointset_Powerset_Uint8_Box_from_constraints(+Constraint_-
System, -Handle)
```

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

```
ppl_new_Pointset_Powerset_Uint8_Box_from_congruences(+Congruence_-
System, -Handle)
```

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of uint8 boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

```
ppl_new_Pointset_Powerset_Uint8_Box_from_Pointset_Powerset_Uint8_-
Box(+Handle_1, -Handle_2)
```

Builds a new pointset powerset `P_1` from the pointset_powerset_uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_Uint8_Box_from_Uint8_Box(+Handle_1,
-Handle_2)
```

Builds a new pointset powerset `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_Uint8_Box_from_Pointset_Powerset_Uint8_Box_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new pointset powerset `P_1` from the pointset_powerset_uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_Uint8_Box_from_Uint8_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new pointset powerset `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Uint8_Box domain.

```
ppl_delete_Pointset_Powerset_Uint8_Box(+Handle)
```

Invalidates the handle referenced by `Handle`*: this makes sure the corresponding resources will eventually be released.*

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Uint8_Box_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is empty.*

`ppl_Pointset_Powerset_Uint8_Box_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is the universe.*

`ppl_Pointset_Powerset_Uint8_Box_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is bounded.*

`ppl_Pointset_Powerset_Uint8_Box_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *contains an integer point.*

`ppl_Pointset_Powerset_Uint8_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is topologically closed.*

`ppl_Pointset_Powerset_Uint8_Box_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is discrete.*

`ppl_Pointset_Powerset_Uint8_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from above in the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_Uint8_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from below in the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_Uint8_Box_contains_Pointset_Powerset_Uint8_-Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in or equal to the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_Uint8_Box_strictly_contains_Pointset_Powerset_-Uint8_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in but not equal to the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_Uint8_Box_is_disjoint_from_Pointset_Powerset_-Uint8_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is disjoint from the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_Uint8_Box_geometrically_covers_Pointset_-Powerset_Uint8_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically covers the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_Uint8_Box_geometrically_equals_Pointset_Powerset_Uint8_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Uint8_Box_equals_Pointset_Powerset_Uint8_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Uint8_Box_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Pointset_Powerset_Uint8_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

`ppl_Pointset_Powerset_Uint8_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Uint8_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Uint8_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Uint8_Box_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Uint8_Box_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Uint8_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and,

otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_Uint8_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

`ppl_Pointset_Powerset_Uint8_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

`ppl_Pointset_Powerset_Uint8_Box_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

`ppl_Pointset_Powerset_Uint8_Box_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`.

`ppl_Pointset_Powerset_Uint8_Box_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the external memory in bytes occupied by the pointset powerset referenced by* `Handle`.

`ppl_Pointset_Powerset_Uint8_Box_size(+Handle, ?Number)`

Unifies `Number` *with the size of the memory in bytes occupied by the pointset powerset referenced by* `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Pointset_Powerset_Uint8_Box_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_Uint8_Box_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_Pointset_Powerset_Uint8_Box_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Pointset_Powerset_Uint8_Box_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Pointset_Powerset_Uint8_Box_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Pointset_Powerset_Uint8_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Pointset_Powerset_Uint8_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Pointset_Powerset_Uint8_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Pointset_Powerset_Uint8_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_Uint8_Box_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its topological closure.

`ppl_Pointset_Powerset_Uint8_Box_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its pairwise reduction.

`ppl_Pointset_Powerset_Uint8_Box_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its omega reduction.

`ppl_Pointset_Powerset_Uint8_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Pointset_Powerset_Uint8_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Uint8_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Uint8_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Uint8_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_Uint8_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_Uint8_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Uint8_Box_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Pointset_Powerset_Uint8_Box_generalized_affine_image_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Pointset_Powerset_Uint8_Box_generalized_affine_preimage_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_Uint8_Box_intersection_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Uint8_Box_upper_bound_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Uint8_Box_difference_assign(+Handle_1, +Handle_-
2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the difference of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Uint8_Box_time_elapse_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Uint8_Box_upper_bound_assign_if_exact(+Handle_1,
+Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` *referenced by* `Handle_1` *with the pointset powerset referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Pointset_Powerset_Uint8_Box_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification*

with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_Uint8_Box_concatenate_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the pointset powerset referenced by* `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_Uint8_Box_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the pointset powerset* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Pointset_Powerset_Uint8_Box_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the pointset powerset* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

```
ppl_Pointset_Powerset_Uint8_Box_remove_space_dimensions(+Handle,
+List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Uint8_Box_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`*.*

```
ppl_Pointset_Powerset_Uint8_Box_expand_space_dimension(+Handle, +PPL_-
Var, +Dimension_Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the pointset powerset referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

```
ppl_Pointset_Powerset_Uint8_Box_fold_space_dimensions(+Handle, +List_-
of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` *by folding the space dimensions contained in* `List_of_PPL_Vars` *into dimension* `PPL_Var`*. The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Uint8_Box_map_space_dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_Uint8_Box_iterator_from_iterator(+Iterator_-
1, -Iterator_2)
```

Builds a new iterator `it` *from the iterator referenced by* `Iterator_1`*.* `Iterator_2` *is unified with the handle for* `it`*.*

```
ppl_Pointset_Powerset_Uint8_Box_begin_iterator(+Handle, -Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Uint8_Box_end_iterator(+Handle, -Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Uint8_Box_iterator_equals_iterator(+Iterator_1,
+Iterator_2)
```

Succeeds if and only if the iterator referenced by `Iterator_1` *is equal to the iterator referenced by* `Iterator_2`*.*

```
ppl_Pointset_Powerset_Uint8_Box_iterator_increment(+Iterator)
```

Increments the iterator referenced by `Iterator` *so that it "points" to the next disjunct.*

```
ppl_Pointset_Powerset_Uint8_Box_iterator_decrement(+Iterator)
```

Decrements the iterator referenced by `Iterator` *so that it "points" to the previous disjunct.*

```
ppl_Pointset_Powerset_Uint8_Box_iterator_get_disjunct(+Iterator,
-Handle)
```

Unifies with `Handle` *a reference to the disjunct referred to by* `Iterator_1`*.*

`ppl_delete_Pointset_Powerset_Uint8_Box_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_Uint8_Box_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_Uint8_Box_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Uint8_Box_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Uint16_Box Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of uint16 boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of uint16 boxes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_Uint16_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Uint16_Box_from_space_dimension(3, universe, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_Uint16_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_Uint16_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of uint16 boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Uint16_Box_from_Pointset_Powerset_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Uint16_Box_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Uint16_Box_from_Pointset_Powerset_Uint16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Uint16_Box_from_Uint16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Uint16_Box domain.

`ppl_delete_Pointset_Powerset_Uint16_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Uint16_Box_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_Uint16_Box_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_Uint16_Box_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_Uint16_Box_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_Uint16_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_Uint16_Box_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_Uint16_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Uint16_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Uint16_Box_contains_Pointset_Powerset_Uint16_-Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Uint16_Box_strictly_contains_Pointset_Powerset_-Uint16_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Uint16_Box_is_disjoint_from_Pointset_Powerset_-Uint16_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Uint16_Box_geometrically_covers_Pointset_-Powerset_Uint16_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Uint16_Box_geometrically_equals_Pointset_-Powerset_Uint16_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Uint16_Box_equals_Pointset_Powerset_Uint16_-Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Uint16_Box_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Pointset_Powerset_Uint16_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

`ppl_Pointset_Powerset_Uint16_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Uint16_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Uint16_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Uint16_Box_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Uint16_Box_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Uint16_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_Uint16_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_Uint16_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

Coeff_1 is unified with the numerator of the supremum value and Coeff_2 with the denominator of the supremum value and Point with a point or closure point where Lin_Expr reaches this value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_Uint16_Box_minimize_with_point(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset P referenced by Handle is not empty and Lin_Expr is bounded from below in P.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value and Point with a point or closure point where Lin_Expr reaches this value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_Uint16_Box_external_memory_in_bytes(+Handle,
?Number)
```

Unifies Number with the size of the total memory in bytes occupied by the pointset powerset referenced by Handle.

```
ppl_Pointset_Powerset_Uint16_Box_total_memory_in_bytes(+Handle,
?Number)
```

Unifies Number with the size of the external memory in bytes occupied by the pointset powerset referenced by Handle.

```
ppl_Pointset_Powerset_Uint16_Box_size(+Handle, ?Number)
```

Unifies Number with the size of the memory in bytes occupied by the pointset powerset referenced by Handle.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_Uint16_Box_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by Handle on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Pointset_Powerset_Uint16_Box_add_constraint(+Handle, +Constraint)
```

Updates the pointset powerset referenced by Handle to one obtained by adding Constraint to its constraint system. For a C polyhedron, Constraint must be an equality or a non-strict inequality.

`ppl_Pointset_Powerset_Uint16_Box_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Pointset_Powerset_Uint16_Box_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Pointset_Powerset_Uint16_Box_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Pointset_Powerset_Uint16_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Pointset_Powerset_Uint16_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Pointset_Powerset_Uint16_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Pointset_Powerset_Uint16_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_Uint16_Box_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its topological closure.

`ppl_Pointset_Powerset_Uint16_Box_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its pairwise reduction.

`ppl_Pointset_Powerset_Uint16_Box_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its omega reduction.

`ppl_Pointset_Powerset_Uint16_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Pointset_Powerset_Uint16_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Uint16_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Uint16_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Uint16_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_Uint16_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_Uint16_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Uint16_Box_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Uint16_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Uint16_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_Uint16_Box_intersection_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Uint16_Box_upper_bound_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Uint16_Box_difference_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the difference of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Uint16_Box_time_elapse_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Uint16_Box_upper_bound_assign_if_exact(+Handle_-
1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` *referenced by* `Handle_1` *with the pointset powerset referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Pointset_Powerset_Uint16_Box_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Pointset_Powerset_Uint16_Box_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Pointset_Powerset_Uint16_Box_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Pointset_Powerset_Uint16_Box_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Pointset_Powerset_Uint16_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Uint16_Box_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Pointset_Powerset_Uint16_Box_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Pointset_Powerset_Uint16_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Uint16_Box_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_Uint16_Box_iterator_from_-
iterator(+Iterator_1, -Iterator_2)
```

Builds a new iterator `it` *from the iterator referenced by* `Iterator_1`*.* `Iterator_2` *is unified with the handle for* `it`*.*

```
ppl_Pointset_Powerset_Uint16_Box_begin_iterator(+Handle, -Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Uint16_Box_end_iterator(+Handle, -Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Uint16_Box_iterator_equals_iterator(+Iterator_1,
+Iterator_2)
```

Succeeds if and only if the iterator referenced by `Iterator_1` *is equal to the iterator referenced by* `Iterator_2`*.*

```
ppl_Pointset_Powerset_Uint16_Box_iterator_increment(+Iterator)
```

Increments the iterator referenced by `Iterator` *so that it "points" to the next disjunct.*

```
ppl_Pointset_Powerset_Uint16_Box_iterator_decrement(+Iterator)
```

Decrements the iterator referenced by `Iterator` *so that it "points" to the previous disjunct.*

```
ppl_Pointset_Powerset_Uint16_Box_iterator_get_disjunct(+Iterator,
-Handle)
```

Unifies with `Handle` *a reference to the disjunct referred to by* `Iterator_1`*.*

```
ppl_delete_Pointset_Powerset_Uint16_Box_iterator(+Iterator)
```

Invalidates the handle referenced by `Iterator`*: this makes sure the corresponding resources will eventually be released.*

```
ppl_Pointset_Powerset_Uint16_Box_add_disjunct(+Handle_1, +Handle_2)
```

Adds to the pointset powerset referenced by `Handle_1` *a disjunct referred to by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Uint16_Box_drop_disjunct(+Handle, +Iterator)
```

If `it` *is the iterator referred to by* `Iterator`*, drops from the pointset powerset referenced by* `Handle` *the disjunct pointed to by* `it` *and assigns to* `it` *an iterator to the next disjunct.*

```
ppl_Pointset_Powerset_Uint16_Box_drop_disjuncts(+Handle, +Iterator_1,
+Iterator_2)
```

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Uint32_Box Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of uint32 boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of uint32 boxes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_Uint32_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Uint32_Box_from_space_dimension(3, universe, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_Uint32_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_Uint32_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of uint32 boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Uint32_Box_from_Pointset_Powerset_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Uint32_Box_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Uint32_Box_from_Pointset_Powerset_Uint32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Uint32_Box_from_Uint32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Uint32_Box domain.

`ppl_delete_Pointset_Powerset_Uint32_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Uint32_Box_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_Uint32_Box_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_Uint32_Box_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_Uint32_Box_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_Uint32_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_Uint32_Box_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is discrete.*

`ppl_Pointset_Powerset_Uint32_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from above in the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_Uint32_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from below in the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_Uint32_Box_contains_Pointset_Powerset_Uint32_-Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in or equal to the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_Uint32_Box_strictly_contains_Pointset_Powerset_-Uint32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in but not equal to the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_Uint32_Box_is_disjoint_from_Pointset_Powerset_-Uint32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is disjoint from the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_Uint32_Box_geometrically_covers_Pointset_-Powerset_Uint32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically covers the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_Uint32_Box_geometrically_equals_Pointset_-Powerset_Uint32_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically equals the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_Uint32_Box_equals_Pointset_Powerset_Uint32_-Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` *is equal to the pointset powerset referenced by* `Handle_2`*.*

`ppl_Pointset_Powerset_Uint32_Box_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

`ppl_Pointset_Powerset_Uint32_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` *constrains the dimension* `PPL_Var`*.*

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

`ppl_Pointset_Powerset_Uint32_Box_space_dimension(+Handle, ?Dimension_-Type)`

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_Uint32_Box_affine_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the affine dimension of the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_Uint32_Box_relation_with_constraint(+Handle,
+Constraint, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Constraint`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Uint32_Box_relation_with_generator(+Handle,
+Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Generator`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Uint32_Box_relation_with_congruence(+Handle,
+Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Congruence`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Uint32_Box_maximize(+Handle, +Lin_Expr, ?Coeff_-
1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_Uint32_Box_minimize(+Handle, +Lin_Expr, ?Coeff_-
1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_Uint32_Box_maximize_with_point(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_Uint32_Box_minimize_with_point(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value and Point with a point or closure point where Lin_Expr reaches this value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_Uint32_Box_external_memory_in_bytes(+Handle,
?Number)
```

Unifies Number with the size of the total memory in bytes occupied by the pointset powerset referenced by Handle.

```
ppl_Pointset_Powerset_Uint32_Box_total_memory_in_bytes(+Handle,
?Number)
```

Unifies Number with the size of the external memory in bytes occupied by the pointset powerset referenced by Handle.

```
ppl_Pointset_Powerset_Uint32_Box_size(+Handle, ?Number)
```

Unifies Number with the size of the memory in bytes occupied by the pointset powerset referenced by Handle.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_Uint32_Box_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by Handle on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Pointset_Powerset_Uint32_Box_add_constraint(+Handle, +Constraint)
```

Updates the pointset powerset referenced by Handle to one obtained by adding Constraint to its constraint system. For a C polyhedron, Constraint must be an equality or a non-strict inequality.

```
ppl_Pointset_Powerset_Uint32_Box_add_congruence(+Handle, +Congruence)
```

Updates the pointset powerset referenced by Handle to one obtained by adding Congruence to its congruence system. For a C polyhedron, Congruence must be an equality.

```
ppl_Pointset_Powerset_Uint32_Box_add_constraints( +Handle,
+Constraint_System)
```

Updates the pointset powerset referenced by Handle to one obtained by adding to its constraint system the constraints in Constraint_System. For a C polyhedron, Constraints must be a list of equalities and non-strict inequalities.

`ppl_Pointset_Powerset_Uint32_Box_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Pointset_Powerset_Uint32_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Pointset_Powerset_Uint32_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Pointset_Powerset_Uint32_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Pointset_Powerset_Uint32_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_Uint32_Box_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its topological closure.

`ppl_Pointset_Powerset_Uint32_Box_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its pairwise reduction.

`ppl_Pointset_Powerset_Uint32_Box_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its omega reduction.

`ppl_Pointset_Powerset_Uint32_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Pointset_Powerset_Uint32_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Uint32_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Uint32_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Uint32_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_Uint32_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_Uint32_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Uint32_Box_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Uint32_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Uint32_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Pointset_Powerset_Uint32_Box_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Uint32_Box_upper_bound_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Uint32_Box_difference_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the difference of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Uint32_Box_time_elapse_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Uint32_Box_upper_bound_assign_if_exact(+Handle_-
1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` *referenced by* `Handle_1` *with the pointset powerset referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Pointset_Powerset_Uint32_Box_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_Uint32_Box_concatenate_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_Uint32_Box_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Pointset_Powerset_Uint32_Box_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Pointset_Powerset_Uint32_Box_remove_space_dimensions(+Handle,
+List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Uint32_Box_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Pointset_Powerset_Uint32_Box_expand_space_dimension(+Handle,
+PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Pointset_Powerset_Uint32_Box_fold_space_dimensions(+Handle,
+List_of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Uint32_Box_map_space_dimensions(+Handle, +P_-
Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

`ppl_new_Pointset_Powerset_Uint32_Box_iterator_from_-iterator(+Iterator_1, -Iterator_2)`

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

`ppl_Pointset_Powerset_Uint32_Box_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Uint32_Box_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Uint32_Box_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_Uint32_Box_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_Uint32_Box_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_Uint32_Box_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_Uint32_Box_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_Uint32_Box_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_Uint32_Box_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Uint32_Box_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Uint64_Box Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of uint64 boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of uint64 boxes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_Uint64_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Uint64_Box_from_space_dimension(3, universe, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_Uint64_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_Uint64_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of uint64 boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Uint64_Box_from_Pointset_Powerset_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Uint64_Box_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` *from the uint64_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

```
ppl_new_Pointset_Powerset_Uint64_Box_from_Pointset_Powerset_Uint64_-
Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new pointset powerset `P_1` *from the pointset_powerset_uint64_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

```
ppl_new_Pointset_Powerset_Uint64_Box_from_Uint64_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new pointset powerset `P_1` *from the uint64_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Uint64_Box domain.

```
ppl_delete_Pointset_Powerset_Uint64_Box(+Handle)
```

Invalidates the handle referenced by `Handle`*: this makes sure the corresponding resources will eventually be released.*

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

```
ppl_Pointset_Powerset_Uint64_Box_is_empty(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is empty.*

```
ppl_Pointset_Powerset_Uint64_Box_is_universe(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is the universe.*

```
ppl_Pointset_Powerset_Uint64_Box_is_bounded(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is bounded.*

```
ppl_Pointset_Powerset_Uint64_Box_contains_integer_point(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *contains an integer point.*

```
ppl_Pointset_Powerset_Uint64_Box_is_topologically_closed(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is topologically closed.*

```
ppl_Pointset_Powerset_Uint64_Box_is_discrete(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is discrete.*

```
ppl_Pointset_Powerset_Uint64_Box_bounds_from_above(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from above in the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Uint64_Box_bounds_from_below(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from below in the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_Uint64_Box_contains_Pointset_Powerset_Uint64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Uint64_Box_strictly_contains_Pointset_Powerset_Uint64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Uint64_Box_is_disjoint_from_Pointset_Powerset_Uint64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Uint64_Box_geometrically_covers_Pointset_Powerset_Uint64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Uint64_Box_geometrically_equals_Pointset_Powerset_Uint64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Uint64_Box_equals_Pointset_Powerset_Uint64_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Uint64_Box_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Pointset_Powerset_Uint64_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

`ppl_Pointset_Powerset_Uint64_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Uint64_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Uint64_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Constraint`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Uint64_Box_relation_with_generator(+Handle,
+Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Generator`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Uint64_Box_relation_with_congruence(+Handle,
+Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Congruence`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Uint64_Box_maximize(+Handle, +Lin_Expr, ?Coeff_-
1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Uint64_Box_minimize(+Handle, +Lin_Expr, ?Coeff_-
1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Uint64_Box_maximize_with_point(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Uint64_Box_minimize_with_point(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Uint64_Box_external_memory_in_bytes(+Handle,
?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_Uint64_Box_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Uint64_Box_size(+Handle, ?Number)`

Unifies `Number` with the size of the memory in bytes occupied by the pointset powerset referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Pointset_Powerset_Uint64_Box_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_Uint64_Box_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_Pointset_Powerset_Uint64_Box_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Pointset_Powerset_Uint64_Box_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Pointset_Powerset_Uint64_Box_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Pointset_Powerset_Uint64_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`.

```
ppl_Pointset_Powerset_Uint64_Box_refine_with_congruence( +Handle,
+Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`.

```
ppl_Pointset_Powerset_Uint64_Box_refine_with_constraints( +Handle,
+Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`.

```
ppl_Pointset_Powerset_Uint64_Box_refine_with_congruences( +Handle,
+Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Pointset_Powerset_Uint64_Box_topological_closure_assign(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

```
ppl_Pointset_Powerset_Uint64_Box_pairwise_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

```
ppl_Pointset_Powerset_Uint64_Box_omega_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

```
ppl_Pointset_Powerset_Uint64_Box_unconstrain_space_dimension(+Handle,
+PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`.

```
ppl_Pointset_Powerset_Uint64_Box_unconstrain_space_dimensions(+Handle,
+List_of_PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Uint64_Box_affine_image(+Handle, +PPL_Var,
+Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_Uint64_Box_affine_preimage(+Handle, +PPL_Var,
+Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_Uint64_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_Uint64_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_Uint64_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Uint64_Box_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Uint64_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Uint64_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_Uint64_Box_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`.

```
ppl_Pointset_Powerset_Uint64_Box_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`.

`ppl_Pointset_Powerset_Uint64_Box_difference_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the difference of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Uint64_Box_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the time elapse of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Uint64_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the pointset powerset `P_1` referenced by `Handle_1` with the pointset powerset referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Pointset_Powerset_Uint64_Box_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of pointset powerset `P_1` referenced by `Handle_1` and the pointset powerset `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Pointset_Powerset_Uint64_Box_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections

Adding New Dimensions to the Vector Space, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_Uint64_Box_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Pointset_Powerset_Uint64_Box_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Pointset_Powerset_Uint64_Box_remove_space_dimensions(+Handle,
+List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Uint64_Box_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Pointset_Powerset_Uint64_Box_expand_space_dimension(+Handle,
+PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Pointset_Powerset_Uint64_Box_fold_space_dimensions(+Handle,
+List_of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Uint64_Box_map_space_dimensions(+Handle, +P_-
Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_Uint64_Box_iterator_from_-
iterator(+Iterator_1, -Iterator_2)
```

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

`ppl_Pointset_Powerset_Uint64_Box_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Uint64_Box_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Uint64_Box_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_Uint64_Box_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_Uint64_Box_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_Uint64_Box_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_Uint64_Box_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_Uint64_Box_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_Uint64_Box_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Uint64_Box_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Rational_Box Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of rational boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of rational boxes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.

- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_Rational_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Rational_Box_from_space_dimension(3, universe, X)
   .
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_Rational_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_Rational_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of rational boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Rational_Box_from_Pointset_Powerset_Rational_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Rational_Box_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Rational_Box_from_Pointset_Powerset_Rational_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Rational_Box_from_Rational_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Rational_Box domain.

`ppl_delete_Pointset_Powerset_Rational_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Rational_Box_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_Rational_Box_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_Rational_Box_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_Rational_Box_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_Rational_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_Rational_Box_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_Rational_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Rational_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Rational_Box_contains_Pointset_Powerset_Rational_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Rational_Box_strictly_contains_Pointset_Powerset_Rational_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Rational_Box_is_disjoint_from_Pointset_Powerset_Rational_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Rational_Box_geometrically_covers_Pointset_Powerset_Rational_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Rational_Box_geometrically_equals_Pointset_Powerset_Rational_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Rational_Box_equals_Pointset_Powerset_Rational_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Rational_Box_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Pointset_Powerset_Rational_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

`ppl_Pointset_Powerset_Rational_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Rational_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Rational_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Rational_Box_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Generator`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Rational_Box_relation_with_congruence(+Handle,
+Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Congruence`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Rational_Box_maximize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Rational_Box_minimize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Rational_Box_maximize_with_point(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Rational_Box_minimize_with_point(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Rational_Box_external_memory_in_bytes(+Handle,
?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Rational_Box_total_memory_in_bytes(+Handle,
?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_Rational_Box_size(+Handle, ?Number)`

Unifies `Number` with the size of the memory in bytes occupied by the pointset powerset referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Pointset_Powerset_Rational_Box_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_Rational_Box_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_Pointset_Powerset_Rational_Box_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Pointset_Powerset_Rational_Box_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Pointset_Powerset_Rational_Box_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Pointset_Powerset_Rational_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Pointset_Powerset_Rational_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Pointset_Powerset_Rational_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Pointset_Powerset_Rational_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_Rational_Box_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its topological closure.

`ppl_Pointset_Powerset_Rational_Box_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its pairwise reduction.

`ppl_Pointset_Powerset_Rational_Box_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its omega reduction.

`ppl_Pointset_Powerset_Rational_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Pointset_Powerset_Rational_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Rational_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Rational_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Rational_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

`ppl_Pointset_Powerset_Rational_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

`ppl_Pointset_Powerset_Rational_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Rational_Box_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Rational_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Rational_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Pointset_Powerset_Rational_Box_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the intersection of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Rational_Box_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the upper bound of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Rational_Box_difference_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the difference of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Rational_Box_time_elapse_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Rational_Box_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` *referenced by* `Handle_1` *with the pointset powerset referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Pointset_Powerset_Rational_Box_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_Rational_Box_concatenate_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Pointset_Powerset_Rational_Box_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Pointset_Powerset_Rational_Box_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Pointset_Powerset_Rational_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Rational_Box_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Pointset_Powerset_Rational_Box_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Pointset_Powerset_Rational_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Rational_Box_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

`ppl_new_Pointset_Powerset_Rational_Box_iterator_from_iterator(+Iterator_1, -Iterator_2)`

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

`ppl_Pointset_Powerset_Rational_Box_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Rational_Box_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Rational_Box_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_Rational_Box_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_Rational_Box_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_Rational_Box_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_Rational_Box_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_Rational_Box_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_Rational_Box_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Rational_Box_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Z_Box Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of z boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of z boxes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_Z_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Z_Box_from_space_dimension(3, universe, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_Z_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_Z_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of z boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Z_Box_from_Pointset_Powerset_Z_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Z_Box_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Z_Box_from_Pointset_Powerset_Z_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Z_Box_from_Z_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Z_Box domain.

`ppl_delete_Pointset_Powerset_Z_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Z_Box_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_Z_Box_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_Z_Box_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_Z_Box_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_Z_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_Z_Box_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_Z_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Z_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Z_Box_contains_Pointset_Powerset_Z_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Z_Box_strictly_contains_Pointset_Powerset_Z_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Z_Box_is_disjoint_from_Pointset_Powerset_Z_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Z_Box_geometrically_covers_Pointset_Powerset_Z_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Z_Box_geometrically_equals_Pointset_Powerset_Z_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Z_Box_equals_Pointset_Powerset_Z_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Z_Box_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Pointset_Powerset_Z_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

`ppl_Pointset_Powerset_Z_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Z_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Z_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Z_Box_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Z_Box_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Z_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

Coeff_1 is unified with the numerator of the supremum value and Coeff_2 with the denominator of the supremum value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_Z_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset P referenced by Handle is not empty and Lin_Expr is bounded from below in P.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_Z_Box_maximize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset P referenced by Handle is not empty and Lin_Expr is bounded from above in P.

Coeff_1 is unified with the numerator of the supremum value and Coeff_2 with the denominator of the supremum value and Point with a point or closure point where Lin_Expr reaches this value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_Z_Box_minimize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset P referenced by Handle is not empty and Lin_Expr is bounded from below in P.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value and Point with a point or closure point where Lin_Expr reaches this value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_Z_Box_external_memory_in_bytes(+Handle, ?Number)
```

Unifies Number with the size of the total memory in bytes occupied by the pointset powerset referenced by Handle.

```
ppl_Pointset_Powerset_Z_Box_total_memory_in_bytes(+Handle, ?Number)
```

Unifies Number with the size of the external memory in bytes occupied by the pointset powerset referenced by Handle.

```
ppl_Pointset_Powerset_Z_Box_size(+Handle, ?Number)
```

Unifies Number with the size of the memory in bytes occupied by the pointset powerset referenced by Handle.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_Z_Box_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by Handle on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_Z_Box_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Pointset_Powerset_Z_Box_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Pointset_Powerset_Z_Box_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Pointset_Powerset_Z_Box_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Pointset_Powerset_Z_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Pointset_Powerset_Z_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Pointset_Powerset_Z_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Pointset_Powerset_Z_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_Z_Box_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its topological closure.

`ppl_Pointset_Powerset_Z_Box_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its pairwise reduction.

`ppl_Pointset_Powerset_Z_Box_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its omega reduction.

`ppl_Pointset_Powerset_Z_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Pointset_Powerset_Z_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Z_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Z_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Z_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_Z_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_Z_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Z_Box_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Pointset_Powerset_Z_Box_generalized_affine_image_lhs_rhs(+Handle,
+Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Pointset_Powerset_Z_Box_generalized_affine_preimage_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_Z_Box_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Z_Box_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Z_Box_difference_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the difference of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Z_Box_time_elapse_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Z_Box_upper_bound_assign_if_exact(+Handle_1,
+Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` *referenced by* `Handle_1` *with the pointset powerset referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Pointset_Powerset_Z_Box_simplify_using_context_assign(+Handle_1,
+Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Pointset_Powerset_Z_Box_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Pointset_Powerset_Z_Box_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Pointset_Powerset_Z_Box_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Pointset_Powerset_Z_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Z_Box_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Pointset_Powerset_Z_Box_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Pointset_Powerset_Z_Box_fold_space_dimensions(+Handle, +List_of_-PPL_Vars, +PPL_Var)`

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Z_Box_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

`ppl_new_Pointset_Powerset_Z_Box_iterator_from_iterator(+Iterator_1, -Iterator_2)`

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

`ppl_Pointset_Powerset_Z_Box_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Z_Box_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Z_Box_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_Z_Box_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_Z_Box_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_Z_Box_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_Z_Box_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_Z_Box_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_Z_Box_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Z_Box_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_BD_Shape_int8_t Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of bd shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_BD_Shape_int8_t_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_BD_Shape_int8_t_from_space_dimension(3, universe,
    X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_BD_Shape_int8_t_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_BD_Shape_int8_t_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_BD_Shape_int8_t_from_Pointset_Powerset_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_BD_Shape_int8_t_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_BD_Shape_int8_t_from_Pointset_Powerset_BD_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_BD_Shape_int8_t_from_BD_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_BD_Shape_int8_t domain.

`ppl_delete_Pointset_Powerset_BD_Shape_int8_t(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_BD_Shape_int8_t_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_BD_Shape_int8_t_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_BD_Shape_int8_t_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_contains_integer_point(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_is_topologically_closed(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_is_discrete(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_bounds_from_above(+Handle,
+Lin_Expr)
```

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_bounds_from_below(+Handle,
+Lin_Expr)
```

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_contains_Pointset_Powerset_BD_-
Shape_int8_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_strictly_contains_Pointset_-
Powerset_BD_Shape_int8_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_is_disjoint_from_Pointset_-
Powerset_BD_Shape_int8_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_geometrically_covers_Pointset_-
Powerset_BD_Shape_int8_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_geometrically_equals_Pointset_-
Powerset_BD_Shape_int8_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_equals_Pointset_Powerset_BD_-
Shape_int8_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_OK(+Handle)
```

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *constrains the dimension* `PPL_Var`*.*

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_space_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_affine_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the affine dimension of the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Constraint`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_relation_with_generator(+Handle,
+Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Generator`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_relation_with_-
congruence(+Handle, +Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Congruence`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_maximize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_minimize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_maximize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_minimize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_external_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the total memory in bytes occupied by the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_total_memory_in_bytes(+Handle,
?Number)
```

Unifies `Number` with the size of the external memory in bytes occupied by the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_size(+Handle, ?Number)
```

Unifies `Number` with the size of the memory in bytes occupied by the pointset powerset referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_add_constraint(+Handle,
+Constraint)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_add_congruence(+Handle,
+Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_add_constraints( +Handle,
+Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_add_congruences( +Handle,
+Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_refine_with_constraint( +Handle,
+Constraint)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_refine_with_congruence( +Handle,
+Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_refine_with_constraints(
+Handle, +Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_refine_with_congruences(
+Handle, +Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_topological_closure_-
assign(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_pairwise_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` its pairwise reduction.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_omega_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` its omega reduction.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_unconstrain_space_-
dimension(+Handle, +PPL_Var)
```

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_unconstrain_space_-
dimensions(+Handle, +List_of_PPL_Var)
```

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_affine_image(+Handle, +PPL_Var,
+Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` assigning the affine expression for `Lin_-Expr/Coeff` to `PPL_Var`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_affine_preimage(+Handle, +PPL_-
Var, +Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` substituting the affine expression for `Lin_-Expr/Coeff` to `PPL_Var`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_bounded_affine_image(+Handle,
+PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_bounded_affine_preimage(+Handle,
+PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_generalized_affine_-
image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_generalized_affine_-
preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_generalized_affine_image_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_generalized_affine_preimage_-
lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_intersection_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_upper_bound_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_difference_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the difference of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_time_elapse_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` *referenced by* `Handle_1` *with the pointset powerset referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Pointset_Powerset_BD_Shape_int8_t_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_BHZ03_H79_BHMZ05_widening_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening certified by the convergence certificate for H79.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_BHZ03_H79_H79_widening_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening certified by the convergence certificate for H79.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_BGP99_BHMZ05_extrapolation_-
assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening and the cardinality threshold `C_unsigned`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_BGP99_H79_extrapolation_-
assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_concatenate_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Pointset_Powerset_BD_Shape_int8_t_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Pointset_Powerset_BD_Shape_int8_t_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_BD_Shape_int8_t_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Pointset_Powerset_BD_Shape_int8_t_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Pointset_Powerset_BD_Shape_int8_t_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Pointset_Powerset_BD_Shape_int8_t_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

`ppl_new_Pointset_Powerset_BD_Shape_int8_t_iterator_from_iterator(+Iterator_1, -Iterator_2)`

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

`ppl_Pointset_Powerset_BD_Shape_int8_t_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_int8_t_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_iterator_equals_-
iterator(+Iterator_1, +Iterator_2)
```

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_iterator_increment(+Iterator)
```

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_iterator_decrement(+Iterator)
```

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_iterator_get_disjunct(+Iterator,
-Handle)
```

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

```
ppl_delete_Pointset_Powerset_BD_Shape_int8_t_iterator(+Iterator)
```

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_add_disjunct(+Handle_1,
+Handle_2)
```

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_drop_disjunct(+Handle,
+Iterator)
```

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

```
ppl_Pointset_Powerset_BD_Shape_int8_t_drop_disjuncts(+Handle,
+Iterator_1, +Iterator_2)
```

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_BD_Shape_int16_t Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of bd shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Pointset_Powerset_BD_Shape_int16_t_from_space_-
dimension(+Dimension_Type, +Universe_or_Empty, -Handle)
```

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_BD_Shape_int16_t_from_space_dimension(3, universe
   , X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_BD_Shape_int16_t_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_BD_Shape_int16_t_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_BD_Shape_int16_t_from_Pointset_Powerset_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_BD_Shape_int16_t_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_BD_Shape_int16_t_from_Pointset_Powerset_BD_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_BD_Shape_int16_t_from_BD_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_BD_Shape_int16_t domain.

`ppl_delete_Pointset_Powerset_BD_Shape_int16_t(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_BD_Shape_int16_t_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_BD_Shape_int16_t_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_BD_Shape_int16_t_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_BD_Shape_int16_t_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_BD_Shape_int16_t_is_topologically_-closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_BD_Shape_int16_t_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_BD_Shape_int16_t_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_contains_Pointset_Powerset_BD_-Shape_int16_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_strictly_contains_Pointset_-Powerset_BD_Shape_int16_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_is_disjoint_from_Pointset_-Powerset_BD_Shape_int16_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_geometrically_covers_Pointset_-
Powerset_BD_Shape_int16_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_geometrically_equals_Pointset_-
Powerset_BD_Shape_int16_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_equals_Pointset_Powerset_BD_-
Shape_int16_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_OK(+Handle)
```

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_space_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_affine_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_relation_with_-
generator(+Handle, +Generator, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_relation_with_-
congruence(+Handle, +Congruence, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_maximize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_minimize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_maximize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_minimize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_external_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the total memory in bytes occupied by the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_total_memory_in_bytes(+Handle,
?Number)
```

Unifies `Number` with the size of the external memory in bytes occupied by the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_size(+Handle, ?Number)
```

Unifies `Number` with the size of the memory in bytes occupied by the pointset powerset referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Pointset_Powerset_BD_Shape_int16_t_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_BD_Shape_int16_t_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_Pointset_Powerset_BD_Shape_int16_t_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Pointset_Powerset_BD_Shape_int16_t_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Pointset_Powerset_BD_Shape_int16_t_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Pointset_Powerset_BD_Shape_int16_t_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_refine_with_congruences(
+Handle, +Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Pointset_Powerset_BD_Shape_int16_t_topological_closure_-
assign(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

```
ppl_Pointset_Powerset_BD_Shape_int16_t_pairwise_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

```
ppl_Pointset_Powerset_BD_Shape_int16_t_omega_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

```
ppl_Pointset_Powerset_BD_Shape_int16_t_unconstrain_space_-
dimension(+Handle, +PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_unconstrain_space_-
dimensions(+Handle, +List_of_PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_BD_Shape_int16_t_affine_image(+Handle, +PPL_Var,
+Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_-Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_affine_preimage(+Handle, +PPL_-
Var, +Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_-Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_bounded_affine_image(+Handle,
+PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_bounded_affine_-
preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_BD_Shape_int16_t_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `PPL_Var` ⋈ `Lin_Expr/Coeff`, where ⋈ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` ⋈ `Lin_Expr/Coeff`, where ⋈ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Pointset_Powerset_BD_Shape_int16_t_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the intersection of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the upper bound of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_difference_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the difference of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the time elapse of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the pointset powerset `P_1` *referenced by* `Handle_1` *with the pointset powerset referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Pointset_Powerset_BD_Shape_int16_t_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_BD_Shape_int16_t_BHZ03_H79_BHMZ05_widening_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` *referenced by* `Handle_1` *the BHZ03-widening between* `P_1` *and the pointset powerset referenced by* `Handle_2`*, using the BHMZ05-widening certified by the convergence certificate for H79.*

```
ppl_Pointset_Powerset_BD_Shape_int16_t_BHZ03_H79_H79_widening_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` *referenced by* `Handle_1` *the BHZ03-widening between* `P_1` *and the pointset powerset referenced by* `Handle_2`*, using the H79-widening certified by the convergence certificate for H79.*

```
ppl_Pointset_Powerset_BD_Shape_int16_t_BGP99_BHMZ05_extrapolation_-
assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` *referenced by* `Handle_1` *the result of applying the BGP99 extrapolation operator between* `P_1` *and the pointset powerset referenced by* `Handle_2`*, using the BHMZ05-widening and the cardinality threshold* `C_unsigned`*.*

```
ppl_Pointset_Powerset_BD_Shape_int16_t_BGP99_H79_extrapolation_-
assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` *referenced by* `Handle_1` *the result of applying the BGP99 extrapolation operator between* `P_1` *and the pointset powerset referenced by* `Handle_2`*, using the H79-widening and the cardinality threshold* `C_unsigned`*.*

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Pointset_Powerset_BD_Shape_int16_t_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Pointset_Powerset_BD_Shape_int16_t_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Pointset_Powerset_BD_Shape_int16_t_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Pointset_Powerset_BD_Shape_int16_t_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_BD_Shape_int16_t_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Pointset_Powerset_BD_Shape_int16_t_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Pointset_Powerset_BD_Shape_int16_t_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

`ppl_new_Pointset_Powerset_BD_Shape_int16_t_iterator_from_iterator(+Iterator_1, -Iterator_2)`

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_BD_Shape_int16_t_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_BD_Shape_int16_t_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_BD_Shape_int16_t_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_BD_Shape_int16_t_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_BD_Shape_int16_t_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_BD_Shape_int16_t_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_BD_Shape_int32_t Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of bd shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_BD_Shape_int32_t_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_BD_Shape_int32_t_from_space_dimension(3, universe
   , X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_BD_Shape_int32_t_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_BD_Shape_int32_t_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_BD_Shape_int32_t_from_Pointset_Powerset_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_BD_Shape_int32_t_from_BD_Shape_int32_-
t(+Handle_1, -Handle_2)
```

Builds a new pointset powerset `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_BD_Shape_int32_t_from_Pointset_Powerset_BD_-
Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new pointset powerset `P_1` from the pointset_powerset_bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_BD_Shape_int32_t_from_BD_Shape_int32_t_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new pointset powerset `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_BD_Shape_int32_t domain.

```
ppl_delete_Pointset_Powerset_BD_Shape_int32_t(+Handle)
```

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_is_empty(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_is_universe(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_is_bounded(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_contains_integer_point(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_is_topologically_-
closed(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_is_discrete(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_BD_Shape_int32_t_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_int32_t_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_int32_t_contains_Pointset_Powerset_BD_Shape_int32_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_BD_Shape_int32_t_strictly_contains_Pointset_Powerset_BD_Shape_int32_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_BD_Shape_int32_t_is_disjoint_from_Pointset_Powerset_BD_Shape_int32_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_BD_Shape_int32_t_geometrically_covers_Pointset_Powerset_BD_Shape_int32_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_BD_Shape_int32_t_geometrically_equals_Pointset_Powerset_BD_Shape_int32_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_BD_Shape_int32_t_equals_Pointset_Powerset_BD_Shape_int32_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_BD_Shape_int32_t_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Pointset_Powerset_BD_Shape_int32_t_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

`ppl_Pointset_Powerset_BD_Shape_int32_t_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_affine_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the affine dimension of the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Constraint`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_relation_with_-
generator(+Handle, +Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Generator`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_relation_with_-
congruence(+Handle, +Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Congruence`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_maximize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_minimize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_maximize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_minimize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value and Point with a point or closure point where Lin_Expr reaches this value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_external_memory_in_-
bytes(+Handle, ?Number)
```

Unifies Number with the size of the total memory in bytes occupied by the pointset powerset referenced by Handle.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_total_memory_in_bytes(+Handle,
?Number)
```

Unifies Number with the size of the external memory in bytes occupied by the pointset powerset referenced by Handle.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_size(+Handle, ?Number)
```

Unifies Number with the size of the memory in bytes occupied by the pointset powerset referenced by Handle.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by Handle on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_add_constraint(+Handle,
+Constraint)
```

Updates the pointset powerset referenced by Handle to one obtained by adding Constraint to its constraint system. For a C polyhedron, Constraint must be an equality or a non-strict inequality.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_add_congruence(+Handle,
+Congruence)
```

Updates the pointset powerset referenced by Handle to one obtained by adding Congruence to its congruence system. For a C polyhedron, Congruence must be an equality.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_add_constraints( +Handle,
+Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_add_congruences( +Handle,
+Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_refine_with_constraint(
+Handle, +Constraint)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_refine_with_congruence(
+Handle, +Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_refine_with_constraints(
+Handle, +Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_refine_with_congruences(
+Handle, +Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_topological_closure_-
assign(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_pairwise_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_omega_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_unconstrain_space_-
dimension(+Handle, +PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_unconstrain_space_-
dimensions(+Handle, +List_of_PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Pointset_Powerset_BD_Shape_int32_t_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

`ppl_Pointset_Powerset_BD_Shape_int32_t_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

`ppl_Pointset_Powerset_BD_Shape_int32_t_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Pointset_Powerset_BD_Shape_int32_t_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Pointset_Powerset_BD_Shape_int32_t_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Pointset_Powerset_BD_Shape_int32_t_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Pointset_Powerset_BD_Shape_int32_t_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Pointset_Powerset_BD_Shape_int32_t_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral*

Difference; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_intersection_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the intersection of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_upper_bound_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the upper bound of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_difference_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the difference of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_time_elapse_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the time elapse of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` referenced by `Handle_1` with the pointset powerset referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` referenced by `Handle_1` and the pointset powerset `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_BHZ03_H79_BHMZ05_widening_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening certified by the convergence certificate for H79.

```
ppl_Pointset_Powerset_BD_Shape_int32_t_BHZ03_H79_H79_widening_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening certified by the convergence certificate for H79.

`ppl_Pointset_Powerset_BD_Shape_int32_t_BGP99_BHMZ05_extrapolation_assign(+Handle_1, +Handle_2, C_unsigned)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening and the cardinality threshold `C_unsigned`.

`ppl_Pointset_Powerset_BD_Shape_int32_t_BGP99_H79_extrapolation_assign(+Handle_1, +Handle_2, C_unsigned)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Pointset_Powerset_BD_Shape_int32_t_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Pointset_Powerset_BD_Shape_int32_t_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Pointset_Powerset_BD_Shape_int32_t_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Pointset_Powerset_BD_Shape_int32_t_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_BD_Shape_int32_t_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Pointset_Powerset_BD_Shape_int32_t_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Pointset_Powerset_BD_Shape_int32_t_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Pointset_Powerset_BD_Shape_int32_t_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

`ppl_new_Pointset_Powerset_BD_Shape_int32_t_iterator_from_iterator(+Iterator_1, -Iterator_2)`

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

`ppl_Pointset_Powerset_BD_Shape_int32_t_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_int32_t_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_int32_t_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_BD_Shape_int32_t_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_BD_Shape_int32_t_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` *so that it "points" to the previous disjunct.*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_iterator_get_-
disjunct(+Iterator, -Handle)
```

Unifies with `Handle` *a reference to the disjunct referred to by* `Iterator_1`*.*

```
ppl_delete_Pointset_Powerset_BD_Shape_int32_t_iterator(+Iterator)
```

Invalidates the handle referenced by `Iterator`*: this makes sure the corresponding resources will eventually be released.*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_add_disjunct(+Handle_1,
+Handle_2)
```

Adds to the pointset powerset referenced by `Handle_1` *a disjunct referred to by* `Handle_2`*.*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_drop_disjunct(+Handle,
+Iterator)
```

If `it` *is the iterator referred to by* `Iterator`*, drops from the pointset powerset referenced by* `Handle` *the disjunct pointed to by* `it` *and assigns to* `it` *an iterator to the next disjunct.*

```
ppl_Pointset_Powerset_BD_Shape_int32_t_drop_disjuncts(+Handle,
+Iterator_1, +Iterator_2)
```

If `it_1` *and* `it_2` *are the iterators referred to by* `Iterator_1` *and* `Iterator_2`*, respectively, drops from the pointset powerset referenced by* `Handle` *all the disjuncts from* `it_1` *to* `it_2` *(excluded).*

Predicates for the Pointset_Powerset_BD_Shape_int64_t Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of bd shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Pointset_Powerset_BD_Shape_int64_t_from_space_-
dimension(+Dimension_Type, +Universe_or_Empty, -Handle)
```

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` *dimensions; it is empty or the universe depending on whether* `Atom` *is* `empty` *or* `universe`*, respectively.* `Handle` *is unified with the handle for $\mathcal{P}$. Thus the query*

```
?- ppl_new_Pointset_Powerset_BD_Shape_int64_t_from_space_dimension(3, universe
   , X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` *bound to a valid handle for accessing it.*

`ppl_new_Pointset_Powerset_BD_Shape_int64_t_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_BD_Shape_int64_t_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_BD_Shape_int64_t_from_Pointset_Powerset_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_BD_Shape_int64_t_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_BD_Shape_int64_t_from_Pointset_Powerset_BD_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_BD_Shape_int64_t_from_BD_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_BD_Shape_int64_t domain.

`ppl_delete_Pointset_Powerset_BD_Shape_int64_t(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_BD_Shape_int64_t_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_BD_Shape_int64_t_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_BD_Shape_int64_t_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_BD_Shape_int64_t_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_BD_Shape_int64_t_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_BD_Shape_int64_t_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_BD_Shape_int64_t_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_int64_t_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_int64_t_contains_Pointset_Powerset_BD_Shape_int64_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_BD_Shape_int64_t_strictly_contains_Pointset_Powerset_BD_Shape_int64_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_BD_Shape_int64_t_is_disjoint_from_Pointset_Powerset_BD_Shape_int64_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_BD_Shape_int64_t_geometrically_covers_Pointset_Powerset_BD_Shape_int64_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_BD_Shape_int64_t_geometrically_equals_Pointset_Powerset_BD_Shape_int64_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_BD_Shape_int64_t_equals_Pointset_Powerset_BD_Shape_int64_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_BD_Shape_int64_t_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Pointset_Powerset_BD_Shape_int64_t_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

`ppl_Pointset_Powerset_BD_Shape_int64_t_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_int64_t_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_int64_t_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_BD_Shape_int64_t_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_BD_Shape_int64_t_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_BD_Shape_int64_t_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and,

otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_minimize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_maximize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_minimize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_external_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_total_memory_in_bytes(+Handle,
?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_size(+Handle, ?Number)
```

Unifies `Number` *with the size of the memory in bytes occupied by the pointset powerset referenced by* `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_BD_Shape_int64_t_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Pointset_Powerset_BD_Shape_int64_t_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Pointset_Powerset_BD_Shape_int64_t_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Pointset_Powerset_BD_Shape_int64_t_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Pointset_Powerset_BD_Shape_int64_t_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Pointset_Powerset_BD_Shape_int64_t_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Pointset_Powerset_BD_Shape_int64_t_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Pointset_Powerset_BD_Shape_int64_t_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_BD_Shape_int64_t_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

`ppl_Pointset_Powerset_BD_Shape_int64_t_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

`ppl_Pointset_Powerset_BD_Shape_int64_t_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

`ppl_Pointset_Powerset_BD_Shape_int64_t_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

`ppl_Pointset_Powerset_BD_Shape_int64_t_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Pointset_Powerset_BD_Shape_int64_t_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff to PPL_Var`*.*

`ppl_Pointset_Powerset_BD_Shape_int64_t_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff to PPL_Var`*.*

`ppl_Pointset_Powerset_BD_Shape_int64_t_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Pointset_Powerset_BD_Shape_int64_t_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Pointset_Powerset_BD_Shape_int64_t_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Pointset_Powerset_BD_Shape_int64_t_generalized_affine_-
preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` ⋈ `Lin_Expr/Coeff`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_generalized_affine_image_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_generalized_affine_preimage_-
lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_intersection_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the intersection of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_upper_bound_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the upper bound of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_difference_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the difference of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_time_elapse_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the time elapse of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` referenced by `Handle_1` with the pointset powerset referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` referenced by `Handle_1` and the pointset powerset `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_BHZ03_H79_BHMZ05_widening_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening certified by the convergence certificate for H79.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_BHZ03_H79_H79_widening_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening certified by the convergence certificate for H79.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_BGP99_BHMZ05_extrapolation_-
assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening and the cardinality threshold `C_unsigned`.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_BGP99_H79_extrapolation_-
assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_concatenate_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_remove_space_-
dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_expand_space_dimension(+Handle,
+PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_fold_space_dimensions(+Handle,
+List_of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_BD_Shape_int64_t_map_space_dimensions(+Handle,
+P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

`ppl_new_Pointset_Powerset_BD_Shape_int64_t_iterator_from_-iterator(+Iterator_1, -Iterator_2)`

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

`ppl_Pointset_Powerset_BD_Shape_int64_t_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_int64_t_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_int64_t_iterator_equals_-iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_BD_Shape_int64_t_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_BD_Shape_int64_t_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_BD_Shape_int64_t_iterator_get_-disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_BD_Shape_int64_t_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_BD_Shape_int64_t_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_BD_Shape_int64_t_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_BD_Shape_int64_t_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_BD_Shape_mpq_class Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of bd shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Pointset_Powerset_BD_Shape_mpq_class_from_space_-
dimension(+Dimension_Type, +Universe_or_Empty, -Handle)
```

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_BD_Shape_mpq_class_from_space_dimension(3, univer
   se, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

```
ppl_new_Pointset_Powerset_BD_Shape_mpq_class_from_-
constraints(+Constraint_System, -Handle)
```

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

```
ppl_new_Pointset_Powerset_BD_Shape_mpq_class_from_-
congruences(+Congruence_System, -Handle)
```

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

```
ppl_new_Pointset_Powerset_BD_Shape_mpq_class_from_Pointset_Powerset_-
BD_Shape_mpq_class(+Handle_1, -Handle_2)
```

Builds a new pointset powerset `P_1` from the pointset_powerset_bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_BD_Shape_mpq_class_from_BD_Shape_mpq_-
class(+Handle_1, -Handle_2)
```

Builds a new pointset powerset `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_BD_Shape_mpq_class_from_Pointset_Powerset_-
BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new pointset powerset `P_1` from the pointset_powerset_bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_BD_Shape_mpq_class_from_BD_Shape_mpq_class_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new pointset powerset `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_BD_Shape_mpq_class domain.

```
ppl_delete_Pointset_Powerset_BD_Shape_mpq_class(+Handle)
```

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_is_empty(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_is_universe(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_is_bounded(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_contains_integer_-
point(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_is_topologically_-
closed(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_is_discrete(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_bounds_from_above(+Handle,
+Lin_Expr)
```

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_bounds_from_below(+Handle,
+Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from below in the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_contains_Pointset_Powerset_-
BD_Shape_mpq_class(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in or equal to the pointset powerset referenced by* `Handle_1`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_strictly_contains_Pointset_-
Powerset_BD_Shape_mpq_class(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in but not equal to the pointset powerset referenced by* `Handle_1`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_is_disjoint_from_Pointset_-
Powerset_BD_Shape_mpq_class(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is disjoint from the pointset powerset referenced by* `Handle_1`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_geometrically_covers_-
Pointset_Powerset_BD_Shape_mpq_class(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically covers the pointset powerset referenced by* `Handle_1`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_geometrically_equals_-
Pointset_Powerset_BD_Shape_mpq_class(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically equals the pointset powerset referenced by* `Handle_1`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_equals_Pointset_Powerset_BD_-
Shape_mpq_class(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_1` *is equal to the pointset powerset referenced by* `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_OK(+Handle)
```

Succeeds only if the pointset powerset referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *constrains the dimension* `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_space_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_affine_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the affine dimension of the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Constraint`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_relation_with_-
generator(+Handle, +Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Generator`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_relation_with_-
congruence(+Handle, +Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Congruence`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_maximize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_minimize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_maximize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_minimize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_external_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the total memory in bytes occupied by the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_total_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the external memory in bytes occupied by the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_size(+Handle, ?Number)
```

Unifies `Number` with the size of the memory in bytes occupied by the pointset powerset referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_add_constraint(+Handle,
+Constraint)
```

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_add_congruence(+Handle,
+Congruence)
```

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_add_constraints( +Handle,
+Constraint_System)
```

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_add_congruences( +Handle,
+Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_refine_with_constraint(
+Handle, +Constraint)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_refine_with_congruence(
+Handle, +Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_refine_with_constraints(
+Handle, +Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_refine_with_congruences(
+Handle, +Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_topological_closure_-
assign(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_pairwise_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_omega_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_unconstrain_space_-
dimension(+Handle, +PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_unconstrain_space_-
dimensions(+Handle, +List_of_PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_affine_image(+Handle, +PPL_-
Var, +Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_affine_preimage(+Handle,
+PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_bounded_affine_image(+Handle,
+PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_bounded_affine_
preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_generalized_affine_
image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_generalized_affine_
preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_generalized_affine_image_
lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_generalized_affine_preimage_
lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_intersection_assign(+Handle_
1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`.

`ppl_Pointset_Powerset_BD_Shape_mpq_class_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the upper bound of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_BD_Shape_mpq_class_difference_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the difference of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_BD_Shape_mpq_class_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the time elapse of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_BD_Shape_mpq_class_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the pointset powerset `P_1` referenced by `Handle_1` with the pointset powerset referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Pointset_Powerset_BD_Shape_mpq_class_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of pointset powerset `P_1` referenced by `Handle_1` and the pointset powerset `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

`ppl_Pointset_Powerset_BD_Shape_mpq_class_BHZ03_H79_BHMZ05_widening_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening certified by the convergence certificate for H79.

`ppl_Pointset_Powerset_BD_Shape_mpq_class_BHZ03_H79_H79_widening_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening certified by the convergence certificate for H79.

`ppl_Pointset_Powerset_BD_Shape_mpq_class_BGP99_BHMZ05_extrapolation_assign(+Handle_1, +Handle_2, C_unsigned)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening and the cardinality threshold `C_unsigned`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_BGP99_H79_extrapolation_-
assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_concatenate_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_remove_space_-
dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_expand_space_-
dimension(+Handle, +PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_fold_space_-
dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_map_space_dimensions(+Handle,
+P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_BD_Shape_mpq_class_iterator_from_-
iterator(+Iterator_1, -Iterator_2)
```

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_begin_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_end_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_iterator_equals_-
iterator(+Iterator_1, +Iterator_2)
```

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_iterator_increment(+Iterator)
```

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_iterator_decrement(+Iterator)
```

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

```
ppl_Pointset_Powerset_BD_Shape_mpq_class_iterator_get_-
disjunct(+Iterator, -Handle)
```

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_BD_Shape_mpq_class_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_BD_Shape_mpq_class_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_BD_Shape_mpq_class_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_BD_Shape_mpq_class_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_BD_Shape_mpz_class Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of bd shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_BD_Shape_mpz_class_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_BD_Shape_mpz_class_from_space_dimension(3, univer
   se, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_BD_Shape_mpz_class_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_BD_Shape_mpz_class_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

```
ppl_new_Pointset_Powerset_BD_Shape_mpz_class_from_Pointset_Powerset_-
BD_Shape_mpz_class(+Handle_1, -Handle_2)
```

Builds a new pointset powerset `P_1` from the pointset_powerset_bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_BD_Shape_mpz_class_from_BD_Shape_mpz_-
class(+Handle_1, -Handle_2)
```

Builds a new pointset powerset `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_BD_Shape_mpz_class_from_Pointset_Powerset_-
BD_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new pointset powerset `P_1` from the pointset_powerset_bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_BD_Shape_mpz_class_from_BD_Shape_mpz_class_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new pointset powerset `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_BD_Shape_mpz_class domain.

```
ppl_delete_Pointset_Powerset_BD_Shape_mpz_class(+Handle)
```

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_is_empty(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is empty.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_is_universe(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is the universe.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_is_bounded(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is bounded.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_contains_integer_-
point(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *contains an integer point.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_is_topologically_-
closed(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is topologically closed.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_is_discrete(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is discrete.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_bounds_from_above(+Handle,
+Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from above in the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_bounds_from_below(+Handle,
+Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from below in the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_contains_Pointset_Powerset_-
BD_Shape_mpz_class(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in or equal to the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_strictly_contains_Pointset_-
Powerset_BD_Shape_mpz_class(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in but not equal to the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_is_disjoint_from_Pointset_-
Powerset_BD_Shape_mpz_class(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is disjoint from the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_geometrically_covers_-
Pointset_Powerset_BD_Shape_mpz_class(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically covers the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_geometrically_equals_-
Pointset_Powerset_BD_Shape_mpz_class(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically equals the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_equals_Pointset_Powerset_BD_-
Shape_mpz_class(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_1` *is equal to the pointset powerset referenced by* `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_OK(+Handle)
```

Succeeds only if the pointset powerset referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *constrains the dimension* `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_space_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_affine_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the affine dimension of the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Constraint`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_relation_with_-
generator(+Handle, +Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Generator`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_relation_with_-
congruence(+Handle, +Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Congruence`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_maximize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_minimize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_maximize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_minimize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_external_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the total memory in bytes occupied by the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_total_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the external memory in bytes occupied by the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_size(+Handle, ?Number)
```

Unifies `Number` with the size of the memory in bytes occupied by the pointset powerset referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_BD_Shape_mpz_class_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Pointset_Powerset_BD_Shape_mpz_class_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Pointset_Powerset_BD_Shape_mpz_class_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Pointset_Powerset_BD_Shape_mpz_class_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Pointset_Powerset_BD_Shape_mpz_class_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Pointset_Powerset_BD_Shape_mpz_class_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Pointset_Powerset_BD_Shape_mpz_class_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Pointset_Powerset_BD_Shape_mpz_class_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the

main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_BD_Shape_mpz_class_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its topological closure.

`ppl_Pointset_Powerset_BD_Shape_mpz_class_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its pairwise reduction.

`ppl_Pointset_Powerset_BD_Shape_mpz_class_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its omega reduction.

`ppl_Pointset_Powerset_BD_Shape_mpz_class_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Pointset_Powerset_BD_Shape_mpz_class_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_BD_Shape_mpz_class_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_BD_Shape_mpz_class_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_BD_Shape_mpz_class_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_BD_Shape_mpz_class_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_BD_Shape_mpz_class_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_BD_Shape_mpz_class_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by

Relation_Symbol.

ppl_Pointset_Powerset_BD_Shape_mpz_class_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)

Assigns to pointset powerset P *referenced by* Handle *the generalized image with respect to the generalized affine transfer relation* Lin_Expr_1 ⋈ Lin_Expr_2*, where* ⋈ *is the symbol represented by* Relation_Symbol*.*

ppl_Pointset_Powerset_BD_Shape_mpz_class_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)

Assigns to pointset powerset P *referenced by* Handle *the generalized preimage with respect to the generalized affine transfer relation* Lin_Expr_1 ⋈ Lin_Expr_2*, where* ⋈ *is the symbol represented by* Relation_Symbol*.*

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

ppl_Pointset_Powerset_BD_Shape_mpz_class_intersection_assign(+Handle_1, +Handle_2)

Assigns to the pointset powerset P *referenced by* Handle_1 *the intersection of* P *and the pointset powerset referenced by* Handle_2*.*

ppl_Pointset_Powerset_BD_Shape_mpz_class_upper_bound_assign(+Handle_1, +Handle_2)

Assigns to the pointset powerset P *referenced by* Handle_1 *the upper bound of* P *and the pointset powerset referenced by* Handle_2*.*

ppl_Pointset_Powerset_BD_Shape_mpz_class_difference_assign(+Handle_1, +Handle_2)

Assigns to the pointset powerset P *referenced by* Handle_1 *the difference of* P *and the pointset powerset referenced by* Handle_2*.*

ppl_Pointset_Powerset_BD_Shape_mpz_class_time_elapse_assign(+Handle_1, +Handle_2)

Assigns to the pointset powerset P *referenced by* Handle_1 *the time elapse of* P *and the pointset powerset referenced by* Handle_2*.*

ppl_Pointset_Powerset_BD_Shape_mpz_class_upper_bound_assign_if_exact(+Handle_1, +Handle_2)

Succeeds if the least upper bound of the pointset powerset P_1 *referenced by* Handle_1 *with the pointset powerset referenced by* Handle_2 *is exact; in which case the least upper bound is assigned to* P_1*; fails otherwise.*

ppl_Pointset_Powerset_BD_Shape_mpz_class_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)

Succeeds if and only if the intersection of pointset powerset P_1 *referenced by* Handle_1 *and the pointset powerset* P_2 *referenced by* Handle_2 *is non-empty. Assigns to* P_1 *its meet-preserving simplification with respect to* P_2*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_BHZ03_H79_BHMZ05_widening_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening certified by the convergence certificate for H79.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_BHZ03_H79_H79_widening_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening certified by the convergence certificate for H79.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_BGP99_BHMZ05_extrapolation_-
assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening and the cardinality threshold `C_unsigned`.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_BGP99_H79_extrapolation_-
assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_concatenate_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections

Adding New Dimensions to the Vector Space, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_remove_space_-
dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_expand_space_-
dimension(+Handle, +PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_fold_space_-
dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_map_space_dimensions(+Handle,
+P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_BD_Shape_mpz_class_iterator_from_-
iterator(+Iterator_1, -Iterator_2)
```

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_begin_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_end_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_iterator_equals_-
iterator(+Iterator_1, +Iterator_2)
```

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_iterator_increment(+Iterator)
```

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_iterator_decrement(+Iterator)
```

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_iterator_get_-
disjunct(+Iterator, -Handle)
```

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

```
ppl_delete_Pointset_Powerset_BD_Shape_mpz_class_iterator(+Iterator)
```

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_add_disjunct(+Handle_1,
+Handle_2)
```

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_drop_disjunct(+Handle,
+Iterator)
```

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

```
ppl_Pointset_Powerset_BD_Shape_mpz_class_drop_disjuncts(+Handle,
+Iterator_1, +Iterator_2)
```

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Octagonal_Shape_int8_t Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of octagonal shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_int8_t_from_space_-
dimension(+Dimension_Type, +Universe_or_Empty, -Handle)
```

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Octagonal_Shape_int8_t_from_space_dimension(3, un
   iverse, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_int8_t_from_-
constraints(+Constraint_System, -Handle)
```

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_int8_t_from_-
congruences(+Congruence_System, -Handle)
```

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_int8_t_from_Pointset_-
Powerset_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)
```

Builds a new pointset powerset `P_1` from the pointset_powerset_octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_int8_t_from_Octagonal_-
Shape_int8_t(+Handle_1, -Handle_2)
```

Builds a new pointset powerset `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_int8_t_from_Pointset_-
Powerset_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new pointset powerset `P_1` from the pointset_powerset_octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_int8_t_from_Octagonal_-
Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new pointset powerset `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Octagonal_Shape_int8_t domain.

```
ppl_delete_Pointset_Powerset_Octagonal_Shape_int8_t(+Handle)
```

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_is_empty(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_is_universe(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_is_bounded(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_contains_integer_-
point(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_is_topologically_-
closed(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_is_discrete(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_bounds_from_-
above(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from above in the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_bounds_from_-
below(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from below in the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_contains_Pointset_-
Powerset_Octagonal_Shape_int8_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in or equal to the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_strictly_contains_-
Pointset_Powerset_Octagonal_Shape_int8_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in but not equal to the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_is_disjoint_from_-
Pointset_Powerset_Octagonal_Shape_int8_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is disjoint from the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_geometrically_covers_-
Pointset_Powerset_Octagonal_Shape_int8_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically covers the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_geometrically_equals_-
Pointset_Powerset_Octagonal_Shape_int8_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically equals the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_equals_Pointset_-
Powerset_Octagonal_Shape_int8_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_1` *is equal to the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_OK(+Handle)
```

Succeeds only if the pointset powerset referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_constrains(+Handle,
+PPL_Var)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *constrains the dimension* `PPL_Var`*.*

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_space_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_affine_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the affine dimension of the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Constraint`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_relation_with_-
generator(+Handle, +Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Generator`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_relation_with_-
congruence(+Handle, +Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Congruence`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_maximize(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_minimize(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_maximize_with_-
point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_minimize_with_-
point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the*

atom `false`.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the external memory in bytes occupied by the pointset powerset referenced by* `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_size(+Handle, ?Number)`

Unifies `Number` *with the size of the memory in bytes occupied by the pointset powerset referenced by* `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its topological closure.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its pairwise reduction.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its omega reduction.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the intersection of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_upper_bound_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the upper bound of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_difference_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the difference of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_time_elapse_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the time elapse of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` referenced by `Handle_1` with the pointset powerset referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` referenced by `Handle_1` and the pointset powerset `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_BHZ03_H79_BHMZ05_-
widening_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening certified by the convergence certificate for H79.

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_BGP99_BHMZ05_-
extrapolation_assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the pointset powerset referenced by `Handle` *by folding the space dimensions contained in* `List_of_PPL_Vars` *into dimension* `PPL_Var`*. The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_map_space_-
dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_int8_t_iterator_from_-
iterator(+Iterator_1, -Iterator_2)
```

Builds a new iterator `it` *from the iterator referenced by* `Iterator_1`*.* `Iterator_2` *is unified with the handle for* `it`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_begin_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_end_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_iterator_equals_-
iterator(+Iterator_1, +Iterator_2)
```

Succeeds if and only if the iterator referenced by `Iterator_1` *is equal to the iterator referenced by* `Iterator_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_iterator_-
increment(+Iterator)
```

Increments the iterator referenced by `Iterator` *so that it "points" to the next disjunct.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_iterator_-
decrement(+Iterator)
```

Decrements the iterator referenced by `Iterator` *so that it "points" to the previous disjunct.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int8_t_iterator_get_-
disjunct(+Iterator, -Handle)
```

Unifies with `Handle` *a reference to the disjunct referred to by* `Iterator_1`*.*

```
ppl_delete_Pointset_Powerset_Octagonal_Shape_int8_t_-
iterator(+Iterator)
```

Invalidates the handle referenced by `Iterator`*: this makes sure the corresponding resources will eventually be released.*

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_int8_t_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Octagonal_Shape_int16_t Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of octagonal shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int16_t_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Octagonal_Shape_int16_t_from_space_dimension(3, u
   niverse, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int16_t_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int16_t_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int16_t_from_Pointset_-Powerset_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int16_t_from_Octagonal_-Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int16_t_from_Pointset_-Powerset_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int16_t_from_Octagonal_-Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Octagonal_Shape_int16_t domain.

`ppl_delete_Pointset_Powerset_Octagonal_Shape_int16_t(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_contains_integer_-point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_is_topologically_-closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_bounds_from_-above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_bounds_from_-below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_contains_Pointset_-Powerset_Octagonal_Shape_int16_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_strictly_contains_-Pointset_Powerset_Octagonal_Shape_int16_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_is_disjoint_from_-Pointset_Powerset_Octagonal_Shape_int16_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_geometrically_covers_-Pointset_Powerset_Octagonal_Shape_int16_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_geometrically_equals_-Pointset_Powerset_Octagonal_Shape_int16_t(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_equals_Pointset_-
Powerset_Octagonal_Shape_int16_t(+Handle_1, +Handle_2)

Succeeds if and only if the pointset powerset referenced by `Handle_1` *is equal to the pointset powerset referenced by* `Handle_2`.

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_OK(+Handle)

Succeeds only if the pointset powerset referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_constrains(+Handle,
+PPL_Var)

Succeeds if and only if the pointset powerset referenced by `Handle` *constrains the dimension* `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_space_dimension(+Handle,
?Dimension_Type)

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the pointset powerset referenced by* `Handle`.

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_affine_-
dimension(+Handle, ?Dimension_Type)

Unifies `Dimension_Type` *with the affine dimension of the pointset powerset referenced by* `Handle`.

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Constraint`. *The possible relations are listed in the grammar rules above.*

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_relation_with_-
generator(+Handle, +Generator, ?Relation_List)

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Generator`. *The possible relations are listed in the grammar rules above.*

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_relation_with_-
congruence(+Handle, +Congruence, ?Relation_List)

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Congruence`. *The possible relations are listed in the grammar rules above.*

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_maximize(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean)

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_minimize(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean)

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_maximize_with_-
point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_minimize_with_-
point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_external_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_total_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_size(+Handle, ?Number)
```

Unifies `Number` *with the size of the memory in bytes occupied by the pointset powerset referenced by* `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the

main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by*

Relation_Symbol.

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_generalized_affine_-
image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)

Assigns to pointset powerset P *referenced by* Handle *the generalized image with respect to the generalized affine transfer relation* Lin_Expr_1 ⋈ Lin_Expr_2*, where* ⋈ *is the symbol represented by* Relation_Symbol*.*

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_generalized_affine_-
preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)

Assigns to pointset powerset P *referenced by* Handle *the generalized preimage with respect to the generalized affine transfer relation* Lin_Expr_1 ⋈ Lin_Expr_2*, where* ⋈ *is the symbol represented by* Relation_Symbol*.*

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_intersection_-
assign(+Handle_1, +Handle_2)

Assigns to the pointset powerset P *referenced by* Handle_1 *the intersection of* P *and the pointset powerset referenced by* Handle_2*.*

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_upper_bound_-
assign(+Handle_1, +Handle_2)

Assigns to the pointset powerset P *referenced by* Handle_1 *the upper bound of* P *and the pointset powerset referenced by* Handle_2*.*

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_difference_-
assign(+Handle_1, +Handle_2)

Assigns to the pointset powerset P *referenced by* Handle_1 *the difference of* P *and the pointset powerset referenced by* Handle_2*.*

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_time_elapse_-
assign(+Handle_1, +Handle_2)

Assigns to the pointset powerset P *referenced by* Handle_1 *the time elapse of* P *and the pointset powerset referenced by* Handle_2*.*

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)

Succeeds if the least upper bound of the pointset powerset P_1 *referenced by* Handle_1 *with the pointset powerset referenced by* Handle_2 *is exact; in which case the least upper bound is assigned to* P_1*; fails otherwise.*

ppl_Pointset_Powerset_Octagonal_Shape_int16_t_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)

Succeeds if and only if the intersection of pointset powerset P_1 *referenced by* Handle_1 *and the pointset powerset* P_2 *referenced by* Handle_2 *is non-empty. Assigns to* P_1 *its meet-preserving simplification with respect to* P_2*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_BHZ03_H79_BHMZ05_-
widening_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening certified by the convergence certificate for H79.

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_BGP99_BHMZ05_-
extrapolation_assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_concatenate_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_add_space_dimensions_-
and_embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_add_space_dimensions_-
and_project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the pointset powerset* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_remove_space_-
dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_expand_space_-
dimension(+Handle, +PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the pointset powerset referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_fold_space_-
dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` *by folding the space dimensions contained in* `List_of_PPL_Vars` *into dimension* `PPL_Var`*. The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_map_space_-
dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_int16_t_iterator_from_-
iterator(+Iterator_1, -Iterator_2)
```

Builds a new iterator `it` *from the iterator referenced by* `Iterator_1`*.* `Iterator_2` *is unified with the handle for* `it`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_begin_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int16_t_end_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_Octagonal_Shape_int16_t_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_int16_t_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Octagonal_Shape_int32_t Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of octagonal shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int32_t_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Octagonal_Shape_int32_t_from_space_dimension(3, u
   niverse, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int32_t_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int32_t_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int32_t_from_Pointset_Powerset_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int32_t_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int32_t_from_Pointset_Powerset_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int32_t_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` *from the octagonal_shape_int32_t referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Octagonal_Shape_int32_t domain.

```
ppl_delete_Pointset_Powerset_Octagonal_Shape_int32_t(+Handle)
```

Invalidates the handle referenced by `Handle`*: this makes sure the corresponding resources will eventually be released.*

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_is_empty(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is empty.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_is_universe(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is the universe.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_is_bounded(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is bounded.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_contains_integer_
point(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *contains an integer point.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_is_topologically_
closed(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is topologically closed.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_is_discrete(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is discrete.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_bounds_from_
above(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from above in the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_bounds_from_
below(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from below in the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_contains_Pointset_
Powerset_Octagonal_Shape_int32_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in or equal to the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_strictly_contains_-
Pointset_Powerset_Octagonal_Shape_int32_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_is_disjoint_from_-
Pointset_Powerset_Octagonal_Shape_int32_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_geometrically_covers_-
Pointset_Powerset_Octagonal_Shape_int32_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_geometrically_equals_-
Pointset_Powerset_Octagonal_Shape_int32_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_equals_Pointset_-
Powerset_Octagonal_Shape_int32_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_OK(+Handle)
```

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_constrains(+Handle,
+PPL_Var)
```

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_space_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_affine_-
dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the total memory in bytes occupied by the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the external memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_size(+Handle, ?Number)
```

Unifies `Number` *with the size of the memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_add_constraint(+Handle,
+Constraint)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_add_congruence(+Handle,
+Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_add_constraints(
+Handle, +Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_add_congruences(
+Handle, +Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_refine_with_constraint(
+Handle, +Constraint)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_refine_with_congruence(
+Handle, +Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_refine_with_constraints(
+Handle, +Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_refine_with_congruences(
+Handle, +Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_topological_closure_-
assign(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_pairwise_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_omega_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_unconstrain_space_-
dimension(+Handle, +PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_unconstrain_space_-
dimensions(+Handle, +List_of_PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_affine_image(+Handle,
+PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_-Expr/Coeff to PPL_Var`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_affine_preimage(+Handle,
+PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_difference_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the difference of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the time elapse of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the pointset powerset `P_1` referenced by `Handle_1` with the pointset powerset referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of pointset powerset `P_1` referenced by `Handle_1` and the pointset powerset `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_BHZ03_H79_BHMZ05_widening_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening certified by the convergence certificate for H79.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_BGP99_BHMZ05_extrapolation_assign(+Handle_1, +Handle_2, C_unsigned)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_concatenate_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_add_space_dimensions_-
and_embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_add_space_dimensions_-
and_project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_remove_space_-
dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_expand_space_-
dimension(+Handle, +PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_fold_space_-
dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Octagonal_Shape_int32_t_map_space_-
dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int32_t_iterator_from_iterator(+Iterator_1, -Iterator_2)`

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_Octagonal_Shape_int32_t_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_int32_t_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Octagonal_Shape_int64_t Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of octagonal shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_int64_t_from_space_-
dimension(+Dimension_Type, +Universe_or_Empty, -Handle)
```

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Octagonal_Shape_int64_t_from_space_dimension(3, u
   niverse, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_int64_t_from_-
constraints(+Constraint_System, -Handle)
```

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_int64_t_from_-
congruences(+Congruence_System, -Handle)
```

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int64_t_from_Pointset_-Powerset_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int64_t_from_Octagonal_-Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int64_t_from_Pointset_-Powerset_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int64_t_from_Octagonal_-Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Octagonal_Shape_int64_t domain.

`ppl_delete_Pointset_Powerset_Octagonal_Shape_int64_t(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_contains_integer_-point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *contains an integer point.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_is_topologically_-
closed(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is topologically closed.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_is_discrete(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is discrete.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_bounds_from_-
above(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from above in the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_bounds_from_-
below(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from below in the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_contains_Pointset_-
Powerset_Octagonal_Shape_int64_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in or equal to the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_strictly_contains_-
Pointset_Powerset_Octagonal_Shape_int64_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in but not equal to the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_is_disjoint_from_-
Pointset_Powerset_Octagonal_Shape_int64_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is disjoint from the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_geometrically_covers_-
Pointset_Powerset_Octagonal_Shape_int64_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically covers the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_geometrically_equals_-
Pointset_Powerset_Octagonal_Shape_int64_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically equals the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_equals_Pointset_-
Powerset_Octagonal_Shape_int64_t(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_1` *is equal to the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_OK(+Handle)
```

Succeeds only if the pointset powerset referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_constrains(+Handle,
+PPL_Var)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *constrains the dimension* `PPL_Var`*.*

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_space_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_affine_-
dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_relation_with_-
generator(+Handle, +Generator, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_relation_with_-
congruence(+Handle, +Congruence, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_maximize(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_minimize(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_maximize_with_-
point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the

atom `false`.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the external memory in bytes occupied by the pointset powerset referenced by* `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_size(+Handle, ?Number)`

Unifies `Number` *with the size of the memory in bytes occupied by the pointset powerset referenced by* `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_add_constraints(
+Handle, +Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_add_congruences(
+Handle, +Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_refine_with_constraint(
+Handle, +Constraint)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_refine_with_congruence(
+Handle, +Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_refine_with_constraints(
+Handle, +Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_refine_with_congruences(
+Handle, +Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_topological_closure_-
assign(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_pairwise_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_omega_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_intersection_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_upper_bound_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_difference_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the difference of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_time_elapse_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` *referenced by* `Handle_1` *with the pointset powerset referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_BHZ03_H79_BHMZ05_-
widening_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` *referenced by* `Handle_1` *the BHZ03-widening between* `P_1` *and the pointset powerset referenced by* `Handle_2`*, using the BHMZ05-widening certified by the convergence certificate for H79.*

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_BGP99_BHMZ05_-
extrapolation_assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_concatenate_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_add_space_dimensions_-
and_embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_add_space_dimensions_-
and_project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_remove_space_-
dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Octagonal_Shape_int64_t_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

`ppl_new_Pointset_Powerset_Octagonal_Shape_int64_t_iterator_from_iterator(+Iterator_1, -Iterator_2)`

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_Octagonal_Shape_int64_t_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_int64_t_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Octagonal_Shape_mpq_class Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of octagonal shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_Octagonal_Shape_mpq_class_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Octagonal_Shape_mpq_class_from_space_dimension(3,
    universe, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_Octagonal_Shape_mpq_class_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` *from* `Constraint_System`*.* `Handle` *is unified with the handle for* `P`*.*

```
ppl_new_Pointset_Powerset_Octagonal_Shape_mpq_class_from_-
congruences(+Congruence_System, -Handle)
```

Builds a new pointset powerset `P` *from* `Congruence_System`*.* `Handle` *is unified with the handle for* `P`*.*

Predicates that build new pointset powersets of octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_mpq_class_from_Pointset_-
Powerset_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)
```

Builds a new pointset powerset `P_1` *from the pointset_powerset_octagonal_shape_mpq_class referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

```
ppl_new_Pointset_Powerset_Octagonal_Shape_mpq_class_from_Octagonal_-
Shape_mpq_class(+Handle_1, -Handle_2)
```

Builds a new pointset powerset `P_1` *from the octagonal_shape_mpq_class referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

```
ppl_new_Pointset_Powerset_Octagonal_Shape_mpq_class_from_Pointset_-
Powerset_Octagonal_Shape_mpq_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new pointset powerset `P_1` *from the pointset_powerset_octagonal_shape_mpq_class referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

```
ppl_new_Pointset_Powerset_Octagonal_Shape_mpq_class_from_Octagonal_-
Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new pointset powerset `P_1` *from the octagonal_shape_mpq_class referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Octagonal_Shape_mpq_class domain.

```
ppl_delete_Pointset_Powerset_Octagonal_Shape_mpq_class(+Handle)
```

Invalidates the handle referenced by `Handle`*: this makes sure the corresponding resources will eventually be released.*

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_contains_Pointset_Powerset_Octagonal_Shape_mpq_class(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_strictly_contains_Pointset_Powerset_Octagonal_Shape_mpq_class(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_is_disjoint_from_Pointset_Powerset_Octagonal_Shape_mpq_class(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_geometrically_covers_Pointset_Powerset_Octagonal_Shape_mpq_class(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_geometrically_equals_Pointset_Powerset_Octagonal_Shape_mpq_class(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_equals_Pointset_Powerset_Octagonal_Shape_mpq_class(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

Coeff_1 is unified with the numerator of the supremum value and Coeff_2 with the denominator of the supremum value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_minimize(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset P referenced by Handle is not empty and Lin_Expr is bounded from below in P.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_maximize_with_-
point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset P referenced by Handle is not empty and Lin_Expr is bounded from above in P.

Coeff_1 is unified with the numerator of the supremum value and Coeff_2 with the denominator of the supremum value and Point with a point or closure point where Lin_Expr reaches this value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_minimize_with_-
point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset P referenced by Handle is not empty and Lin_Expr is bounded from below in P.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value and Point with a point or closure point where Lin_Expr reaches this value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_external_memory_in_-
bytes(+Handle, ?Number)
```

Unifies Number with the size of the total memory in bytes occupied by the pointset powerset referenced by Handle.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_total_memory_in_-
bytes(+Handle, ?Number)
```

Unifies Number with the size of the external memory in bytes occupied by the pointset powerset referenced by Handle.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_size(+Handle, ?Number)
```

Unifies Number with the size of the memory in bytes occupied by the pointset powerset referenced by Handle.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by Handle on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_add_constraints(+Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_add_congruences(+Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff to PPL_Var`*.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff to PPL_Var`*.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` ⋈ `Lin_Expr/Coeff`, where ⋈ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the intersection of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the upper bound of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_difference_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the difference of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the time elapse of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the pointset powerset `P_1` referenced by `Handle_1` with the pointset powerset referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_BHZ03_H79_BHMZ05_-
widening_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` *referenced by* `Handle_1` *the BHZ03-widening between* `P_1` *and the pointset powerset referenced by* `Handle_2`*, using the BHMZ05-widening certified by the convergence certificate for H79.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_BGP99_BHMZ05_-
extrapolation_assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` *referenced by* `Handle_1` *the result of applying the BGP99 extrapolation operator between* `P_1` *and the pointset powerset referenced by* `Handle_2`*, using the BHMZ05-widening and the cardinality threshold* `C_unsigned`*.*

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_concatenate_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_add_space_dimensions_-
and_embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_add_space_dimensions_-
and_project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_remove_space_-
dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_expand_space_-
dimension(+Handle, +PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_fold_space_-
dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_map_space_-
dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_mpq_class_iterator_from_-
iterator(+Iterator_1, -Iterator_2)
```

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_begin_-
iterator(+Handle, -Iterator)
```

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_Octagonal_Shape_mpq_class_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_mpq_class_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Octagonal_Shape_mpz_class Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of octagonal shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_mpz_class_from_space_-
dimension(+Dimension_Type, +Universe_or_Empty, -Handle)
```

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Octagonal_Shape_mpz_class_from_space_dimension(3,
    universe, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_mpz_class_from_-
constraints(+Constraint_System, -Handle)
```

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_mpz_class_from_-
congruences(+Congruence_System, -Handle)
```

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_mpz_class_from_Pointset_-
Powerset_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)
```

Builds a new pointset powerset `P_1` from the pointset_powerset_octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_mpz_class_from_Octagonal_-
Shape_mpz_class(+Handle_1, -Handle_2)
```

Builds a new pointset powerset `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_mpz_class_from_Pointset_-
Powerset_Octagonal_Shape_mpz_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new pointset powerset `P_1` from the pointset_powerset_octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_mpz_class_from_Octagonal_-
Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new pointset powerset `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Octagonal_Shape_mpz_class domain.

```
ppl_delete_Pointset_Powerset_Octagonal_Shape_mpz_class(+Handle)
```

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_is_empty(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_is_universe(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_is_bounded(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_contains_integer_-
point(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_is_topologically_-
closed(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_is_discrete(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_bounds_from_-
above(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_bounds_from_-
below(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_contains_Pointset_-Powerset_Octagonal_Shape_mpz_class(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_strictly_contains_-Pointset_Powerset_Octagonal_Shape_mpz_class(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_is_disjoint_from_-Pointset_Powerset_Octagonal_Shape_mpz_class(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_geometrically_covers_-Pointset_Powerset_Octagonal_Shape_mpz_class(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_geometrically_equals_-Pointset_Powerset_Octagonal_Shape_mpz_class(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_equals_Pointset_-Powerset_Octagonal_Shape_mpz_class(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

`ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_space_-dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_affine_-dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Constraint`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_relation_with_-
generator(+Handle, +Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Generator`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_relation_with_-
congruence(+Handle, +Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Congruence`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_maximize(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_minimize(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_maximize_with_-
point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_minimize_with_-
point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_external_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_total_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_size(+Handle, ?Number)
```

Unifies `Number` *with the size of the memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_add_-
constraint(+Handle, +Constraint)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_add_-
congruence(+Handle, +Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_add_constraints(
+Handle, +Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_add_congruences(
+Handle, +Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of*

equalities.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_refine_with_-
constraint( +Handle, +Constraint)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_refine_with_-
congruence( +Handle, +Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_refine_with_-
constraints( +Handle, +Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_refine_with_-
congruences( +Handle, +Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_topological_closure_-
assign(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_pairwise_-
reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_omega_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_unconstrain_space_-
dimension(+Handle, +PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_unconstrain_space_-
dimensions(+Handle, +List_of_PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_affine_image(+Handle,
+PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_upper_bound_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_difference_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the difference of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_time_elapse_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_upper_bound_assign_-
if_exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` *referenced by* `Handle_1` *with the pointset powerset referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_simplify_using_-
context_assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_BHZ03_H79_BHMZ05_-
widening_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` *referenced by* `Handle_1` *the BHZ03-widening between* `P_1` *and the pointset powerset referenced by* `Handle_2`*, using the BHMZ05-widening certified by the convergence certificate for H79.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_BGP99_BHMZ05_-
extrapolation_assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` *referenced by* `Handle_1` *the result of applying the BGP99 extrapolation operator between* `P_1` *and the pointset powerset referenced by* `Handle_2`*, using the BHMZ05-widening and the cardinality threshold* `C_unsigned`*.*

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_concatenate_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_add_space_dimensions_-
and_embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the pointset powerset* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_add_space_dimensions_-
and_project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the pointset powerset* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_remove_space_-
dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_expand_space_-
dimension(+Handle, +PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the pointset powerset referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_fold_space_-
dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` *by folding the space dimensions contained in* `List_of_PPL_Vars` *into dimension* `PPL_Var`*. The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_map_space_-
dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_mpz_class_iterator_from_-
iterator(+Iterator_1, -Iterator_2)
```

Builds a new iterator `it` *from the iterator referenced by* `Iterator_1`*.* `Iterator_2` *is unified with the handle for* `it`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_begin_-
iterator(+Handle, -Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_end_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_iterator_equals_-
iterator(+Iterator_1, +Iterator_2)
```

Succeeds if and only if the iterator referenced by `Iterator_1` *is equal to the iterator referenced by* `Iterator_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_iterator_-
increment(+Iterator)
```

Increments the iterator referenced by `Iterator` *so that it "points" to the next disjunct.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_iterator_-
decrement(+Iterator)
```

Decrements the iterator referenced by `Iterator` *so that it "points" to the previous disjunct.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_iterator_get_-
disjunct(+Iterator, -Handle)
```

Unifies with `Handle` *a reference to the disjunct referred to by* `Iterator_1`*.*

```
ppl_delete_Pointset_Powerset_Octagonal_Shape_mpz_class_-
iterator(+Iterator)
```

Invalidates the handle referenced by `Iterator`*: this makes sure the corresponding resources will eventually be released.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_add_disjunct(+Handle_-
1, +Handle_2)
```

Adds to the pointset powerset referenced by `Handle_1` *a disjunct referred to by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_drop_disjunct(+Handle,
+Iterator)
```

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

```
ppl_Pointset_Powerset_Octagonal_Shape_mpz_class_drop_-
disjuncts(+Handle, +Iterator_1, +Iterator_2)
```

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_C_Polyhedron Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of c polyhedra.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of c polyhedra

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Pointset_Powerset_C_Polyhedron_from_space_-
dimension(+Dimension_Type, +Universe_or_Empty, -Handle)
```

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_C_Polyhedron_from_space_dimension(3, universe, X)
   .
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

```
ppl_new_Pointset_Powerset_C_Polyhedron_from_constraints(+Constraint_-
System, -Handle)
```

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

```
ppl_new_Pointset_Powerset_C_Polyhedron_from_congruences(+Congruence_-
System, -Handle)
```

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of c polyhedra by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions

- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_C_Polyhedron_from_Pointset_Powerset_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_c_polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_C_Polyhedron_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_C_Polyhedron_from_Pointset_Powerset_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_c_polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_C_Polyhedron_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_C_Polyhedron domain.

`ppl_delete_Pointset_Powerset_C_Polyhedron(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_C_Polyhedron_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_C_Polyhedron_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_C_Polyhedron_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_C_Polyhedron_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *contains an integer point.*

`ppl_Pointset_Powerset_C_Polyhedron_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is topologically closed.*

`ppl_Pointset_Powerset_C_Polyhedron_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is discrete.*

`ppl_Pointset_Powerset_C_Polyhedron_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from above in the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_C_Polyhedron_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from below in the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_C_Polyhedron_contains_Pointset_Powerset_C_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in or equal to the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_C_Polyhedron_strictly_contains_Pointset_Powerset_C_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in but not equal to the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_C_Polyhedron_is_disjoint_from_Pointset_Powerset_C_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is disjoint from the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_C_Polyhedron_geometrically_covers_Pointset_Powerset_C_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically covers the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_C_Polyhedron_geometrically_equals_Pointset_Powerset_C_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically equals the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_C_Polyhedron_equals_Pointset_Powerset_C_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` *is equal to the pointset powerset referenced by* `Handle_2`*.*

`ppl_Pointset_Powerset_C_Polyhedron_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

`ppl_Pointset_Powerset_C_Polyhedron_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` *constrains the dimension* `PPL_Var`*.*

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_C_Polyhedron_space_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_C_Polyhedron_affine_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_C_Polyhedron_relation_with_constraint(+Handle,
+Constraint, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_C_Polyhedron_relation_with_generator(+Handle,
+Generator, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_C_Polyhedron_relation_with_congruence(+Handle,
+Congruence, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_C_Polyhedron_maximize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_C_Polyhedron_minimize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_C_Polyhedron_maximize_with_point(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the

atom `false`.

`ppl_Pointset_Powerset_C_Polyhedron_minimize_with_point(+Handle, +Lin_-Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

`ppl_Pointset_Powerset_C_Polyhedron_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`.

`ppl_Pointset_Powerset_C_Polyhedron_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the external memory in bytes occupied by the pointset powerset referenced by* `Handle`.

`ppl_Pointset_Powerset_C_Polyhedron_size(+Handle, ?Number)`

Unifies `Number` *with the size of the memory in bytes occupied by the pointset powerset referenced by* `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Pointset_Powerset_C_Polyhedron_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_C_Polyhedron_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Pointset_Powerset_C_Polyhedron_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

```
ppl_Pointset_Powerset_C_Polyhedron_add_constraints( +Handle,
+Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

```
ppl_Pointset_Powerset_C_Polyhedron_add_congruences( +Handle,
+Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

```
ppl_Pointset_Powerset_C_Polyhedron_refine_with_constraint( +Handle,
+Constraint)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

```
ppl_Pointset_Powerset_C_Polyhedron_refine_with_congruence( +Handle,
+Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

```
ppl_Pointset_Powerset_C_Polyhedron_refine_with_constraints( +Handle,
+Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

```
ppl_Pointset_Powerset_C_Polyhedron_refine_with_congruences( +Handle,
+Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Pointset_Powerset_C_Polyhedron_topological_closure_assign(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

```
ppl_Pointset_Powerset_C_Polyhedron_pairwise_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

```
ppl_Pointset_Powerset_C_Polyhedron_omega_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

```
ppl_Pointset_Powerset_C_Polyhedron_unconstrain_space_-
dimension(+Handle, +PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`.

```
ppl_Pointset_Powerset_C_Polyhedron_unconstrain_space_-
dimensions(+Handle, +List_of_PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_C_Polyhedron_affine_image(+Handle, +PPL_Var,
+Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_-Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_C_Polyhedron_affine_preimage(+Handle, +PPL_Var,
+Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_-Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_C_Polyhedron_bounded_affine_image(+Handle,
+PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_C_Polyhedron_bounded_affine_preimage(+Handle,
+PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_C_Polyhedron_generalized_affine_image(+Handle,
+PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_C_Polyhedron_generalized_affine_-
preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_C_Polyhedron_generalized_affine_image_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

```
ppl_Pointset_Powerset_C_Polyhedron_generalized_affine_preimage_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, *where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_C_Polyhedron_intersection_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_C_Polyhedron_upper_bound_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_C_Polyhedron_difference_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the difference of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_C_Polyhedron_time_elapse_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_C_Polyhedron_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` *referenced by* `Handle_1` *with the pointset powerset referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Pointset_Powerset_C_Polyhedron_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_C_Polyhedron_BHZ03_BHRZ03_BHRZ03_widening_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` *referenced by* `Handle_1` *the BHZ03-widening between* `P_1` *and the pointset powerset referenced by* `Handle_2`*, using the BHRZ03-widening certified by the convergence certificate for BHRZ03.*

`ppl_Pointset_Powerset_C_Polyhedron_BHZ03_H79_H79_widening_-assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening certified by the convergence certificate for H79.

`ppl_Pointset_Powerset_C_Polyhedron_BGP99_BHRZ03_extrapolation_-assign(+Handle_1, +Handle_2, C_unsigned)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHRZ03-widening and the cardinality threshold `C_unsigned`.

`ppl_Pointset_Powerset_C_Polyhedron_BGP99_H79_extrapolation_-assign(+Handle_1, +Handle_2, C_unsigned)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Pointset_Powerset_C_Polyhedron_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Pointset_Powerset_C_Polyhedron_add_space_dimensions_and_-embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Pointset_Powerset_C_Polyhedron_add_space_dimensions_and_-project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Pointset_Powerset_C_Polyhedron_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_C_Polyhedron_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Pointset_Powerset_C_Polyhedron_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Pointset_Powerset_C_Polyhedron_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Pointset_Powerset_C_Polyhedron_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

`ppl_new_Pointset_Powerset_C_Polyhedron_iterator_from_iterator(+Iterator_1, -Iterator_2)`

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

`ppl_Pointset_Powerset_C_Polyhedron_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_C_Polyhedron_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_C_Polyhedron_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_C_Polyhedron_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_C_Polyhedron_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_C_Polyhedron_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_C_Polyhedron_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_C_Polyhedron_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_C_Polyhedron_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_C_Polyhedron_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_NNC_Polyhedron Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of nnc polyhedra.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of nnc polyhedra

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_NNC_Polyhedron_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_NNC_Polyhedron_from_space_dimension(3, universe,
   X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ *with* `X` *bound to a valid handle for accessing it.*

`ppl_new_Pointset_Powerset_NNC_Polyhedron_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` *from* `Constraint_System`*.* `Handle` *is unified with the handle for* `P`*.*

`ppl_new_Pointset_Powerset_NNC_Polyhedron_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` *from* `Congruence_System`*.* `Handle` *is unified with the handle for* `P`*.*

Predicates that build new pointset powersets of nnc polyhedra by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_NNC_Polyhedron_from_Pointset_Powerset_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` *from the pointset_powerset_nnc_polyhedron referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Pointset_Powerset_NNC_Polyhedron_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` *from the polyhedron referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Pointset_Powerset_NNC_Polyhedron_from_Pointset_Powerset_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` *from the pointset_powerset_nnc_polyhedron referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Pointset_Powerset_NNC_Polyhedron_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` *from the polyhedron referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_NNC_Polyhedron domain.

`ppl_delete_Pointset_Powerset_NNC_Polyhedron(+Handle)`

Invalidates the handle referenced by `Handle`*: this makes sure the corresponding resources will eventually be released.*

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_NNC_Polyhedron_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is empty.*

`ppl_Pointset_Powerset_NNC_Polyhedron_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is the universe.*

`ppl_Pointset_Powerset_NNC_Polyhedron_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is bounded.*

`ppl_Pointset_Powerset_NNC_Polyhedron_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *contains an integer point.*

`ppl_Pointset_Powerset_NNC_Polyhedron_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is topologically closed.*

`ppl_Pointset_Powerset_NNC_Polyhedron_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is discrete.*

`ppl_Pointset_Powerset_NNC_Polyhedron_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from above in the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_NNC_Polyhedron_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from below in the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_NNC_Polyhedron_contains_Pointset_Powerset_NNC_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in or equal to the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_NNC_Polyhedron_strictly_contains_Pointset_Powerset_NNC_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in but not equal to the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_NNC_Polyhedron_is_disjoint_from_Pointset_Powerset_NNC_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is disjoint from the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_NNC_Polyhedron_geometrically_covers_Pointset_Powerset_NNC_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_NNC_Polyhedron_geometrically_equals_Pointset_-
Powerset_NNC_Polyhedron(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_NNC_Polyhedron_equals_Pointset_Powerset_NNC_-
Polyhedron(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_NNC_Polyhedron_OK(+Handle)
```

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

```
ppl_Pointset_Powerset_NNC_Polyhedron_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_NNC_Polyhedron_space_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_NNC_Polyhedron_affine_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_NNC_Polyhedron_relation_with_constraint(+Handle,
+Constraint, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_NNC_Polyhedron_relation_with_generator(+Handle,
+Generator, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_NNC_Polyhedron_relation_with_congruence(+Handle,
+Congruence, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_NNC_Polyhedron_maximize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

Coeff_1 is unified with the numerator of the supremum value and Coeff_2 with the denominator of the supremum value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_NNC_Polyhedron_minimize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset P referenced by Handle is not empty and Lin_Expr is bounded from below in P.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_NNC_Polyhedron_maximize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset P referenced by Handle is not empty and Lin_Expr is bounded from above in P.

Coeff_1 is unified with the numerator of the supremum value and Coeff_2 with the denominator of the supremum value and Point with a point or closure point where Lin_Expr reaches this value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_NNC_Polyhedron_minimize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset P referenced by Handle is not empty and Lin_Expr is bounded from below in P.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value and Point with a point or closure point where Lin_Expr reaches this value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_NNC_Polyhedron_external_memory_in_bytes(+Handle,
?Number)
```

Unifies Number with the size of the total memory in bytes occupied by the pointset powerset referenced by Handle.

```
ppl_Pointset_Powerset_NNC_Polyhedron_total_memory_in_bytes(+Handle,
?Number)
```

Unifies Number with the size of the external memory in bytes occupied by the pointset powerset referenced by Handle.

```
ppl_Pointset_Powerset_NNC_Polyhedron_size(+Handle, ?Number)
```

Unifies Number with the size of the memory in bytes occupied by the pointset powerset referenced by Handle.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_NNC_Polyhedron_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by Handle on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_NNC_Polyhedron_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_Pointset_Powerset_NNC_Polyhedron_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Pointset_Powerset_NNC_Polyhedron_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Pointset_Powerset_NNC_Polyhedron_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Pointset_Powerset_NNC_Polyhedron_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Pointset_Powerset_NNC_Polyhedron_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Pointset_Powerset_NNC_Polyhedron_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Pointset_Powerset_NNC_Polyhedron_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_NNC_Polyhedron_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

`ppl_Pointset_Powerset_NNC_Polyhedron_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

`ppl_Pointset_Powerset_NNC_Polyhedron_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

`ppl_Pointset_Powerset_NNC_Polyhedron_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

`ppl_Pointset_Powerset_NNC_Polyhedron_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Pointset_Powerset_NNC_Polyhedron_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

`ppl_Pointset_Powerset_NNC_Polyhedron_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

`ppl_Pointset_Powerset_NNC_Polyhedron_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Pointset_Powerset_NNC_Polyhedron_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Pointset_Powerset_NNC_Polyhedron_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Pointset_Powerset_NNC_Polyhedron_generalized_affine_-
preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` ⋈ `Lin_Expr/Coeff`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Pointset_Powerset_NNC_Polyhedron_generalized_affine_image_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Pointset_Powerset_NNC_Polyhedron_generalized_affine_preimage_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_NNC_Polyhedron_intersection_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the intersection of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_NNC_Polyhedron_upper_bound_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the upper bound of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_NNC_Polyhedron_difference_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the difference of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_NNC_Polyhedron_time_elapse_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the time elapse of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_NNC_Polyhedron_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` referenced by `Handle_1` with the pointset powerset referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_Pointset_Powerset_NNC_Polyhedron_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` referenced by `Handle_1` and the pointset powerset `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_NNC_Polyhedron_BHZ03_BHRZ03_BHRZ03_widening_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHRZ03-widening certified by the convergence certificate for BHRZ03.

```
ppl_Pointset_Powerset_NNC_Polyhedron_BHZ03_H79_H79_widening_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening certified by the convergence certificate for H79.

```
ppl_Pointset_Powerset_NNC_Polyhedron_BGP99_BHRZ03_extrapolation_-
assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHRZ03-widening and the cardinality threshold `C_unsigned`.

```
ppl_Pointset_Powerset_NNC_Polyhedron_BGP99_H79_extrapolation_-
assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_NNC_Polyhedron_concatenate_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_NNC_Polyhedron_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Pointset_Powerset_NNC_Polyhedron_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Pointset_Powerset_NNC_Polyhedron_remove_space_dimensions(+Handle,
+List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_NNC_Polyhedron_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Pointset_Powerset_NNC_Polyhedron_expand_space_dimension(+Handle,
+PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Pointset_Powerset_NNC_Polyhedron_fold_space_dimensions(+Handle,
+List_of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_NNC_Polyhedron_map_space_dimensions(+Handle,
+P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

`ppl_new_Pointset_Powerset_NNC_Polyhedron_iterator_from_iterator(+Iterator_1, -Iterator_2)`

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

`ppl_Pointset_Powerset_NNC_Polyhedron_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_NNC_Polyhedron_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_NNC_Polyhedron_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_NNC_Polyhedron_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_NNC_Polyhedron_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_NNC_Polyhedron_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_NNC_Polyhedron_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_NNC_Polyhedron_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_NNC_Polyhedron_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_NNC_Polyhedron_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Grid Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of grids.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of grids

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_Grid_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Grid_from_space_dimension(3, universe, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_Grid_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_Grid_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of grids by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Grid_from_Pointset_Powerset_Grid(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Grid_from_Grid(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Grid_from_Pointset_Powerset_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Grid_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Grid domain.

`ppl_delete_Pointset_Powerset_Grid(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Grid_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_Grid_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_Grid_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_Grid_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_Grid_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_Grid_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_Grid_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Grid_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Grid_contains_Pointset_Powerset_Grid(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_Grid_strictly_contains_Pointset_Powerset_-
Grid(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_Grid_is_disjoint_from_Pointset_Powerset_-
Grid(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_Grid_geometrically_covers_Pointset_Powerset_-
Grid(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_Grid_geometrically_equals_Pointset_Powerset_-
Grid(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_Grid_equals_Pointset_Powerset_Grid(+Handle_1,
+Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_Grid_OK(+Handle)
```

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

```
ppl_Pointset_Powerset_Grid_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_Grid_space_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_Grid_affine_dimension(+Handle, ?Dimension_Type)
```

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_Grid_relation_with_constraint(+Handle,
+Constraint, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_Grid_relation_with_generator(+Handle,
+Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Generator`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Grid_relation_with_congruence(+Handle,
+Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Congruence`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Grid_maximize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Grid_minimize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Grid_maximize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Grid_minimize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Grid_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Grid_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_Grid_size(+Handle, ?Number)
```

Unifies `Number` *with the size of the memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_Grid_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Pointset_Powerset_Grid_add_constraint(+Handle, +Constraint)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

```
ppl_Pointset_Powerset_Grid_add_congruence(+Handle, +Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

```
ppl_Pointset_Powerset_Grid_add_constraints( +Handle, +Constraint_-
System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

```
ppl_Pointset_Powerset_Grid_add_congruences( +Handle, +Congruence_-
System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

```
ppl_Pointset_Powerset_Grid_refine_with_constraint( +Handle,
+Constraint)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

```
ppl_Pointset_Powerset_Grid_refine_with_congruence( +Handle,
+Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Pointset_Powerset_Grid_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Pointset_Powerset_Grid_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_Grid_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its topological closure.

`ppl_Pointset_Powerset_Grid_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its pairwise reduction.

`ppl_Pointset_Powerset_Grid_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its omega reduction.

`ppl_Pointset_Powerset_Grid_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Pointset_Powerset_Grid_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Grid_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Grid_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Grid_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

`ppl_Pointset_Powerset_Grid_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

`ppl_Pointset_Powerset_Grid_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Grid_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Grid_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Grid_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Pointset_Powerset_Grid_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the intersection of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Grid_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the upper bound of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Grid_difference_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the difference of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Grid_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the time elapse of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Grid_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the pointset powerset `P_1` referenced by `Handle_1` with the pointset powerset referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Pointset_Powerset_Grid_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of pointset powerset `P_1` referenced by `Handle_1` and the pointset powerset `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

`ppl_Pointset_Powerset_Grid_BHZ03_Grid_congruence_widening_-assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the congruence-widening certified by the convergence certificate for Grid.

`ppl_Pointset_Powerset_Grid_BHZ03_Grid_generator_widening_-assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the generator-widening certified by the convergence certificate for Grid.

`ppl_Pointset_Powerset_Grid_BGP99_congruence_extrapolation_-assign(+Handle_1, +Handle_2, C_unsigned)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the congruence-widening and the cardinality threshold `C_unsigned`.

`ppl_Pointset_Powerset_Grid_BGP99_generator_extrapolation_-assign(+Handle_1, +Handle_2, C_unsigned)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the generator-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Pointset_Powerset_Grid_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Pointset_Powerset_Grid_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Pointset_Powerset_Grid_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Pointset_Powerset_Grid_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Grid_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Pointset_Powerset_Grid_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Pointset_Powerset_Grid_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Grid_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial

function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

`ppl_new_Pointset_Powerset_Grid_iterator_from_iterator(+Iterator_1, -Iterator_2)`

Builds a new iterator `it` *from the iterator referenced by* `Iterator_1`*.* `Iterator_2` *is unified with the handle for* `it`*.*

`ppl_Pointset_Powerset_Grid_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` *with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

`ppl_Pointset_Powerset_Grid_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` *with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

`ppl_Pointset_Powerset_Grid_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` *is equal to the iterator referenced by* `Iterator_2`*.*

`ppl_Pointset_Powerset_Grid_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` *so that it "points" to the next disjunct.*

`ppl_Pointset_Powerset_Grid_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` *so that it "points" to the previous disjunct.*

`ppl_Pointset_Powerset_Grid_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` *a reference to the disjunct referred to by* `Iterator_1`*.*

`ppl_delete_Pointset_Powerset_Grid_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`*: this makes sure the corresponding resources will eventually be released.*

`ppl_Pointset_Powerset_Grid_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` *a disjunct referred to by* `Handle_2`*.*

`ppl_Pointset_Powerset_Grid_drop_disjunct(+Handle, +Iterator)`

If `it` *is the iterator referred to by* `Iterator`*, drops from the pointset powerset referenced by* `Handle` *the disjunct pointed to by* `it` *and assigns to* `it` *an iterator to the next disjunct.*

`ppl_Pointset_Powerset_Grid_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` *and* `it_2` *are the iterators referred to by* `Iterator_1` *and* `Iterator_2`*, respectively, drops from the pointset powerset referenced by* `Handle` *all the disjuncts from* `it_1` *to* `it_2` *(excluded).*

Predicates for the Constraints_Product_C_Polyhedron_Grid Domain

Here we provide a short description for each of the predicates available for the domain of constraints products.

Constructor, copy, conversion and destructor predicates

Constructor predicates for constraints products

The constructor predicates build a constraints product from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new constraints product $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Constraints_Product_C_Polyhedron_Grid_from_space_dimension(3, unive
   rse, X).
```

creates the constraints product defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_constraints(+Constraint_System, -Handle)`

Builds a new constraints product `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_congruences(+Congruence_System, -Handle)`

Builds a new constraints product `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new constraints products by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_int16_-
t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_int32_-
t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_int64_-
t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_mpz_-
class(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_mpq_-
class(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_-
int8_t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_-
int16_t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_-
int32_t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_-
int64_t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_-
mpz_class(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_-
mpq_class(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_C_-Polyhedron(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_NNC_-Polyhedron(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Grid(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Float_Box(+Handle_-1, -Handle_2)`

Builds a new constraints product `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_-float(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_-float(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Double_-Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_-double(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_-double(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Long_Double_-Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_long_-double(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_-long_double(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Constraints_-Product_C_Polyhedron_Grid(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the constraints_product_c_polyhedron_grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Int8_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Int16_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Int32_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Int64_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Uint8_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Uint16_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Uint32_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Uint64_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Z_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Rational_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

ppl_new_Constraints_Product_C_Polyhedron_Grid_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

ppl_new_Constraints_Product_C_Polyhedron_Grid_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Grid_with_complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_-
float_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Double_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_double_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_-
double_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Long_Double_Box_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_BD_Shape_long_-
double_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Octagonal_Shape_-
long_double_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_C_Polyhedron_Grid_from_Constraints_-
Product_C_Polyhedron_Grid_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new constraints product `P_1` from the constraints_product_c_polyhedron_grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Constraints_Product_C_Polyhedron_Grid domain.

`ppl_delete_Constraints_Product_C_Polyhedron_Grid(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the constraints product

Test Predicates

These predicates test the constraints product for different properties and succeed or fail depending on the outcome.

`ppl_Constraints_Product_C_Polyhedron_Grid_is_empty(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` is empty.

`ppl_Constraints_Product_C_Polyhedron_Grid_is_universe(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` is the universe.

`ppl_Constraints_Product_C_Polyhedron_Grid_is_bounded(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` is bounded.

`ppl_Constraints_Product_C_Polyhedron_Grid_is_topologically_closed(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` is topologically closed.

`ppl_Constraints_Product_C_Polyhedron_Grid_is_discrete(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` is discrete.

`ppl_Constraints_Product_C_Polyhedron_Grid_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the constraints product referenced by `Handle`.

`ppl_Constraints_Product_C_Polyhedron_Grid_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the constraints product referenced by `Handle`.

`ppl_Constraints_Product_C_Polyhedron_Grid_contains_Constraints_Product_C_Polyhedron_Grid(+Handle_1, +Handle_2)`

Succeeds if and only if the constraints product referenced by `Handle_2` is included in or equal to the constraints product referenced by `Handle_1`.

`ppl_Constraints_Product_C_Polyhedron_Grid_strictly_contains_Constraints_Product_C_Polyhedron_Grid(+Handle_1, +Handle_2)`

Succeeds if and only if the constraints product referenced by `Handle_2` is included in but not equal to the constraints product referenced by `Handle_1`.

`ppl_Constraints_Product_C_Polyhedron_Grid_is_disjoint_from_Constraints_Product_C_Polyhedron_Grid(+Handle_1, +Handle_2)`

Succeeds if and only if the constraints product referenced by `Handle_2` *is disjoint from the constraints product referenced by* `Handle_1`.

```
ppl_Constraints_Product_C_Polyhedron_Grid_equals_Constraints_Product_-
C_Polyhedron_Grid(+Handle_1, +Handle_2)
```

Succeeds if and only if the constraints product referenced by `Handle_1` *is equal to the constraints product referenced by* `Handle_2`.

```
ppl_Constraints_Product_C_Polyhedron_Grid_OK(+Handle)
```

Succeeds only if the constraints product referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_Constraints_Product_C_Polyhedron_Grid_constrains(+Handle, +PPL_-
Var)
```

Succeeds if and only if the constraints product referenced by `Handle` *constrains the dimension* `PPL_Var`.

Predicates that return information about the constraints product

These predicates will obtain more detailed information about the constraints product unifying some of their arguments with the results.

```
ppl_Constraints_Product_C_Polyhedron_Grid_space_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the constraints product referenced by* `Handle`.

```
ppl_Constraints_Product_C_Polyhedron_Grid_affine_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the affine dimension of the constraints product referenced by* `Handle`.

```
ppl_Constraints_Product_C_Polyhedron_Grid_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the constraints product referenced by* `Handle` *has with* `Constraint`. *The possible relations are listed in the grammar rules above.*

```
ppl_Constraints_Product_C_Polyhedron_Grid_relation_with_-
generator(+Handle, +Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the constraints product referenced by* `Handle` *has with* `Generator`. *The possible relations are listed in the grammar rules above.*

```
ppl_Constraints_Product_C_Polyhedron_Grid_relation_with_-
congruence(+Handle, +Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the constraints product referenced by* `Handle` *has with* `Congruence`. *The possible relations are listed in the grammar rules above.*

```
ppl_Constraints_Product_C_Polyhedron_Grid_maximize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if constraints product `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Constraints_Product_C_Polyhedron_Grid_minimize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if constraints product `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Constraints_Product_C_Polyhedron_Grid_maximize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if constraints product `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Constraints_Product_C_Polyhedron_Grid_minimize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if constraints product `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Constraints_Product_C_Polyhedron_Grid_external_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the total memory in bytes occupied by the constraints product referenced by `Handle`.

```
ppl_Constraints_Product_C_Polyhedron_Grid_total_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the external memory in bytes occupied by the constraints product referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Constraints_Product_C_Polyhedron_Grid_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the constraints product referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the constraints product

These predicates may modify the constraints product referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the constraints product by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Constraints_Product_C_Polyhedron_Grid_add_constraint(+Handle,
+Constraint)
```

Updates the constraints product referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

```
ppl_Constraints_Product_C_Polyhedron_Grid_add_congruence(+Handle,
+Congruence)
```

Updates the constraints product referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

```
ppl_Constraints_Product_C_Polyhedron_Grid_add_constraints( +Handle,
+Constraint_System)
```

Updates the constraints product referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

```
ppl_Constraints_Product_C_Polyhedron_Grid_add_congruences( +Handle,
+Congruence_System)
```

Updates the constraints product referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

```
ppl_Constraints_Product_C_Polyhedron_Grid_refine_with_constraint(
+Handle, +Constraint)
```

Updates the constraints product referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

```
ppl_Constraints_Product_C_Polyhedron_Grid_refine_with_congruence(
+Handle, +Congruence)
```

Updates the constraints product referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

```
ppl_Constraints_Product_C_Polyhedron_Grid_refine_with_constraints(
+Handle, +Constraint_System)
```

Updates the constraints product referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

```
ppl_Constraints_Product_C_Polyhedron_Grid_refine_with_congruences(
+Handle, +Congruence_System)
```

Updates the constraints product referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the constraints product

These predicates enable transformations such as taking the topological closure (which for the domain of constraints product is the identity transformation), unconstraining a specified dimension as explained in

the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Constraints_Product_C_Polyhedron_Grid_topological_closure_assign(+Handle)`

Assigns to the constraints product referenced by `Handle` *its topological closure.*

`ppl_Constraints_Product_C_Polyhedron_Grid_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the constraints product `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

`ppl_Constraints_Product_C_Polyhedron_Grid_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the constraints product `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Constraints_Product_C_Polyhedron_Grid_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the constraints product referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff to PPL_Var`*.*

`ppl_Constraints_Product_C_Polyhedron_Grid_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the constraints product referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff to PPL_Var`*.*

`ppl_Constraints_Product_C_Polyhedron_Grid_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to constraints product `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Constraints_Product_C_Polyhedron_Grid_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to constraints product `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Constraints_Product_C_Polyhedron_Grid_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to constraints product `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Constraints_Product_C_Polyhedron_Grid_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to constraints product `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Constraints_Product_C_Polyhedron_Grid_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to constraints product `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Constraints_Product_C_Polyhedron_Grid_generalized_affine_-
preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to constraints product `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one constraints product

These predicates include the binary operators which will assign to the constraints product referred to by the first argument its combination with the constraints product referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Constraints_Product_C_Polyhedron_Grid_intersection_-
assign(+Handle_1, +Handle_2)
```

Assigns to the constraints product `P` referenced by `Handle_1` the intersection of `P` and the constraints product referenced by `Handle_2`.

```
ppl_Constraints_Product_C_Polyhedron_Grid_upper_bound_assign(+Handle_-
1, +Handle_2)
```

Assigns to the constraints product `P` referenced by `Handle_1` the upper bound of `P` and the constraints product referenced by `Handle_2`.

```
ppl_Constraints_Product_C_Polyhedron_Grid_difference_assign(+Handle_1,
+Handle_2)
```

Assigns to the constraints product `P` referenced by `Handle_1` the difference of `P` and the constraints product referenced by `Handle_2`.

```
ppl_Constraints_Product_C_Polyhedron_Grid_time_elapse_assign(+Handle_-
1, +Handle_2)
```

Assigns to the constraints product `P` referenced by `Handle_1` the time elapse of `P` and the constraints product referenced by `Handle_2`.

```
ppl_Constraints_Product_C_Polyhedron_Grid_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the constraints product `P_1` referenced by `Handle_1` with the constraints product referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the constraints product referred to by the second argument has to be contained in (or equal to) the constraints product referred to by the first argument.

```
ppl_Constraints_Product_C_Polyhedron_Grid_widening_assign_with_-
tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Same as predicate

`ppl_Constraints_Product_C_Polyhedron_Grid_widening_assign(+Handle_1, +Handle_2)`

Same as predicate

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the constraints product referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Constraints_Product_C_Polyhedron_Grid_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the constraints product `P` referenced by `Handle_1` the concatenation of `P` and the constraints product referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the constraints product referred to in the first argument. These predicates enable the modification of the vector space of the constraints product referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Constraints_Product_C_Polyhedron_Grid_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the constraints product `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Constraints_Product_C_Polyhedron_Grid_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the constraints product `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Constraints_Product_C_Polyhedron_Grid_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the constraints product `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Constraints_Product_C_Polyhedron_Grid_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the constraints product `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Constraints_Product_C_Polyhedron_Grid_expand_space_-
dimension(+Handle, +PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the constraints product referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Constraints_Product_C_Polyhedron_Grid_fold_space_-
dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)
```

Modifies the constraints product referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Constraints_Product_C_Polyhedron_Grid_map_space_-
dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Constraints_Product_NNC_Polyhedron_Grid Domain

Here we provide a short description for each of the predicates available for the domain of constraints products.

Constructor, copy, conversion and destructor predicates

Constructor predicates for constraints products

The constructor predicates build a constraints product from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_space_-
dimension(+Dimension_Type, +Universe_or_Empty, -Handle)
```

Builds a new constraints product $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_space_dimension(3, uni
   verse, X).
```

creates the constraints product defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_-
constraints(+Constraint_System, -Handle)
```

Builds a new constraints product `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_-
congruences(+Congruence_System, -Handle)
```

Builds a new constraints product `P` *from* `Congruence_System`. `Handle` *is unified with the handle for* `P`.

Predicates that build new constraints products by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Int8_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` *from the int8_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Int16_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` *from the int16_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Int32_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` *from the int32_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Int64_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` *from the int64_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Uint8_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` *from the uint8_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Uint16_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` *from the uint16_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Uint32_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` *from the uint32_box referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Uint64_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Z_Box(+Handle_1,
-Handle_2)
```

Builds a new constraints product `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Rational_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_int8_-
t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_int16_-
t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_int32_-
t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_int64_-
t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_mpz_-
class(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_mpq_-
class(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_-
int8_t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_-
int16_t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_-
int32_t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_-
int64_t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_-
mpz_class(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_-
mpq_class(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_C_-
Polyhedron(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_NNC_-
Polyhedron(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Grid(+Handle_1,
-Handle_2)
```

Builds a new constraints product `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Float_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_-
float(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_-
float(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Double_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_-
double(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_-
double(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Long_Double_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_long_-
double(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_-
long_double(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Constraints_-
Product_NNC_Polyhedron_Grid(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the constraints_product_nnc_polyhedron_grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Int8_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Int16_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Int32_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Int64_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Uint8_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Uint16_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Uint32_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Uint64_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Z_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Rational_Box_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_int8_t_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_int16_-
t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_int32_-
t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_int64_-
t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_mpz_-
class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Grid_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Float_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_float_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_-
float_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the octagonal_shape_float referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Double_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_double_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_-
double_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the octagonal_shape_double referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Long_Double_Box_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_BD_Shape_long_-
double_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the bd_shape_long_double referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the

handle for `P_1`.

`ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Octagonal_Shape_-long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` *from the octagonal_shape_long_double referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`.

`ppl_new_Constraints_Product_NNC_Polyhedron_Grid_from_Constraints_-Product_NNC_Polyhedron_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` *from the constraints_product_nnc_polyhedron_grid referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`.

Destructor predicate

Below is the destructor predicate for the Constraints_Product_NNC_Polyhedron_Grid domain.

`ppl_delete_Constraints_Product_NNC_Polyhedron_Grid(+Handle)`

Invalidates the handle referenced by `Handle`*: this makes sure the corresponding resources will eventually be released.*

Predicates that do not change the constraints product

Test Predicates

These predicates test the constraints product for different properties and succeed or fail depending on the outcome.

`ppl_Constraints_Product_NNC_Polyhedron_Grid_is_empty(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` *is empty.*

`ppl_Constraints_Product_NNC_Polyhedron_Grid_is_universe(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` *is the universe.*

`ppl_Constraints_Product_NNC_Polyhedron_Grid_is_bounded(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` *is bounded.*

`ppl_Constraints_Product_NNC_Polyhedron_Grid_is_topologically_-closed(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` *is topologically closed.*

`ppl_Constraints_Product_NNC_Polyhedron_Grid_is_discrete(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` *is discrete.*

`ppl_Constraints_Product_NNC_Polyhedron_Grid_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from above in the constraints product referenced by* `Handle`.

`ppl_Constraints_Product_NNC_Polyhedron_Grid_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from below in the constraints product referenced by* `Handle`.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_contains_Constraints_-
Product_NNC_Polyhedron_Grid(+Handle_1, +Handle_2)
```

Succeeds if and only if the constraints product referenced by `Handle_2` *is included in or equal to the constraints product referenced by* `Handle_1`.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_strictly_contains_-
Constraints_Product_NNC_Polyhedron_Grid(+Handle_1, +Handle_2)
```

Succeeds if and only if the constraints product referenced by `Handle_2` *is included in but not equal to the constraints product referenced by* `Handle_1`.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_is_disjoint_from_-
Constraints_Product_NNC_Polyhedron_Grid(+Handle_1, +Handle_2)
```

Succeeds if and only if the constraints product referenced by `Handle_2` *is disjoint from the constraints product referenced by* `Handle_1`.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_equals_Constraints_-
Product_NNC_Polyhedron_Grid(+Handle_1, +Handle_2)
```

Succeeds if and only if the constraints product referenced by `Handle_1` *is equal to the constraints product referenced by* `Handle_2`.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_OK(+Handle)
```

Succeeds only if the constraints product referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_constrains(+Handle, +PPL_-
Var)
```

Succeeds if and only if the constraints product referenced by `Handle` *constrains the dimension* `PPL_Var`.

Predicates that return information about the constraints product

These predicates will obtain more detailed information about the constraints product unifying some of their arguments with the results.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_space_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the constraints product referenced by* `Handle`.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_affine_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the affine dimension of the constraints product referenced by* `Handle`.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the constraints product referenced by* `Handle` *has with* `Constraint`. *The possible relations are listed in the grammar rules above.*

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_relation_with_-
generator(+Handle, +Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the constraints product referenced by* `Handle` *has with* `Generator`*. The possible relations are listed in the grammar rules above.*

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_relation_with_-
congruence(+Handle, +Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the constraints product referenced by* `Handle` *has with* `Congruence`*. The possible relations are listed in the grammar rules above.*

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_maximize(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if constraints product `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_minimize(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if constraints product `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_maximize_with_-
point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if constraints product `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_minimize_with_-
point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if constraints product `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_external_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the constraints product referenced by* `Handle`*.*

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_total_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the constraints product referenced by* `Handle`*.*

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Constraints_Product_NNC_Polyhedron_Grid_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the constraints product referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the constraints product

These predicates may modify the constraints product referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the constraints product by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Constraints_Product_NNC_Polyhedron_Grid_add_constraint(+Handle, +Constraint)`

Updates the constraints product referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Constraints_Product_NNC_Polyhedron_Grid_add_congruence(+Handle, +Congruence)`

Updates the constraints product referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Constraints_Product_NNC_Polyhedron_Grid_add_constraints( +Handle, +Constraint_System)`

Updates the constraints product referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Constraints_Product_NNC_Polyhedron_Grid_add_congruences( +Handle, +Congruence_System)`

Updates the constraints product referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Constraints_Product_NNC_Polyhedron_Grid_refine_with_constraint( +Handle, +Constraint)`

Updates the constraints product referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Constraints_Product_NNC_Polyhedron_Grid_refine_with_congruence( +Handle, +Congruence)`

Updates the constraints product referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Constraints_Product_NNC_Polyhedron_Grid_refine_with_constraints(+Handle, +Constraint_System)`

Updates the constraints product referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Constraints_Product_NNC_Polyhedron_Grid_refine_with_congruences(+Handle, +Congruence_System)`

Updates the constraints product referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the constraints product

These predicates enable transformations such as taking the topological closure (which for the domain of constraints product is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Constraints_Product_NNC_Polyhedron_Grid_topological_closure_assign(+Handle)`

Assigns to the constraints product referenced by `Handle` its topological closure.

`ppl_Constraints_Product_NNC_Polyhedron_Grid_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the constraints product `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Constraints_Product_NNC_Polyhedron_Grid_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the constraints product `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Constraints_Product_NNC_Polyhedron_Grid_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the constraints product referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Constraints_Product_NNC_Polyhedron_Grid_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the constraints product referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Constraints_Product_NNC_Polyhedron_Grid_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to constraints product `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $Lin_Expr_1/Coeff \leq PPL_Var \leq Lin_Expr_2/Coeff$.

`ppl_Constraints_Product_NNC_Polyhedron_Grid_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to constraints product `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $Lin_Expr_1/Coeff \leq PPL_Var \leq Lin_Expr_2/Coeff$.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_generalized_affine_-
image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to constraints product `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `PPL_Var` ⋈ `Lin_Expr/Coeff`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_generalized_affine_-
preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to constraints product `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` ⋈ `Lin_Expr/Coeff`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_generalized_affine_image_-
lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to constraints product `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_generalized_affine_-
preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to constraints product `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one constraints product

These predicates include the binary operators which will assign to the constraints product referred to by the first argument its combination with the constraints product referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_intersection_-
assign(+Handle_1, +Handle_2)
```

Assigns to the constraints product `P` referenced by `Handle_1` the intersection of `P` and the constraints product referenced by `Handle_2`.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_upper_bound_-
assign(+Handle_1, +Handle_2)
```

Assigns to the constraints product `P` referenced by `Handle_1` the upper bound of `P` and the constraints product referenced by `Handle_2`.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_difference_-
assign(+Handle_1, +Handle_2)
```

Assigns to the constraints product `P` referenced by `Handle_1` the difference of `P` and the constraints product referenced by `Handle_2`.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_time_elapse_-
assign(+Handle_1, +Handle_2)
```

Assigns to the constraints product `P` referenced by `Handle_1` the time elapse of `P` and the constraints product referenced by `Handle_2`.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the constraints product `P_1` *referenced by* `Handle_1` *with the constraints product referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the constraints product referred to by the second argument has to be contained in (or equal to) the constraints product referred to by the first argument.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_widening_assign_with_-
tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Same as predicate

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_widening_assign(+Handle_1,
+Handle_2)
```

Same as predicate

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the constraints product referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_concatenate_-
assign(+Handle_1, +Handle_2)
```

Assigns to the constraints product `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the constraints product referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the constraints product referred to in the first argument. These predicates enable the modification of the vector space of the constraints product referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the constraints product* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the constraints product `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_remove_space_-
dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the constraints product `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the constraints product `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_expand_space_-
dimension(+Handle, +PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the constraints product referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_fold_space_-
dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)
```

Modifies the constraints product referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Constraints_Product_NNC_Polyhedron_Grid_map_space_-
dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Constraints_Product_Grid_C_Polyhedron Domain

Here we provide a short description for each of the predicates available for the domain of constraints products.

Constructor, copy, conversion and destructor predicates

Constructor predicates for constraints products

The constructor predicates build a constraints product from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_space_-
dimension(+Dimension_Type, +Universe_or_Empty, -Handle)
```

Builds a new constraints product $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Constraints_Product_Grid_C_Polyhedron_from_space_dimension(3, unive
   rse, X).
```

creates the constraints product defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_constraints(+Constraint_System, -Handle)`

Builds a new constraints product `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_congruences(+Congruence_System, -Handle)`

Builds a new constraints product `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new constraints products by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Uint16_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Uint32_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Uint64_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Z_Box(+Handle_1,
-Handle_2)
```

Builds a new constraints product `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Rational_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_int8_-
t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_int16_-
t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_int32_-
t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_int64_-
t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_mpz_-
class(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_mpq_-
class(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_-
int8_t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_-
int16_t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_-
int32_t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_-
int64_t(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_-
mpz_class(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_-
mpq_class(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_C_-
Polyhedron(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_NNC_-
Polyhedron(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Grid(+Handle_1,
-Handle_2)
```

Builds a new constraints product `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Float_Box(+Handle_-
1, -Handle_2)
```

Builds a new constraints product `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_-
float(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_-
float(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Double_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_-
double(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_-
double(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Long_Double_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_long_-
double(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_-
long_double(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Constraints_-
Product_Grid_C_Polyhedron(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the constraints_product_grid_c_polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Int8_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Int16_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Int32_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Int64_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Uint8_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Uint16_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Uint32_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Uint64_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Z_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Rational_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_int8_t_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_int16_t_-
with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_-mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_C_Polyhedron_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_NNC_Polyhedron_-with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Grid_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Float_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_float_-with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_-float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the octagonal_shape_float referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Double_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_double_-with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_-double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_C_Polyhedron_from_Constraints_Product_Grid_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the constraints_product_grid_c_polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Constraints_Product_Grid_C_Polyhedron domain.

```
ppl_delete_Constraints_Product_Grid_C_Polyhedron(+Handle)
```

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the constraints product

Test Predicates

These predicates test the constraints product for different properties and succeed or fail depending on the outcome.

```
ppl_Constraints_Product_Grid_C_Polyhedron_is_empty(+Handle)
```

Succeeds if and only if the constraints product referenced by `Handle` is empty.

```
ppl_Constraints_Product_Grid_C_Polyhedron_is_universe(+Handle)
```

Succeeds if and only if the constraints product referenced by `Handle` is the universe.

```
ppl_Constraints_Product_Grid_C_Polyhedron_is_bounded(+Handle)
```

Succeeds if and only if the constraints product referenced by `Handle` is bounded.

`ppl_Constraints_Product_Grid_C_Polyhedron_is_topologically_closed(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` is topologically closed.

`ppl_Constraints_Product_Grid_C_Polyhedron_is_discrete(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` is discrete.

`ppl_Constraints_Product_Grid_C_Polyhedron_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the constraints product referenced by `Handle`.

`ppl_Constraints_Product_Grid_C_Polyhedron_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the constraints product referenced by `Handle`.

`ppl_Constraints_Product_Grid_C_Polyhedron_contains_Constraints_Product_Grid_C_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the constraints product referenced by `Handle_2` is included in or equal to the constraints product referenced by `Handle_1`.

`ppl_Constraints_Product_Grid_C_Polyhedron_strictly_contains_Constraints_Product_Grid_C_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the constraints product referenced by `Handle_2` is included in but not equal to the constraints product referenced by `Handle_1`.

`ppl_Constraints_Product_Grid_C_Polyhedron_is_disjoint_from_Constraints_Product_Grid_C_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the constraints product referenced by `Handle_2` is disjoint from the constraints product referenced by `Handle_1`.

`ppl_Constraints_Product_Grid_C_Polyhedron_equals_Constraints_Product_Grid_C_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the constraints product referenced by `Handle_1` is equal to the constraints product referenced by `Handle_2`.

`ppl_Constraints_Product_Grid_C_Polyhedron_OK(+Handle)`

Succeeds only if the constraints product referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Constraints_Product_Grid_C_Polyhedron_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the constraints product referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the constraints product

These predicates will obtain more detailed information about the constraints product unifying some of their arguments with the results.

`ppl_Constraints_Product_Grid_C_Polyhedron_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the constraints product referenced by* `Handle`.

```
ppl_Constraints_Product_Grid_C_Polyhedron_affine_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the affine dimension of the constraints product referenced by* `Handle`.

```
ppl_Constraints_Product_Grid_C_Polyhedron_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the constraints product referenced by* `Handle` *has with* `Constraint`. *The possible relations are listed in the grammar rules above.*

```
ppl_Constraints_Product_Grid_C_Polyhedron_relation_with_-
generator(+Handle, +Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the constraints product referenced by* `Handle` *has with* `Generator`. *The possible relations are listed in the grammar rules above.*

```
ppl_Constraints_Product_Grid_C_Polyhedron_relation_with_-
congruence(+Handle, +Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the constraints product referenced by* `Handle` *has with* `Congruence`. *The possible relations are listed in the grammar rules above.*

```
ppl_Constraints_Product_Grid_C_Polyhedron_maximize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if constraints product `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Constraints_Product_Grid_C_Polyhedron_minimize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if constraints product `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Constraints_Product_Grid_C_Polyhedron_maximize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if constraints product `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Constraints_Product_Grid_C_Polyhedron_minimize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if constraints product `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Constraints_Product_Grid_C_Polyhedron_external_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the total memory in bytes occupied by the constraints product referenced by `Handle`.

```
ppl_Constraints_Product_Grid_C_Polyhedron_total_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the external memory in bytes occupied by the constraints product referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Constraints_Product_Grid_C_Polyhedron_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the constraints product referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the constraints product

These predicates may modify the constraints product referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the constraints product by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Constraints_Product_Grid_C_Polyhedron_add_constraint(+Handle,
+Constraint)
```

Updates the constraints product referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

```
ppl_Constraints_Product_Grid_C_Polyhedron_add_congruence(+Handle,
+Congruence)
```

Updates the constraints product referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

```
ppl_Constraints_Product_Grid_C_Polyhedron_add_constraints( +Handle,
+Constraint_System)
```

Updates the constraints product referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Constraints_Product_Grid_C_Polyhedron_add_congruences( +Handle, +Congruence_System)`

Updates the constraints product referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Constraints_Product_Grid_C_Polyhedron_refine_with_constraint( +Handle, +Constraint)`

Updates the constraints product referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Constraints_Product_Grid_C_Polyhedron_refine_with_congruence( +Handle, +Congruence)`

Updates the constraints product referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Constraints_Product_Grid_C_Polyhedron_refine_with_constraints( +Handle, +Constraint_System)`

Updates the constraints product referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Constraints_Product_Grid_C_Polyhedron_refine_with_congruences( +Handle, +Congruence_System)`

Updates the constraints product referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the constraints product

These predicates enable transformations such as taking the topological closure (which for the domain of constraints product is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Constraints_Product_Grid_C_Polyhedron_topological_closure_assign(+Handle)`

Assigns to the constraints product referenced by `Handle` its topological closure.

`ppl_Constraints_Product_Grid_C_Polyhedron_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the constraints product `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Constraints_Product_Grid_C_Polyhedron_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the constraints product `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Constraints_Product_Grid_C_Polyhedron_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the constraints product referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Constraints_Product_Grid_C_Polyhedron_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the constraints product referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Constraints_Product_Grid_C_Polyhedron_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to constraints product `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

`ppl_Constraints_Product_Grid_C_Polyhedron_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to constraints product `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

`ppl_Constraints_Product_Grid_C_Polyhedron_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to constraints product `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Constraints_Product_Grid_C_Polyhedron_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to constraints product `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Constraints_Product_Grid_C_Polyhedron_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to constraints product `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Constraints_Product_Grid_C_Polyhedron_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to constraints product `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one constraints product

These predicates include the binary operators which will assign to the constraints product referred to by the first argument its combination with the constraints product referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Constraints_Product_Grid_C_Polyhedron_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the constraints product `P` referenced by `Handle_1` the intersection of `P` and the constraints product referenced by `Handle_2`.

`ppl_Constraints_Product_Grid_C_Polyhedron_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the constraints product `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the constraints product referenced by* `Handle_2`*.*

```
ppl_Constraints_Product_Grid_C_Polyhedron_difference_assign(+Handle_1,
+Handle_2)
```

Assigns to the constraints product `P` *referenced by* `Handle_1` *the difference of* `P` *and the constraints product referenced by* `Handle_2`*.*

```
ppl_Constraints_Product_Grid_C_Polyhedron_time_elapse_assign(+Handle_-
1, +Handle_2)
```

Assigns to the constraints product `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the constraints product referenced by* `Handle_2`*.*

```
ppl_Constraints_Product_Grid_C_Polyhedron_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the constraints product `P_1` *referenced by* `Handle_1` *with the constraints product referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the constraints product referred to by the second argument has to be contained in (or equal to) the constraints product referred to by the first argument.

```
ppl_Constraints_Product_Grid_C_Polyhedron_widening_assign_with_-
tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Same as predicate

```
ppl_Constraints_Product_Grid_C_Polyhedron_widening_assign(+Handle_1,
+Handle_2)
```

Same as predicate

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the constraints product referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Constraints_Product_Grid_C_Polyhedron_concatenate_assign(+Handle_-
1, +Handle_2)
```

Assigns to the constraints product `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the constraints product referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the constraints product referred to in the first argument. These predicates enable the modification of the vector space of the constraints product referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Constraints_Product_Grid_C_Polyhedron_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the constraints product `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Constraints_Product_Grid_C_Polyhedron_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the constraints product `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Constraints_Product_Grid_C_Polyhedron_remove_space_-
dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the constraints product `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Constraints_Product_Grid_C_Polyhedron_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the constraints product `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Constraints_Product_Grid_C_Polyhedron_expand_space_-
dimension(+Handle, +PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the constraints product referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Constraints_Product_Grid_C_Polyhedron_fold_space_-
dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)
```

Modifies the constraints product referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Constraints_Product_Grid_C_Polyhedron_map_space_-
dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Constraints_Product_Grid_NNC_Polyhedron Domain

Here we provide a short description for each of the predicates available for the domain of constraints products.

Constructor, copy, conversion and destructor predicates

Constructor predicates for constraints products

The constructor predicates build a constraints product from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_space_-
dimension(+Dimension_Type, +Universe_or_Empty, -Handle)
```

Builds a new constraints product $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_space_dimension(3, uni
   verse, X).
```

creates the constraints product defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

```
ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_-
constraints(+Constraint_System, -Handle)
```

Builds a new constraints product `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

```
ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_-
congruences(+Congruence_System, -Handle)
```

Builds a new constraints product `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new constraints products by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

```
ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Int8_-
Box(+Handle_1, -Handle_2)
```

Builds a new constraints product `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Int16_-Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Int32_-Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Int64_-Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Uint8_-Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Uint16_-Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Uint32_-Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Uint64_-Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Rational_-Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_int8_-t(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_int16_-t(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_NNC_-Polyhedron(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Grid(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Float_-Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_-float(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_-float(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Double_-Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_-double(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_-double(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Long_Double_-Box(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_long_-double(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_-long_double(+Handle_1, -Handle_2)`

Builds a new constraints product `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Constraints_-
Product_Grid_NNC_Polyhedron(+Handle_1, -Handle_2)

Builds a new constraints product `P_1` *from the constraints_product_grid_nnc_polyhedron referenced by handle* `Handle_1`. `Handle_2` *is unified with the handle for* `P_1`.

ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Int8_Box_with_-
complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` *from the int8_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`; `Handle_2` *is unified with the handle for* `P_1`.

ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Int16_Box_with_-
complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` *from the int16_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`; `Handle_2` *is unified with the handle for* `P_1`.

ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Int32_Box_with_-
complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` *from the int32_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`; `Handle_2` *is unified with the handle for* `P_1`.

ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Int64_Box_with_-
complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` *from the int64_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`; `Handle_2` *is unified with the handle for* `P_1`.

ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Uint8_Box_with_-
complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` *from the uint8_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`; `Handle_2` *is unified with the handle for* `P_1`.

ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Uint16_Box_with_-
complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` *from the uint16_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`; `Handle_2` *is unified with the handle for* `P_1`.

ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Uint32_Box_with_-
complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` *from the uint32_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`; `Handle_2` *is unified with the handle for* `P_1`.

ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Uint64_Box_with_-
complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` *from the uint64_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`; `Handle_2` *is unified with the handle for* `P_1`.

ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Z_Box_with_-
complexity(+Handle, +Complexity, -Handle)

Builds a new constraints product `P_1` *from the z_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`; `Handle_2` *is unified with the handle for* `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Rational_Box_-with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_int8_t_-with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_int16_-t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_int32_-t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_int64_-t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_mpz_-class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_mpz_class referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_mpq_-class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_mpq_class referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_-int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_-int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new constraints product `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Constraints_Product_Grid_NNC_Polyhedron_from_Constraints_Product_Grid_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new constraints product `P_1` from the constraints_product_grid_nnc_polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Constraints_Product_Grid_NNC_Polyhedron domain.

```
ppl_delete_Constraints_Product_Grid_NNC_Polyhedron(+Handle)
```

Invalidates the handle referenced by `Handle`*: this makes sure the corresponding resources will eventually be released.*

Predicates that do not change the constraints product

Test Predicates

These predicates test the constraints product for different properties and succeed or fail depending on the outcome.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_is_empty(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` *is empty.*

`ppl_Constraints_Product_Grid_NNC_Polyhedron_is_universe(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` *is the universe.*

`ppl_Constraints_Product_Grid_NNC_Polyhedron_is_bounded(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` *is bounded.*

`ppl_Constraints_Product_Grid_NNC_Polyhedron_is_topologically_closed(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` *is topologically closed.*

`ppl_Constraints_Product_Grid_NNC_Polyhedron_is_discrete(+Handle)`

Succeeds if and only if the constraints product referenced by `Handle` *is discrete.*

`ppl_Constraints_Product_Grid_NNC_Polyhedron_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from above in the constraints product referenced by* `Handle`*.*

`ppl_Constraints_Product_Grid_NNC_Polyhedron_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from below in the constraints product referenced by* `Handle`*.*

`ppl_Constraints_Product_Grid_NNC_Polyhedron_contains_Constraints_Product_Grid_NNC_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the constraints product referenced by `Handle_2` *is included in or equal to the constraints product referenced by* `Handle_1`*.*

`ppl_Constraints_Product_Grid_NNC_Polyhedron_strictly_contains_Constraints_Product_Grid_NNC_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the constraints product referenced by `Handle_2` *is included in but not equal to the constraints product referenced by* `Handle_1`*.*

`ppl_Constraints_Product_Grid_NNC_Polyhedron_is_disjoint_from_Constraints_Product_Grid_NNC_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the constraints product referenced by `Handle_2` *is disjoint from the constraints product referenced by* `Handle_1`*.*

`ppl_Constraints_Product_Grid_NNC_Polyhedron_equals_Constraints_Product_Grid_NNC_Polyhedron(+Handle_1, +Handle_2)`

Succeeds if and only if the constraints product referenced by `Handle_1` is equal to the constraints product referenced by `Handle_2`.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_OK(+Handle)`

Succeeds only if the constraints product referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the constraints product referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the constraints product

These predicates will obtain more detailed information about the constraints product unifying some of their arguments with the results.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the constraints product referenced by `Handle`.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the constraints product referenced by `Handle`.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the constraints product referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the constraints product referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the constraints product referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if constraints product `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if constraints product `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_maximize_with_-
point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if constraints product P referenced by Handle is not empty and Lin_Expr is bounded from above in P.

Coeff_1 is unified with the numerator of the supremum value and Coeff_2 with the denominator of the supremum value and Point with a point or closure point where Lin_Expr reaches this value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_minimize_with_-
point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if constraints product P referenced by Handle is not empty and Lin_Expr is bounded from below in P.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value and Point with a point or closure point where Lin_Expr reaches this value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_external_memory_in_-
bytes(+Handle, ?Number)
```

Unifies Number with the size of the total memory in bytes occupied by the constraints product referenced by Handle.

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_total_memory_in_-
bytes(+Handle, ?Number)
```

Unifies Number with the size of the external memory in bytes occupied by the constraints product referenced by Handle.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the constraints product referenced by Handle on the standard output.

Space-dimension preserving predicates that may change the constraints product

These predicates may modify the constraints product referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the constraints product by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_add_constraint(+Handle, +Constraint)`

Updates the constraints product referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_add_congruence(+Handle, +Congruence)`

Updates the constraints product referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_add_constraints( +Handle, +Constraint_System)`

Updates the constraints product referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_add_congruences( +Handle, +Congruence_System)`

Updates the constraints product referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_refine_with_constraint( +Handle, +Constraint)`

Updates the constraints product referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_refine_with_congruence( +Handle, +Congruence)`

Updates the constraints product referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_refine_with_constraints( +Handle, +Constraint_System)`

Updates the constraints product referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_refine_with_congruences( +Handle, +Congruence_System)`

Updates the constraints product referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the constraints product

These predicates enable transformations such as taking the topological closure (which for the domain of constraints product is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Constraints_Product_Grid_NNC_Polyhedron_topological_closure_assign(+Handle)`

Assigns to the constraints product referenced by `Handle` its topological closure.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the constraints product `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the constraints product `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the constraints product referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the constraints product referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to constraints product `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to constraints product `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to constraints product `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to constraints product `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to constraints product `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Constraints_Product_Grid_NNC_Polyhedron_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to constraints product `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by

Relation_Symbol.

Predicates whose results depend on more than one constraints product

These predicates include the binary operators which will assign to the constraints product referred to by the first argument its combination with the constraints product referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_intersection_-
assign(+Handle_1, +Handle_2)
```

Assigns to the constraints product `P` *referenced by* `Handle_1` *the intersection of* `P` *and the constraints product referenced by* `Handle_2`.

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_upper_bound_-
assign(+Handle_1, +Handle_2)
```

Assigns to the constraints product `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the constraints product referenced by* `Handle_2`.

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_difference_-
assign(+Handle_1, +Handle_2)
```

Assigns to the constraints product `P` *referenced by* `Handle_1` *the difference of* `P` *and the constraints product referenced by* `Handle_2`.

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_time_elapse_-
assign(+Handle_1, +Handle_2)
```

Assigns to the constraints product `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the constraints product referenced by* `Handle_2`.

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the constraints product `P_1` *referenced by* `Handle_1` *with the constraints product referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the constraints product referred to by the second argument has to be contained in (or equal to) the constraints product referred to by the first argument.

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_widening_assign_with_-
tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Same as predicate

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_widening_assign(+Handle_1,
+Handle_2)
```

Same as predicate

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the constraints product referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_concatenate_-
assign(+Handle_1, +Handle_2)
```

Assigns to the constraints product `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the constraints product referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the constraints product referred to in the first argument. These predicates enable the modification of the vector space of the constraints product referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the constraints product* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the constraints product* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_remove_space_-
dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the constraints product `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the constraints product `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`*.*

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_expand_space_-
dimension(+Handle, +PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the constraints product referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_fold_space_-
dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)
```

Modifies the constraints product referenced by `Handle` *by folding the space dimensions contained in* `List_of_PPL_Vars` *into dimension* `PPL_Var`*. The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_Constraints_Product_Grid_NNC_Polyhedron_map_space_-
dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for the Float_Box Domain

Here we provide a short description for each of the predicates available for the domain of float boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for float boxes

The constructor predicates build a float box from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Float_Box_from_space_dimension(+Dimension_Type, +Universe_or_-
Empty, -Handle)
```

Builds a new float box $\mathcal{P}$ *with* `Dimension_Type` *dimensions; it is empty or the universe depending on whether* `Atom` *is* `empty` *or* `universe`*, respectively.* `Handle` *is unified with the handle for* $\mathcal{P}$*. Thus the query*

```
?- ppl_new_Float_Box_from_space_dimension(3, universe, X).
```

creates the float box defining the 3-dimensional vector space $\mathbb{R}^3$ *with* `X` *bound to a valid handle for accessing it.*

```
ppl_new_Float_Box_from_constraints(+Constraint_System, -Handle)
```

Builds a new float box `P` *from* `Constraint_System`*.* `Handle` *is unified with the handle for* `P`*.*

```
ppl_new_Float_Box_from_congruences(+Congruence_System, -Handle)
```

Builds a new float box `P` *from* `Congruence_System`*.* `Handle` *is unified with the handle for* `P`*.*

```
ppl_new_Float_Box_from_generators(+Generator_System, -Handle)
```

Builds a new float box `P` *from* `Generator_System`*.* `Handle` *is unified with the handle for* `P`*.*

Predicates that build new float boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions

- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Float_Box_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Grid(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Float_Box_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new float box `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_BD_Shape_float(+Handle_1, -Handle_2)
```

Builds a new float box `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Octagonal_Shape_float(+Handle_1, -Handle_2)
```

Builds a new float box `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Double_Box(+Handle_1, -Handle_2)
```

Builds a new float box `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_BD_Shape_double(+Handle_1, -Handle_2)
```

Builds a new float box `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Octagonal_Shape_double(+Handle_1, -Handle_2)
```

Builds a new float box `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Long_Double_Box(+Handle_1, -Handle_2)
```

Builds a new float box `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_BD_Shape_long_double(+Handle_1, -Handle_2)
```

Builds a new float box `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Octagonal_Shape_long_double(+Handle_1,
-Handle_2)
```

Builds a new float box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Int8_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new float box `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Int16_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new float box `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Int32_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new float box `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Int64_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new float box `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Uint8_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new float box `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Uint16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Uint32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Uint64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Z_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new float box `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Rational_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_BD_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_BD_Shape_int16_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_BD_Shape_int32_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_BD_Shape_int64_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_BD_Shape_mpz_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_BD_Shape_mpq_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Octagonal_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Octagonal_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new float box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Octagonal_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new float box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Octagonal_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new float box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Octagonal_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new float box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Octagonal_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new float box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_C_Polyhedron_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_NNC_Polyhedron_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Grid_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new float box `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Float_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new float box `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_BD_Shape_float_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Octagonal_Shape_float_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Double_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_BD_Shape_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Octagonal_Shape_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Long_Double_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_BD_Shape_long_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new float box `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Float_Box_from_Octagonal_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new float box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Float_Box domain.

`ppl_delete_Float_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the float box

Test Predicates

These predicates test the float box for different properties and succeed or fail depending on the outcome.

`ppl_Float_Box_is_empty(+Handle)`

Succeeds if and only if the float box referenced by `Handle` is empty.

`ppl_Float_Box_is_universe(+Handle)`

Succeeds if and only if the float box referenced by `Handle` is the universe.

`ppl_Float_Box_is_bounded(+Handle)`

Succeeds if and only if the float box referenced by `Handle` is bounded.

`ppl_Float_Box_contains_integer_point(+Handle)`

Succeeds if and only if the float box referenced by `Handle` contains an integer point.

`ppl_Float_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the float box referenced by `Handle` is topologically closed.

`ppl_Float_Box_is_discrete(+Handle)`

Succeeds if and only if the float box referenced by `Handle` is discrete.

`ppl_Float_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the float box referenced by `Handle`.

`ppl_Float_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the float box referenced by `Handle`.

`ppl_Float_Box_contains_Float_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the float box referenced by `Handle_2` is included in or equal to the float box referenced by `Handle_1`.

`ppl_Float_Box_strictly_contains_Float_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the float box referenced by `Handle_2` is included in but not equal to the float box referenced by `Handle_1`.

`ppl_Float_Box_is_disjoint_from_Float_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the float box referenced by `Handle_2` is disjoint from the float box referenced by `Handle_1`.

`ppl_Float_Box_equals_Float_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the float box referenced by `Handle_1` is equal to the float box referenced by `Handle_2`.

`ppl_Float_Box_OK(+Handle)`

Succeeds only if the float box referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Float_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the float box referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the float box

These predicates will obtain more detailed information about the float box unifying some of their arguments with the results.

`ppl_Float_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the float box referenced by `Handle`.

`ppl_Float_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the float box referenced by `Handle`.

`ppl_Float_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the float box referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Float_Box_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the float box referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Float_Box_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the float box referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Float_Box_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the float box referenced by `Handle`.

`ppl_Float_Box_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the float box referenced by `Handle`.

`ppl_Float_Box_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the float box referenced by `Handle`.

`ppl_Float_Box_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the float box referenced by `Handle`.

```
ppl_Float_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean)
```

Succeeds if and only if float box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Float_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean)
```

Succeeds if and only if float box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Float_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if float box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Float_Box_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if float box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Float_Box_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the total memory in bytes occupied by the float box referenced by `Handle`.

```
ppl_Float_Box_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the external memory in bytes occupied by the float box referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Float_Box_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the float box referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the float box

These predicates may modify the float box referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the float box by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Float_Box_add_constraint(+Handle, +Constraint)`

Updates the float box referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Float_Box_add_congruence(+Handle, +Congruence)`

Updates the float box referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Float_Box_add_constraints( +Handle, +Constraint_System)`

Updates the float box referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Float_Box_add_congruences( +Handle, +Congruence_System)`

Updates the float box referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Float_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the float box referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Float_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the float box referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Float_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the float box referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Float_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the float box referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the float box

These predicates enable transformations such as taking the topological closure (which for the domain of float box is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Float_Box_topological_closure_assign(+Handle)`

Assigns to the float box referenced by `Handle` *its topological closure.*

```
ppl_Float_Box_unconstrain_space_dimension(+Handle, +PPL_Var)
```

Modifies the float box `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

```
ppl_Float_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)
```

Modifies the float box `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Float_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the float box referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_Float_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the float box referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_Float_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1,
+Lin_Expr_2, +Coeff)
```

Assigns to float box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_Float_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1,
+Lin_Expr_2, +Coeff)
```

Assigns to float box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_Float_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_-
Symbol, +Lin_Expr, +Coeff)
```

Assigns to float box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`*.*

```
ppl_Float_Box_generalized_affine_preimage(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to float box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`*.*

```
ppl_Float_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1,
+Relation_Symbol, +Lin_Expr_2)
```

Assigns to float box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`*.*

```
ppl_Float_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_-
1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to float box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`*.*

```
ppl_Float_Box_wrap_assign(+Handle, +List_of_PPL_Var, +Width,
+Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)
```

Transforms the float box `P` *referenced by* `Handle` *by wrapping the dimensions given by* `List_of_PPL_Vars` *while respecting the specified* `Width`, `Representation` *and* `Overflow` *behavior of all these variables. The* `Constraint_System` *represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer* `C_unsigned` *and* `Boolean` *allow control of the complexity/precision ratio; higher values for* `C_unsigned` *will lead to possibly greater precision while a true value for* `Boolean` *indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.*

Predicates whose results depend on more than one float box

These predicates include the binary operators which will assign to the float box referred to by the first argument its combination with the float box referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Float_Box_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the float box `P` *referenced by* `Handle_1` *the intersection of* `P` *and the float box referenced by* `Handle_2`.

```
ppl_Float_Box_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the float box `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the float box referenced by* `Handle_2`.

```
ppl_Float_Box_difference_assign(+Handle_1, +Handle_2)
```

Assigns to the float box `P` *referenced by* `Handle_1` *the difference of* `P` *and the float box referenced by* `Handle_2`.

```
ppl_Float_Box_time_elapse_assign(+Handle_1, +Handle_2)
```

Assigns to the float box `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the float box referenced by* `Handle_2`.

```
ppl_Float_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the float box `P_1` *referenced by* `Handle_1` *with the float box referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Float_Box_simplify_using_context_assign(+Handle_1, +Handle_2,
?Boolean)
```

Succeeds if and only if the intersection of float box `P_1` *referenced by* `Handle_1` *and the float box* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`.

```
ppl_Float_Box_linear_partition(+Handle_1, +Handle_2, -Handle_3,
-Handle_4)
```

`Handle_1` *and* `Handle_2` *are handles for elements* `P_1` *and* `P_2` *in the Float_Box domain. The predicate unifies handle* `Handle_3` *to a reference to the intersection of* `P_1` *and* `P_2` *and* `Handle_4` *to a reference to a pointset powerset of nnc polyhedra* `P_4`*; where* `P_4` *is the linear partition of* `P_1` *with respect to* `P_2`*. This predicate is only provided if the class* `Pointset_Powerset_NNC_Polyhedron` *has been enabled when configuring the library.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with*

Tokens and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the float box referred to by the second argument has to be contained in (or equal to) the float box referred to by the first argument.

`ppl_Float_Box_CC76_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the float box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the float box referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_Float_Box_CC76_widening_assign(+Handle_1, +Handle_2)`

Assigns to the float box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the float box referenced by `Handle_2`.

`ppl_Float_Box_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Same as predicate `ppl_Float_Box_CC76_widening_assign_with_tokens/4`

`ppl_Float_Box_widening_assign(+Handle_1, +Handle_2)`

Same as predicate `ppl_Float_Box_CC76_widening_assign/2`

`ppl_Float_Box_limited_CC76_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the float box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the float box referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_Float_Box_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the float box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the float box referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the float box referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Float_Box_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the float box `P` referenced by `Handle_1` the concatenation of `P` and the float box referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the float box referred to in the first argument. These predicates enable the modification of the vector space of the float box referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections

Adding New Dimensions to the Vector Space, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Float_Box_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the float box `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Float_Box_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the float box `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Float_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the float box `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Float_Box_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the float box `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Float_Box_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the float box referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Float_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the float box referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Float_Box_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the BD_Shape_float Domain

Here we provide a short description for each of the predicates available for the domain of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for bd shapes

The constructor predicates build a bd shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_BD_Shape_float_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new bd shape $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_BD_Shape_float_from_space_dimension(3, universe, X).
```

creates the bd shape defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_BD_Shape_float_from_constraints(+Constraint_System, -Handle)`

Builds a new bd shape `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_BD_Shape_float_from_congruences(+Congruence_System, -Handle)`

Builds a new bd shape `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_BD_Shape_float_from_generators(+Generator_System, -Handle)`

Builds a new bd shape `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_BD_Shape_float_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Grid(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Uint8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Uint16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Uint32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_Uint64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_Z_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_Rational_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_BD_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_BD_Shape_int16_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_BD_Shape_int32_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_BD_Shape_int64_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_BD_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_BD_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_Octagonal_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Octagonal_Shape_int16_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Octagonal_Shape_int32_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Octagonal_Shape_int64_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Octagonal_Shape_mpz_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Octagonal_Shape_mpq_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_float_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_Octagonal_Shape_float_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_Double_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_BD_Shape_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_Octagonal_Shape_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_Long_Double_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_BD_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_float_from_Octagonal_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the BD_Shape_float domain.

```
ppl_delete_BD_Shape_float(+Handle)
```

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the bd shape

Test Predicates

These predicates test the bd shape for different properties and succeed or fail depending on the outcome.

`ppl_BD_Shape_float_is_empty(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is empty.

`ppl_BD_Shape_float_is_universe(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is the universe.

`ppl_BD_Shape_float_is_bounded(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is bounded.

`ppl_BD_Shape_float_contains_integer_point(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` contains an integer point.

`ppl_BD_Shape_float_is_topologically_closed(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is topologically closed.

`ppl_BD_Shape_float_is_discrete(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is discrete.

`ppl_BD_Shape_float_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the bd shape referenced by `Handle`.

`ppl_BD_Shape_float_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the bd shape referenced by `Handle`.

`ppl_BD_Shape_float_contains_BD_Shape_float(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is included in or equal to the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_float_strictly_contains_BD_Shape_float(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is included in but not equal to the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_float_is_disjoint_from_BD_Shape_float(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is disjoint from the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_float_equals_BD_Shape_float(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_1` is equal to the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_float_OK(+Handle)`

Succeeds only if the bd shape referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_BD_Shape_float_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the bd shape referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the bd shape

These predicates will obtain more detailed information about the bd shape unifying some of their arguments with the results.

`ppl_BD_Shape_float_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the bd shape referenced by `Handle`.

`ppl_BD_Shape_float_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the bd shape referenced by `Handle`.

`ppl_BD_Shape_float_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_BD_Shape_float_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_BD_Shape_float_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_BD_Shape_float_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_float_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_float_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_float_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_float_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_float_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_float_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_float_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_float_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the total memory in bytes occupied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_float_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the bd shape referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_BD_Shape_float_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the bd shape referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the bd shape

These predicates may modify the bd shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the bd shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual*

for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_BD_Shape_float_add_constraint(+Handle, +Constraint)`

Updates the bd shape referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_BD_Shape_float_add_congruence(+Handle, +Congruence)`

Updates the bd shape referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_BD_Shape_float_add_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_BD_Shape_float_add_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_BD_Shape_float_refine_with_constraint( +Handle, +Constraint)`

Updates the bd shape referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_BD_Shape_float_refine_with_congruence( +Handle, +Congruence)`

Updates the bd shape referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_BD_Shape_float_refine_with_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_BD_Shape_float_refine_with_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the bd shape

These predicates enable transformations such as taking the topological closure (which for the domain of bd shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_BD_Shape_float_topological_closure_assign(+Handle)`

Assigns to the bd shape referenced by `Handle` its topological closure.

`ppl_BD_Shape_float_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the bd shape `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_BD_Shape_float_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the bd shape `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_BD_Shape_float_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the bd shape referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_BD_Shape_float_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the bd shape referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_BD_Shape_float_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_BD_Shape_float_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_BD_Shape_float_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_float_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_float_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to bd shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_float_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to bd shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_BD_Shape_float_wrap_assign(+Handle, +List_of_PPL_Var, +Width, +Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)`

Transforms the bd shape `P` referenced by `Handle` by wrapping the dimensions given by `List_of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater

precision while a true value for `Boolean` *indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.*

Predicates whose results depend on more than one bd shape

These predicates include the binary operators which will assign to the bd shape referred to by the first argument its combination with the bd shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_BD_Shape_float_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` *referenced by* `Handle_1` *the intersection of* `P` *and the bd shape referenced by* `Handle_2`*.*

```
ppl_BD_Shape_float_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the bd shape referenced by* `Handle_2`*.*

```
ppl_BD_Shape_float_difference_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` *referenced by* `Handle_1` *the difference of* `P` *and the bd shape referenced by* `Handle_2`*.*

```
ppl_BD_Shape_float_time_elapse_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the bd shape referenced by* `Handle_2`*.*

```
ppl_BD_Shape_float_upper_bound_assign_if_exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the bd shape `P_1` *referenced by* `Handle_1` *with the bd shape referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_BD_Shape_float_simplify_using_context_assign(+Handle_1, +Handle_2,
?Boolean)
```

Succeeds if and only if the intersection of bd shape `P_1` *referenced by* `Handle_1` *and the bd shape* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

```
ppl_BD_Shape_float_linear_partition(+Handle_1, +Handle_2, -Handle_3,
-Handle_4)
```

`Handle_1` *and* `Handle_2` *are handles for elements* `P_1` *and* `P_2` *in the BD_Shape_float domain. The predicate unifies handle* `Handle_3` *to a reference to the intersection of* `P_1` *and* `P_2` *and* `Handle_4` *to a reference to a pointset powerset of nnc polyhedra* `P_4`*; where* `P_4` *is the linear partition of* `P_1` *with respect to* `P_2`*. This predicate is only provided if the class* `Pointset_Powerset_NNC_Polyhedron` *has been enabled when configuring the library.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the bd shape referred to by the second argument has to be contained in (or equal to) the bd shape referred to by the first argument.

ppl_BD_Shape_float_BHMZ05_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

ppl_BD_Shape_float_H79_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

ppl_BD_Shape_float_BHMZ05_widening_assign(+Handle_1, +Handle_2)

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`.

ppl_BD_Shape_float_H79_widening_assign(+Handle_1, +Handle_2)

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`.

ppl_BD_Shape_float_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)

Same as predicate `ppl_BD_Shape_float_H79_widening_assign_with_tokens/4`

ppl_BD_Shape_float_widening_assign(+Handle_1, +Handle_2)

Same as predicate `ppl_BD_Shape_float_H79_widening_assign/2`

ppl_BD_Shape_float_limited_BHMZ05_extrapolation_assign_with_-tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-unsigned_2)

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

ppl_BD_Shape_float_limited_H79_extrapolation_assign_with_-tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-unsigned_2)

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

ppl_BD_Shape_float_limited_CC76_extrapolation_assign_with_-tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-unsigned_2)

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

ppl_BD_Shape_float_limited_BHMZ05_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_BD_Shape_float_limited_H79_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_BD_Shape_float_limited_CC76_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_BD_Shape_float_CC76_narrowing_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-narrowing of `P_1` and the bd shape referenced by `Handle_2`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_BD_Shape_float_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` referenced by `Handle_1` the concatenation of `P` and the bd shape referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument. These predicates enable the modification of the vector space of the bd shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_BD_Shape_float_add_space_dimensions_and_embed(+Handle,
+Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_BD_Shape_float_add_space_dimensions_and_project(+Handle,
+Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_BD_Shape_float_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the bd shape `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_BD_Shape_float_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the bd shape `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_BD_Shape_float_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the bd shape referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_BD_Shape_float_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the bd shape referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_BD_Shape_float_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Octagonal_Shape_float Domain

Here we provide a short description for each of the predicates available for the domain of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for octagonal shapes

The constructor predicates build a octagonal shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Octagonal_Shape_float_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new octagonal shape $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Octagonal_Shape_float_from_space_dimension(3, universe, X).
```

creates the octagonal shape defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Octagonal_Shape_float_from_constraints(+Constraint_System, -Handle)`

Builds a new octagonal shape `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Octagonal_Shape_float_from_congruences(+Congruence_System, -Handle)`

Builds a new octagonal shape `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Octagonal_Shape_float_from_generators(+Generator_System, -Handle)`

Builds a new octagonal shape `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Octagonal_Shape_float_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_int64_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_mpz_class(+Handle_-
1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_-1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_mpq_class(+Handle_-
1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_-1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_C_Polyhedron(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_NNC_Polyhedron(+Handle_1, -Handle_-
2)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Grid(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Float_Box(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_BD_Shape_float(+Handle_1, -Handle_-
2)
```

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Double_Box(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_BD_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_long_-double(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Uint8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Uint16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Uint32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Uint64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Z_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Rational_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_BD_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_BD_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_BD_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_BD_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_BD_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Grid_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_BD_Shape_float_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_float_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Double_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_BD_Shape_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Long_Double_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_BD_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_float_from_Octagonal_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Octagonal_Shape_float domain.

```
ppl_delete_Octagonal_Shape_float(+Handle)
```

Invalidates the handle referenced by `Handle`*: this makes sure the corresponding resources will eventually be released.*

Predicates that do not change the octagonal shape

Test Predicates

These predicates test the octagonal shape for different properties and succeed or fail depending on the outcome.

`ppl_Octagonal_Shape_float_is_empty(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is empty.*

`ppl_Octagonal_Shape_float_is_universe(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is the universe.*

`ppl_Octagonal_Shape_float_is_bounded(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is bounded.*

`ppl_Octagonal_Shape_float_contains_integer_point(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *contains an integer point.*

`ppl_Octagonal_Shape_float_is_topologically_closed(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is topologically closed.*

`ppl_Octagonal_Shape_float_is_discrete(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is discrete.*

`ppl_Octagonal_Shape_float_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from above in the octagonal shape referenced by* `Handle`*.*

`ppl_Octagonal_Shape_float_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from below in the octagonal shape referenced by* `Handle`*.*

`ppl_Octagonal_Shape_float_contains_Octagonal_Shape_float(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` *is included in or equal to the octagonal shape referenced by* `Handle_1`*.*

`ppl_Octagonal_Shape_float_strictly_contains_Octagonal_Shape_float(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` *is included in but not equal to the octagonal shape referenced by* `Handle_1`*.*

`ppl_Octagonal_Shape_float_is_disjoint_from_Octagonal_Shape_float(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` *is disjoint from the octagonal shape referenced by* `Handle_1`*.*

`ppl_Octagonal_Shape_float_equals_Octagonal_Shape_float(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_1` *is equal to the octagonal shape referenced by* `Handle_2`*.*

`ppl_Octagonal_Shape_float_OK(+Handle)`

Succeeds only if the octagonal shape referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Octagonal_Shape_float_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the octagonal shape referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the octagonal shape

These predicates will obtain more detailed information about the octagonal shape unifying some of their arguments with the results.

`ppl_Octagonal_Shape_float_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_float_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_float_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the octagonal shape referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Octagonal_Shape_float_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the octagonal shape referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Octagonal_Shape_float_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the octagonal shape referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Octagonal_Shape_float_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_float_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_float_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_float_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_float_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Octagonal_Shape_float_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Octagonal_Shape_float_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Octagonal_Shape_float_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Octagonal_Shape_float_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the total memory in bytes occupied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_float_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the octagonal shape referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Octagonal_Shape_float_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the octagonal shape referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the octagonal shape

These predicates may modify the octagonal shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the octagonal shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Octagonal_Shape_float_add_constraint(+Handle, +Constraint)`

Updates the octagonal shape referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Octagonal_Shape_float_add_congruence(+Handle, +Congruence)`

Updates the octagonal shape referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Octagonal_Shape_float_add_constraints( +Handle, +Constraint_System)`

Updates the octagonal shape referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Octagonal_Shape_float_add_congruences( +Handle, +Congruence_System)`

Updates the octagonal shape referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Octagonal_Shape_float_refine_with_constraint( +Handle, +Constraint)`

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Octagonal_Shape_float_refine_with_congruence( +Handle, +Congruence)`

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Octagonal_Shape_float_refine_with_constraints( +Handle, +Constraint_System)`

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Octagonal_Shape_float_refine_with_congruences( +Handle, +Congruence_System)`

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the octagonal shape

These predicates enable transformations such as taking the topological closure (which for the domain of octagonal shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Octagonal_Shape_float_topological_closure_assign(+Handle)`

Assigns to the octagonal shape referenced by `Handle` *its topological closure.*

`ppl_Octagonal_Shape_float_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the octagonal shape `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

`ppl_Octagonal_Shape_float_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the octagonal shape `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Octagonal_Shape_float_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the octagonal shape referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

`ppl_Octagonal_Shape_float_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the octagonal shape referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

`ppl_Octagonal_Shape_float_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Octagonal_Shape_float_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Octagonal_Shape_float_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Octagonal_Shape_float_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Octagonal_Shape_float_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_float_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_float_wrap_assign(+Handle, +List_of_PPL_Var, +Width, +Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)`

Transforms the octagonal shape `P` referenced by `Handle` by wrapping the dimensions given by `List_of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one octagonal shape

These predicates include the binary operators which will assign to the octagonal shape referred to by the first argument its combination with the octagonal shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Octagonal_Shape_float_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the intersection of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_float_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the upper bound of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_float_difference_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the difference of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_float_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the time elapse of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_float_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the octagonal shape `P_1` referenced by `Handle_1` with the octagonal shape referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_Octagonal_Shape_float_simplify_using_context_assign(+Handle_1,
+Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of octagonal shape `P_1` referenced by `Handle_1` and the octagonal shape `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

```
ppl_Octagonal_Shape_float_linear_partition(+Handle_1, +Handle_2,
-Handle_3, -Handle_4)
```

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Octagonal_Shape_float domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the octagonal shape referred to by the second argument has to be contained in (or equal to) the octagonal shape referred to by the first argument.

```
ppl_Octagonal_Shape_float_BHMZ05_widening_assign_with_tokens(+Handle_-
1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_-unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_float_BHMZ05_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_float_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_Octagonal_Shape_float_H79_widening_assign_with_tokens/4`

```
ppl_Octagonal_Shape_float_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_Octagonal_Shape_float_H79_widening_assign/2`

```
ppl_Octagonal_Shape_float_limited_BHMZ05_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_-unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_float_limited_CC76_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the octagonal shape `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the octagonal shape referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*. The widening with tokens delay technique is applied with* `C_unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Octagonal_Shape_float_limited_BHMZ05_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)
```

Assigns to the octagonal shape `P_1` *referenced by* `Handle_1` *the BHMZ05-widening of* `P_1` *with the octagonal shape referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*.*

```
ppl_Octagonal_Shape_float_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)
```

Assigns to the octagonal shape `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the octagonal shape referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*.*

```
ppl_Octagonal_Shape_float_CC76_narrowing_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P_1` *referenced by* `Handle_1` *the CC76-narrowing of* `P_1` *and the octagonal shape referenced by* `Handle_2`*.*

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Octagonal_Shape_float_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the octagonal shape referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Octagonal_Shape_float_add_space_dimensions_and_embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the octagonal shape* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Octagonal_Shape_float_add_space_dimensions_and_project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the octagonal shape `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Octagonal_Shape_float_remove_space_dimensions(+Handle, +List_of_-
PPL_Vars)
```

Removes from the vector space enclosing the octagonal shape `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Octagonal_Shape_float_remove_higher_space_dimensions(+Handle,
+Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the octagonal shape `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Octagonal_Shape_float_expand_space_dimension(+Handle, +PPL_Var,
+Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the octagonal shape referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Octagonal_Shape_float_fold_space_dimensions(+Handle, +List_of_-
PPL_Vars, +PPL_Var)
```

Modifies the octagonal shape referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Octagonal_Shape_float_map_space_dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Pointset_Powerset_Float_Box Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of float boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of float boxes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Pointset_Powerset_Float_Box_from_space_dimension(+Dimension_-
Type, +Universe_or_Empty, -Handle)
```

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Float_Box_from_space_dimension(3, universe, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_Float_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_Float_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of float boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Float_Box_from_Pointset_Powerset_Float_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Float_Box_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Float_Box_from_Pointset_Powerset_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Float_Box_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Float_Box domain.

`ppl_delete_Pointset_Powerset_Float_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Float_Box_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_Float_Box_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_Float_Box_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_Float_Box_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_Float_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_Float_Box_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_Float_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Float_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Float_Box_contains_Pointset_Powerset_Float_-Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Float_Box_strictly_contains_Pointset_Powerset_-Float_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Float_Box_is_disjoint_from_Pointset_Powerset_-Float_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Float_Box_geometrically_covers_Pointset_-Powerset_Float_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_Float_Box_geometrically_equals_Pointset_-
Powerset_Float_Box(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

```
ppl_Pointset_Powerset_Float_Box_equals_Pointset_Powerset_Float_-
Box(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_Float_Box_OK(+Handle)
```

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

```
ppl_Pointset_Powerset_Float_Box_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_Float_Box_space_dimension(+Handle, ?Dimension_-
Type)
```

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_Float_Box_affine_dimension(+Handle, ?Dimension_-
Type)
```

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_Float_Box_relation_with_constraint(+Handle,
+Constraint, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_Float_Box_relation_with_generator(+Handle,
+Generator, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_Float_Box_relation_with_congruence(+Handle,
+Congruence, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_Float_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

Coeff_1 is unified with the numerator of the supremum value and Coeff_2 with the denominator of the supremum value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_Float_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset P referenced by Handle is not empty and Lin_Expr is bounded from below in P.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_Float_Box_maximize_with_point(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset P referenced by Handle is not empty and Lin_Expr is bounded from above in P.

Coeff_1 is unified with the numerator of the supremum value and Coeff_2 with the denominator of the supremum value and Point with a point or closure point where Lin_Expr reaches this value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_Float_Box_minimize_with_point(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset P referenced by Handle is not empty and Lin_Expr is bounded from below in P.

Coeff_1 is unified with the numerator of the infinum value and Coeff_2 with the denominator of the infinum value and Point with a point or closure point where Lin_Expr reaches this value. If the infinum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.

```
ppl_Pointset_Powerset_Float_Box_external_memory_in_bytes(+Handle,
?Number)
```

Unifies Number with the size of the total memory in bytes occupied by the pointset powerset referenced by Handle.

```
ppl_Pointset_Powerset_Float_Box_total_memory_in_bytes(+Handle,
?Number)
```

Unifies Number with the size of the external memory in bytes occupied by the pointset powerset referenced by Handle.

```
ppl_Pointset_Powerset_Float_Box_size(+Handle, ?Number)
```

Unifies Number with the size of the memory in bytes occupied by the pointset powerset referenced by Handle.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_Float_Box_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by Handle on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_Float_Box_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Pointset_Powerset_Float_Box_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Pointset_Powerset_Float_Box_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Pointset_Powerset_Float_Box_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Pointset_Powerset_Float_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Pointset_Powerset_Float_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Pointset_Powerset_Float_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Pointset_Powerset_Float_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_Float_Box_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

`ppl_Pointset_Powerset_Float_Box_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

`ppl_Pointset_Powerset_Float_Box_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

`ppl_Pointset_Powerset_Float_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

`ppl_Pointset_Powerset_Float_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Pointset_Powerset_Float_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

`ppl_Pointset_Powerset_Float_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

`ppl_Pointset_Powerset_Float_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Pointset_Powerset_Float_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Pointset_Powerset_Float_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Pointset_Powerset_Float_Box_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Pointset_Powerset_Float_Box_generalized_affine_image_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Pointset_Powerset_Float_Box_generalized_affine_preimage_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_Float_Box_intersection_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Float_Box_upper_bound_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Float_Box_difference_assign(+Handle_1, +Handle_-
2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the difference of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Float_Box_time_elapse_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Float_Box_upper_bound_assign_if_exact(+Handle_1,
+Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` *referenced by* `Handle_1` *with the pointset powerset referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Pointset_Powerset_Float_Box_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification*

with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_Float_Box_concatenate_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the pointset powerset referenced by* `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_Float_Box_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the pointset powerset* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Pointset_Powerset_Float_Box_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the pointset powerset* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

```
ppl_Pointset_Powerset_Float_Box_remove_space_dimensions(+Handle,
+List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Float_Box_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`*.*

```
ppl_Pointset_Powerset_Float_Box_expand_space_dimension(+Handle, +PPL_-
Var, +Dimension_Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the pointset powerset referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

```
ppl_Pointset_Powerset_Float_Box_fold_space_dimensions(+Handle, +List_-
of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` *by folding the space dimensions contained in* `List_of_PPL_Vars` *into dimension* `PPL_Var`*. The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Float_Box_map_space_dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_Float_Box_iterator_from_iterator(+Iterator_-
1, -Iterator_2)
```

Builds a new iterator `it` *from the iterator referenced by* `Iterator_1`*.* `Iterator_2` *is unified with the handle for* `it`*.*

```
ppl_Pointset_Powerset_Float_Box_begin_iterator(+Handle, -Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Float_Box_end_iterator(+Handle, -Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Float_Box_iterator_equals_iterator(+Iterator_1,
+Iterator_2)
```

Succeeds if and only if the iterator referenced by `Iterator_1` *is equal to the iterator referenced by* `Iterator_2`*.*

```
ppl_Pointset_Powerset_Float_Box_iterator_increment(+Iterator)
```

Increments the iterator referenced by `Iterator` *so that it "points" to the next disjunct.*

```
ppl_Pointset_Powerset_Float_Box_iterator_decrement(+Iterator)
```

Decrements the iterator referenced by `Iterator` *so that it "points" to the previous disjunct.*

```
ppl_Pointset_Powerset_Float_Box_iterator_get_disjunct(+Iterator,
-Handle)
```

Unifies with `Handle` *a reference to the disjunct referred to by* `Iterator_1`*.*

`ppl_delete_Pointset_Powerset_Float_Box_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_Float_Box_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_Float_Box_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Float_Box_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_BD_Shape_float Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of bd shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_BD_Shape_float_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_BD_Shape_float_from_space_dimension(3, universe,
   X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_BD_Shape_float_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_BD_Shape_float_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_BD_Shape_float_from_Pointset_Powerset_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_BD_Shape_float_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_BD_Shape_float_from_Pointset_Powerset_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_BD_Shape_float_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_BD_Shape_float domain.

`ppl_delete_Pointset_Powerset_BD_Shape_float(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_BD_Shape_float_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is empty.*

`ppl_Pointset_Powerset_BD_Shape_float_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is the universe.*

`ppl_Pointset_Powerset_BD_Shape_float_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is bounded.*

`ppl_Pointset_Powerset_BD_Shape_float_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *contains an integer point.*

`ppl_Pointset_Powerset_BD_Shape_float_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is topologically closed.*

`ppl_Pointset_Powerset_BD_Shape_float_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *is discrete.*

`ppl_Pointset_Powerset_BD_Shape_float_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from above in the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_BD_Shape_float_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from below in the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_BD_Shape_float_contains_Pointset_Powerset_BD_Shape_float(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in or equal to the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_BD_Shape_float_strictly_contains_Pointset_Powerset_BD_Shape_float(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in but not equal to the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_BD_Shape_float_is_disjoint_from_Pointset_Powerset_BD_Shape_float(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is disjoint from the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_BD_Shape_float_geometrically_covers_Pointset_Powerset_BD_Shape_float(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically covers the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_BD_Shape_float_geometrically_equals_Pointset_Powerset_BD_Shape_float(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically equals the pointset powerset referenced by* `Handle_1`*.*

`ppl_Pointset_Powerset_BD_Shape_float_equals_Pointset_Powerset_BD_Shape_float(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` *is equal to the pointset powerset referenced by* `Handle_2`*.*

`ppl_Pointset_Powerset_BD_Shape_float_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Pointset_Powerset_BD_Shape_float_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

`ppl_Pointset_Powerset_BD_Shape_float_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_float_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_float_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_BD_Shape_float_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_BD_Shape_float_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_BD_Shape_float_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_BD_Shape_float_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_BD_Shape_float_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_BD_Shape_float_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_BD_Shape_float_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the total memory in bytes occupied by the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_float_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_float_size(+Handle, ?Number)`

Unifies `Number` with the size of the memory in bytes occupied by the pointset powerset referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Pointset_Powerset_BD_Shape_float_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual*

for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_BD_Shape_float_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_Pointset_Powerset_BD_Shape_float_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Pointset_Powerset_BD_Shape_float_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Pointset_Powerset_BD_Shape_float_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Pointset_Powerset_BD_Shape_float_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Pointset_Powerset_BD_Shape_float_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Pointset_Powerset_BD_Shape_float_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Pointset_Powerset_BD_Shape_float_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_BD_Shape_float_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

```
ppl_Pointset_Powerset_BD_Shape_float_pairwise_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

```
ppl_Pointset_Powerset_BD_Shape_float_omega_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

```
ppl_Pointset_Powerset_BD_Shape_float_unconstrain_space_-
dimension(+Handle, +PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

```
ppl_Pointset_Powerset_BD_Shape_float_unconstrain_space_-
dimensions(+Handle, +List_of_PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_BD_Shape_float_affine_image(+Handle, +PPL_Var,
+Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_-Expr/Coeff to PPL_Var`*.*

```
ppl_Pointset_Powerset_BD_Shape_float_affine_preimage(+Handle, +PPL_-
Var, +Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_-Expr/Coeff to PPL_Var`*.*

```
ppl_Pointset_Powerset_BD_Shape_float_bounded_affine_image(+Handle,
+PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_Pointset_Powerset_BD_Shape_float_bounded_affine_preimage(+Handle,
+PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_Pointset_Powerset_BD_Shape_float_generalized_affine_image(+Handle,
+PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Pointset_Powerset_BD_Shape_float_generalized_affine_-
preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Pointset_Powerset_BD_Shape_float_generalized_affine_image_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by*

Relation_Symbol.

```
ppl_Pointset_Powerset_BD_Shape_float_generalized_affine_preimage_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_BD_Shape_float_intersection_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_BD_Shape_float_upper_bound_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_BD_Shape_float_difference_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the difference of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_BD_Shape_float_time_elapse_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_BD_Shape_float_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` *referenced by* `Handle_1` *with the pointset powerset referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Pointset_Powerset_BD_Shape_float_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to

behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

`ppl_Pointset_Powerset_BD_Shape_float_BHZ03_H79_BHMZ05_widening_-assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening certified by the convergence certificate for H79.

`ppl_Pointset_Powerset_BD_Shape_float_BHZ03_H79_H79_widening_-assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening certified by the convergence certificate for H79.

`ppl_Pointset_Powerset_BD_Shape_float_BGP99_BHMZ05_extrapolation_-assign(+Handle_1, +Handle_2, C_unsigned)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening and the cardinality threshold `C_unsigned`.

`ppl_Pointset_Powerset_BD_Shape_float_BGP99_H79_extrapolation_-assign(+Handle_1, +Handle_2, C_unsigned)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Pointset_Powerset_BD_Shape_float_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_BD_Shape_float_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Pointset_Powerset_BD_Shape_float_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Pointset_Powerset_BD_Shape_float_remove_space_dimensions(+Handle,
+List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_BD_Shape_float_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Pointset_Powerset_BD_Shape_float_expand_space_dimension(+Handle,
+PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Pointset_Powerset_BD_Shape_float_fold_space_dimensions(+Handle,
+List_of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_BD_Shape_float_map_space_dimensions(+Handle,
+P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_BD_Shape_float_iterator_from_-
iterator(+Iterator_1, -Iterator_2)
```

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

```
ppl_Pointset_Powerset_BD_Shape_float_begin_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_float_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_float_iterator_equals_-iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_BD_Shape_float_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_BD_Shape_float_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_BD_Shape_float_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_BD_Shape_float_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_BD_Shape_float_add_disjunct(+Handle_1, +Handle_-2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_BD_Shape_float_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_BD_Shape_float_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Octagonal_Shape_float Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of octagonal shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.

- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_float_from_space_-
dimension(+Dimension_Type, +Universe_or_Empty, -Handle)
```

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Octagonal_Shape_float_from_space_dimension(3, uni
   verse, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_float_from_-
constraints(+Constraint_System, -Handle)
```

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_float_from_-
congruences(+Congruence_System, -Handle)
```

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_float_from_Pointset_-
Powerset_Octagonal_Shape_float(+Handle_1, -Handle_2)
```

Builds a new pointset powerset `P_1` from the pointset_powerset_octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_float_from_Octagonal_Shape_-
float(+Handle_1, -Handle_2)
```

Builds a new pointset powerset `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_float_from_Pointset_-
Powerset_Octagonal_Shape_float_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new pointset powerset `P_1` from the pointset_powerset_octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Octagonal_Shape_float_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Octagonal_Shape_float domain.

`ppl_delete_Pointset_Powerset_Octagonal_Shape_float(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Octagonal_Shape_float_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_Octagonal_Shape_float_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_Octagonal_Shape_float_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_Octagonal_Shape_float_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_Octagonal_Shape_float_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_Octagonal_Shape_float_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_Octagonal_Shape_float_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_float_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_float_contains_Pointset_Powerset_Octagonal_Shape_float(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in or equal to the pointset powerset referenced by* `Handle_1`.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_strictly_contains_-
Pointset_Powerset_Octagonal_Shape_float(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in but not equal to the pointset powerset referenced by* `Handle_1`.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_is_disjoint_from_-
Pointset_Powerset_Octagonal_Shape_float(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is disjoint from the pointset powerset referenced by* `Handle_1`.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_geometrically_covers_-
Pointset_Powerset_Octagonal_Shape_float(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically covers the pointset powerset referenced by* `Handle_1`.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_geometrically_equals_-
Pointset_Powerset_Octagonal_Shape_float(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically equals the pointset powerset referenced by* `Handle_1`.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_equals_Pointset_Powerset_-
Octagonal_Shape_float(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_1` *is equal to the pointset powerset referenced by* `Handle_2`.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_OK(+Handle)
```

Succeeds only if the pointset powerset referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_Pointset_Powerset_Octagonal_Shape_float_constrains(+Handle, +PPL_-
Var)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *constrains the dimension* `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_space_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_affine_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the affine dimension of the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Constraint`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Octagonal_Shape_float_relation_with_-
generator(+Handle, +Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Generator`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Octagonal_Shape_float_relation_with_-
congruence(+Handle, +Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Congruence`*. The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_Octagonal_Shape_float_maximize(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_float_minimize(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_float_maximize_with_-
point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`*.*

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_float_minimize_with_-
point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`*.*

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_float_external_memory_in_-
bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

`ppl_Pointset_Powerset_Octagonal_Shape_float_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_float_size(+Handle, ?Number)`

Unifies `Number` with the size of the memory in bytes occupied by the pointset powerset referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Pointset_Powerset_Octagonal_Shape_float_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_Octagonal_Shape_float_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_Pointset_Powerset_Octagonal_Shape_float_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Pointset_Powerset_Octagonal_Shape_float_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Pointset_Powerset_Octagonal_Shape_float_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Pointset_Powerset_Octagonal_Shape_float_refine_with_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Pointset_Powerset_Octagonal_Shape_float_refine_with_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Pointset_Powerset_Octagonal_Shape_float_refine_with_constraints(+Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Pointset_Powerset_Octagonal_Shape_float_refine_with_congruences(+Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_Octagonal_Shape_float_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its topological closure.

`ppl_Pointset_Powerset_Octagonal_Shape_float_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its pairwise reduction.

`ppl_Pointset_Powerset_Octagonal_Shape_float_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its omega reduction.

`ppl_Pointset_Powerset_Octagonal_Shape_float_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Pointset_Powerset_Octagonal_Shape_float_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Octagonal_Shape_float_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Octagonal_Shape_float_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Octagonal_Shape_float_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $Lin_Expr_1/Coeff \leq PPL_Var \leq Lin_Expr_2/Coeff$.

`ppl_Pointset_Powerset_Octagonal_Shape_float_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $Lin_Expr_1/Coeff \leq PPL_Var \leq Lin_Expr_2/Coeff$.

`ppl_Pointset_Powerset_Octagonal_Shape_float_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $PPL_Var \bowtie Lin_Expr/Coeff$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_float_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $PPL_Var \bowtie Lin_Expr/Coeff$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_float_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $Lin_Expr_1 \bowtie Lin_Expr_2$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_float_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $Lin_Expr_1 \bowtie Lin_Expr_2$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Pointset_Powerset_Octagonal_Shape_float_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the intersection of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_float_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the upper bound of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_difference_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the difference of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_time_elapse_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the time elapse of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` referenced by `Handle_1` with the pointset powerset referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` referenced by `Handle_1` and the pointset powerset `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_BHZ03_H79_BHMZ05_-
widening_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening certified by the convergence certificate for H79.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_BGP99_BHMZ05_-
extrapolation_assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_concatenate_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_remove_space_-
dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_expand_space_-
dimension(+Handle, +PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_fold_space_-
dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_Pointset_Powerset_Octagonal_Shape_float_map_space_-
dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_float_iterator_from_-
iterator(+Iterator_1, -Iterator_2)
```

Builds a new iterator `it` *from the iterator referenced by* `Iterator_1`*.* `Iterator_2` *is unified with the handle for* `it`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_float_begin_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_float_end_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_float_iterator_equals_-
iterator(+Iterator_1, +Iterator_2)
```

Succeeds if and only if the iterator referenced by `Iterator_1` *is equal to the iterator referenced by* `Iterator_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_float_iterator_-
increment(+Iterator)
```

Increments the iterator referenced by `Iterator` *so that it "points" to the next disjunct.*

```
ppl_Pointset_Powerset_Octagonal_Shape_float_iterator_-
decrement(+Iterator)
```

Decrements the iterator referenced by `Iterator` *so that it "points" to the previous disjunct.*

```
ppl_Pointset_Powerset_Octagonal_Shape_float_iterator_get_-
disjunct(+Iterator, -Handle)
```

Unifies with `Handle` *a reference to the disjunct referred to by* `Iterator_1`*.*

```
ppl_delete_Pointset_Powerset_Octagonal_Shape_float_iterator(+Iterator)
```

Invalidates the handle referenced by `Iterator`*: this makes sure the corresponding resources will eventually be released.*

```
ppl_Pointset_Powerset_Octagonal_Shape_float_add_disjunct(+Handle_1,
+Handle_2)
```

Adds to the pointset powerset referenced by `Handle_1` *a disjunct referred to by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_float_drop_disjunct(+Handle,
+Iterator)
```

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_float_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Double_Box Domain

Here we provide a short description for each of the predicates available for the domain of double boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for double boxes

The constructor predicates build a double box from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Double_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new double box $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Double_Box_from_space_dimension(3, universe, X).
```

creates the double box defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Double_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new double box `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Double_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new double box `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Double_Box_from_generators(+Generator_System, -Handle)`

Builds a new double box `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new double boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions

- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Double_Box_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_-2)`

Builds a new double box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_-2)`

Builds a new double box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Grid(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new double box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new double box `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new double box `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new double box `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Double_Box_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new double box `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Uint8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Uint16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Uint32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Uint64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Z_Box_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new double box `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Rational_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_BD_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_BD_Shape_int16_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_BD_Shape_int32_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_BD_Shape_int64_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_BD_Shape_mpz_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_BD_Shape_mpq_class_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Octagonal_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new double box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Octagonal_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new double box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Octagonal_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new double box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Octagonal_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new double box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Octagonal_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new double box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Octagonal_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new double box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_C_Polyhedron_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_NNC_Polyhedron_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Grid_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new double box `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Float_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_BD_Shape_float_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Octagonal_Shape_float_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Double_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_BD_Shape_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Octagonal_Shape_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new double box `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Long_Double_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_BD_Shape_long_double_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new double box `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Double_Box_from_Octagonal_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new double box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Double_Box domain.

`ppl_delete_Double_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the double box

Test Predicates

These predicates test the double box for different properties and succeed or fail depending on the outcome.

`ppl_Double_Box_is_empty(+Handle)`

Succeeds if and only if the double box referenced by `Handle` is empty.

`ppl_Double_Box_is_universe(+Handle)`

Succeeds if and only if the double box referenced by `Handle` is the universe.

`ppl_Double_Box_is_bounded(+Handle)`

Succeeds if and only if the double box referenced by `Handle` is bounded.

`ppl_Double_Box_contains_integer_point(+Handle)`

Succeeds if and only if the double box referenced by `Handle` contains an integer point.

`ppl_Double_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the double box referenced by `Handle` is topologically closed.

`ppl_Double_Box_is_discrete(+Handle)`

Succeeds if and only if the double box referenced by `Handle` is discrete.

`ppl_Double_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the double box referenced by `Handle`.

`ppl_Double_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the double box referenced by `Handle`.

`ppl_Double_Box_contains_Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the double box referenced by `Handle_2` is included in or equal to the double box referenced by `Handle_1`.

`ppl_Double_Box_strictly_contains_Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the double box referenced by `Handle_2` is included in but not equal to the double box referenced by `Handle_1`.

`ppl_Double_Box_is_disjoint_from_Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the double box referenced by `Handle_2` is disjoint from the double box referenced by `Handle_1`.

`ppl_Double_Box_equals_Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the double box referenced by `Handle_1` is equal to the double box referenced by `Handle_2`.

`ppl_Double_Box_OK(+Handle)`

Succeeds only if the double box referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Double_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the double box referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the double box

These predicates will obtain more detailed information about the double box unifying some of their arguments with the results.

`ppl_Double_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the double box referenced by `Handle`.

`ppl_Double_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the double box referenced by `Handle`.

`ppl_Double_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the double box referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Double_Box_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the double box referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Double_Box_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the double box referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Double_Box_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the double box referenced by `Handle`.

`ppl_Double_Box_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the double box referenced by `Handle`.

`ppl_Double_Box_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the double box referenced by `Handle`.

`ppl_Double_Box_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the double box referenced by `Handle`.

`ppl_Double_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if double box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Double_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if double box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Double_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if double box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Double_Box_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if double box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Double_Box_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the total memory in bytes occupied by the double box referenced by `Handle`.

`ppl_Double_Box_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the double box referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Double_Box_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the double box referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the double box

These predicates may modify the double box referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the double box by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Double_Box_add_constraint(+Handle, +Constraint)`

Updates the double box referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Double_Box_add_congruence(+Handle, +Congruence)`

Updates the double box referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Double_Box_add_constraints( +Handle, +Constraint_System)`

Updates the double box referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Double_Box_add_congruences( +Handle, +Congruence_System)`

Updates the double box referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Double_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the double box referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Double_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the double box referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Double_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the double box referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Double_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the double box referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the double box

These predicates enable transformations such as taking the topological closure (which for the domain of double box is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Double_Box_topological_closure_assign(+Handle)`

Assigns to the double box referenced by `Handle` *its topological closure.*

`ppl_Double_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the double box `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

`ppl_Double_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the double box `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Double_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the double box referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff to PPL_Var`*.*

`ppl_Double_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the double box referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff to PPL_Var`*.*

`ppl_Double_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to double box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Double_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to double box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Double_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to double box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Double_Box_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to double box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Double_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to double box `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Double_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to double box `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

`ppl_Double_Box_wrap_assign(+Handle, +List_of_PPL_Var, +Width, +Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)`

Transforms the double box `P` *referenced by* `Handle` *by wrapping the dimensions given by* `List_of_PPL_Vars` *while respecting the specified* `Width`, `Representation` *and* `Overflow` *behavior of all these variables. The* `Constraint_System` *represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer* `C_unsigned` *and* `Boolean` *allow control of the complexity/precision ratio; higher values for* `C_unsigned` *will lead to possibly greater precision while a true value for* `Boolean` *indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.*

Predicates whose results depend on more than one double box

These predicates include the binary operators which will assign to the double box referred to by the first argument its combination with the double box referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Double_Box_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the double box `P` *referenced by* `Handle_1` *the intersection of* `P` *and the double box referenced by* `Handle_2`.

```
ppl_Double_Box_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the double box `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the double box referenced by* `Handle_2`.

```
ppl_Double_Box_difference_assign(+Handle_1, +Handle_2)
```

Assigns to the double box `P` *referenced by* `Handle_1` *the difference of* `P` *and the double box referenced by* `Handle_2`.

```
ppl_Double_Box_time_elapse_assign(+Handle_1, +Handle_2)
```

Assigns to the double box `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the double box referenced by* `Handle_2`.

```
ppl_Double_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the double box `P_1` *referenced by* `Handle_1` *with the double box referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Double_Box_simplify_using_context_assign(+Handle_1, +Handle_2,
?Boolean)
```

Succeeds if and only if the intersection of double box `P_1` *referenced by* `Handle_1` *and the double box* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`.

```
ppl_Double_Box_linear_partition(+Handle_1, +Handle_2, -Handle_3,
-Handle_4)
```

`Handle_1` *and* `Handle_2` *are handles for elements* `P_1` *and* `P_2` *in the Double_Box domain. The predicate unifies handle* `Handle_3` *to a reference to the intersection of* `P_1` *and* `P_2` *and* `Handle_4` *to a reference to a pointset powerset of nnc polyhedra* `P_4`*; where* `P_4` *is the linear partition of* `P_1` *with respect to* `P_2`*. This predicate is only provided if the class* `Pointset_Powerset_NNC_Polyhedron` *has been enabled when configuring the library.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with*

Tokens and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the double box referred to by the second argument has to be contained in (or equal to) the double box referred to by the first argument.

`ppl_Double_Box_CC76_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the double box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the double box referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_Double_Box_CC76_widening_assign(+Handle_1, +Handle_2)`

Assigns to the double box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the double box referenced by `Handle_2`.

`ppl_Double_Box_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Same as predicate `ppl_Double_Box_CC76_widening_assign_with_tokens/4`

`ppl_Double_Box_widening_assign(+Handle_1, +Handle_2)`

Same as predicate `ppl_Double_Box_CC76_widening_assign/2`

`ppl_Double_Box_limited_CC76_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the double box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the double box referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_Double_Box_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the double box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the double box referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the double box referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Double_Box_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the double box `P` referenced by `Handle_1` the concatenation of `P` and the double box referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the double box referred to in the first argument. These predicates enable the modification of the vector space of the double box referred to in

the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Double_Box_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the double box `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Double_Box_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the double box `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Double_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the double box `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Double_Box_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the double box `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Double_Box_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the double box referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Double_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the double box referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Double_Box_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the BD_Shape_double Domain

Here we provide a short description for each of the predicates available for the domain of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for bd shapes

The constructor predicates build a bd shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.

- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_BD_Shape_double_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new bd shape $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_BD_Shape_double_from_space_dimension(3, universe, X).
```

creates the bd shape defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_BD_Shape_double_from_constraints(+Constraint_System, -Handle)`

Builds a new bd shape `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_BD_Shape_double_from_congruences(+Congruence_System, -Handle)`

Builds a new bd shape `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_BD_Shape_double_from_generators(+Generator_System, -Handle)`

Builds a new bd shape `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_BD_Shape_double_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Grid(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Uint8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Uint16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Uint32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Uint64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Z_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Rational_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_mpz_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_mpq_class_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Octagonal_Shape_int8_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_double_from_Octagonal_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_double_from_Octagonal_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_double_from_Octagonal_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_double_from_Octagonal_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_double_from_Octagonal_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_double_from_C_Polyhedron_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_double_from_NNC_Polyhedron_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_double_from_Grid_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_double_from_Float_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_double_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the BD_Shape_double domain.

`ppl_delete_BD_Shape_double(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the bd shape

Test Predicates

These predicates test the bd shape for different properties and succeed or fail depending on the outcome.

`ppl_BD_Shape_double_is_empty(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is empty.

`ppl_BD_Shape_double_is_universe(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is the universe.

`ppl_BD_Shape_double_is_bounded(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is bounded.

`ppl_BD_Shape_double_contains_integer_point(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` contains an integer point.

`ppl_BD_Shape_double_is_topologically_closed(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is topologically closed.

`ppl_BD_Shape_double_is_discrete(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is discrete.

`ppl_BD_Shape_double_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the bd shape referenced by `Handle`.

`ppl_BD_Shape_double_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the bd shape referenced by `Handle`.

`ppl_BD_Shape_double_contains_BD_Shape_double(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is included in or equal to the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_double_strictly_contains_BD_Shape_double(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is included in but not equal to the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_double_is_disjoint_from_BD_Shape_double(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is disjoint from the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_double_equals_BD_Shape_double(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_1` is equal to the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_double_OK(+Handle)`

Succeeds only if the bd shape referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_BD_Shape_double_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the bd shape referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the bd shape

These predicates will obtain more detailed information about the bd shape unifying some of their arguments with the results.

`ppl_BD_Shape_double_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the bd shape referenced by `Handle`.

`ppl_BD_Shape_double_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the bd shape referenced by `Handle`.

`ppl_BD_Shape_double_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_BD_Shape_double_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_BD_Shape_double_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_BD_Shape_double_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_double_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_double_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_double_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_double_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_double_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_double_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_double_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_BD_Shape_double_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the total memory in bytes occupied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_double_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the bd shape referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_BD_Shape_double_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the bd shape referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the bd shape

These predicates may modify the bd shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the bd shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual*

for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_BD_Shape_double_add_constraint(+Handle, +Constraint)`

Updates the bd shape referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_BD_Shape_double_add_congruence(+Handle, +Congruence)`

Updates the bd shape referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_BD_Shape_double_add_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_BD_Shape_double_add_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_BD_Shape_double_refine_with_constraint( +Handle, +Constraint)`

Updates the bd shape referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_BD_Shape_double_refine_with_congruence( +Handle, +Congruence)`

Updates the bd shape referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_BD_Shape_double_refine_with_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_BD_Shape_double_refine_with_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the bd shape

These predicates enable transformations such as taking the topological closure (which for the domain of bd shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_BD_Shape_double_topological_closure_assign(+Handle)`

Assigns to the bd shape referenced by `Handle` its topological closure.

`ppl_BD_Shape_double_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the bd shape `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_BD_Shape_double_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the bd shape `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_BD_Shape_double_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the bd shape referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_BD_Shape_double_affine_preimage(+Handle, +PPL_Var, +Lin_Expr,
+Coeff)
```

Transforms the bd shape referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_BD_Shape_double_bounded_affine_image(+Handle, +PPL_Var, +Lin_-
Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_BD_Shape_double_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_-
Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_BD_Shape_double_generalized_affine_image(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`*.*

```
ppl_BD_Shape_double_generalized_affine_preimage(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_BD_Shape_double_generalized_affine_image_lhs_rhs(+Handle, +Lin_-
Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_-Symbol`*.*

```
ppl_BD_Shape_double_generalized_affine_preimage_lhs_rhs(+Handle,
+Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_BD_Shape_double_wrap_assign(+Handle, +List_of_PPL_Var, +Width,
+Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)
```

Transforms the bd shape `P` *referenced by* `Handle` *by wrapping the dimensions given by* `List_of_-PPL_Vars` *while respecting the specified* `Width`*,* `Representation` *and* `Overflow` *behavior of all these variables. The* `Constraint_System` *represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer* `C_unsigned` *and* `Boolean` *allow control of the complexity/precision ratio; higher values for* `C_unsigned` *will lead to possibly greater*

precision while a true value for `Boolean` *indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.*

Predicates whose results depend on more than one bd shape

These predicates include the binary operators which will assign to the bd shape referred to by the first argument its combination with the bd shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_BD_Shape_double_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` *referenced by* `Handle_1` *the intersection of* `P` *and the bd shape referenced by* `Handle_2`*.*

```
ppl_BD_Shape_double_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the bd shape referenced by* `Handle_2`*.*

```
ppl_BD_Shape_double_difference_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` *referenced by* `Handle_1` *the difference of* `P` *and the bd shape referenced by* `Handle_2`*.*

```
ppl_BD_Shape_double_time_elapse_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the bd shape referenced by* `Handle_2`*.*

```
ppl_BD_Shape_double_upper_bound_assign_if_exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the bd shape `P_1` *referenced by* `Handle_1` *with the bd shape referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_BD_Shape_double_simplify_using_context_assign(+Handle_1, +Handle_-
2, ?Boolean)
```

Succeeds if and only if the intersection of bd shape `P_1` *referenced by* `Handle_1` *and the bd shape* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

```
ppl_BD_Shape_double_linear_partition(+Handle_1, +Handle_2, -Handle_3,
-Handle_4)
```

`Handle_1` *and* `Handle_2` *are handles for elements* `P_1` *and* `P_2` *in the BD_Shape_double domain. The predicate unifies handle* `Handle_3` *to a reference to the intersection of* `P_1` *and* `P_2` *and* `Handle_4` *to a reference to a pointset powerset of nnc polyhedra* `P_4`*; where* `P_4` *is the linear partition of* `P_1` *with respect to* `P_2`*. This predicate is only provided if the class* `Pointset_Powerset_NNC_Polyhedron` *has been enabled when configuring the library.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the bd shape referred to by the second argument has to be contained in (or equal to) the bd shape referred to by the first argument.

`ppl_BD_Shape_double_BHMZ05_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_BD_Shape_double_H79_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_BD_Shape_double_BHMZ05_widening_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_double_H79_widening_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_double_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)`

Same as predicate `ppl_BD_Shape_double_H79_widening_assign_with_tokens/4`

`ppl_BD_Shape_double_widening_assign(+Handle_1, +Handle_2)`

Same as predicate `ppl_BD_Shape_double_H79_widening_assign/2`

`ppl_BD_Shape_double_limited_BHMZ05_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_BD_Shape_double_limited_H79_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_BD_Shape_double_limited_CC76_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_BD_Shape_double_limited_BHMZ05_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_BD_Shape_double_limited_H79_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_BD_Shape_double_limited_CC76_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_BD_Shape_double_CC76_narrowing_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-narrowing of `P_1` and the bd shape referenced by `Handle_2`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_BD_Shape_double_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` referenced by `Handle_1` the concatenation of `P` and the bd shape referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument. These predicates enable the modification of the vector space of the bd shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_BD_Shape_double_add_space_dimensions_and_embed(+Handle,
+Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_embeds `P` in this space.

```
ppl_BD_Shape_double_add_space_dimensions_and_project(+Handle,
+Dimension_Type)
```

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_BD_Shape_double_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the bd shape `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_BD_Shape_double_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the bd shape `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_BD_Shape_double_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the bd shape referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_BD_Shape_double_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the bd shape referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_BD_Shape_double_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Octagonal_Shape_double Domain

Here we provide a short description for each of the predicates available for the domain of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for octagonal shapes

The constructor predicates build a octagonal shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Octagonal_Shape_double_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new octagonal shape $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Octagonal_Shape_double_from_space_dimension(3, universe, X).
```

creates the octagonal shape defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Octagonal_Shape_double_from_constraints(+Constraint_System, -Handle)`

Builds a new octagonal shape `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Octagonal_Shape_double_from_congruences(+Congruence_System, -Handle)`

Builds a new octagonal shape `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Octagonal_Shape_double_from_generators(+Generator_System, -Handle)`

Builds a new octagonal shape `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Octagonal_Shape_double_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_int64_t(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_mpz_-
class(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_-1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_mpq_-
class(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_-1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_C_Polyhedron(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_NNC_Polyhedron(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Grid(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Float_Box(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_BD_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Double_Box(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_BD_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_long_-double(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Uint8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Uint16_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_double_from_Uint32_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Uint64_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Z_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Rational_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_BD_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_BD_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_BD_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_BD_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_BD_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_BD_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Grid_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_BD_Shape_float_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_float_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Double_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_BD_Shape_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Long_Double_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_BD_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_double_from_Octagonal_Shape_long_double_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Octagonal_Shape_double domain.

```
ppl_delete_Octagonal_Shape_double(+Handle)
```

Invalidates the handle referenced by `Handle`*: this makes sure the corresponding resources will eventually be released.*

Predicates that do not change the octagonal shape

Test Predicates

These predicates test the octagonal shape for different properties and succeed or fail depending on the outcome.

`ppl_Octagonal_Shape_double_is_empty(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is empty.*

`ppl_Octagonal_Shape_double_is_universe(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is the universe.*

`ppl_Octagonal_Shape_double_is_bounded(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is bounded.*

`ppl_Octagonal_Shape_double_contains_integer_point(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *contains an integer point.*

`ppl_Octagonal_Shape_double_is_topologically_closed(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is topologically closed.*

`ppl_Octagonal_Shape_double_is_discrete(+Handle)`

Succeeds if and only if the octagonal shape referenced by `Handle` *is discrete.*

`ppl_Octagonal_Shape_double_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from above in the octagonal shape referenced by* `Handle`*.*

`ppl_Octagonal_Shape_double_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` *is bounded from below in the octagonal shape referenced by* `Handle`*.*

`ppl_Octagonal_Shape_double_contains_Octagonal_Shape_double(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` *is included in or equal to the octagonal shape referenced by* `Handle_1`*.*

`ppl_Octagonal_Shape_double_strictly_contains_Octagonal_Shape_double(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` *is included in but not equal to the octagonal shape referenced by* `Handle_1`*.*

`ppl_Octagonal_Shape_double_is_disjoint_from_Octagonal_Shape_double(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` *is disjoint from the octagonal shape referenced by* `Handle_1`*.*

`ppl_Octagonal_Shape_double_equals_Octagonal_Shape_double(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_1` *is equal to the octagonal shape referenced by* `Handle_2`*.*

`ppl_Octagonal_Shape_double_OK(+Handle)`

Succeeds only if the octagonal shape referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Octagonal_Shape_double_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the octagonal shape referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the octagonal shape

These predicates will obtain more detailed information about the octagonal shape unifying some of their arguments with the results.

`ppl_Octagonal_Shape_double_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_double_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_double_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the octagonal shape referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Octagonal_Shape_double_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the octagonal shape referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Octagonal_Shape_double_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the octagonal shape referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Octagonal_Shape_double_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_double_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_double_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_double_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the octagonal shape referenced by `Handle`.

```
ppl_Octagonal_Shape_double_maximize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Octagonal_Shape_double_minimize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Octagonal_Shape_double_maximize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Octagonal_Shape_double_minimize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if octagonal shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Octagonal_Shape_double_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the total memory in bytes occupied by the octagonal shape referenced by `Handle`.

```
ppl_Octagonal_Shape_double_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the external memory in bytes occupied by the octagonal shape referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Octagonal_Shape_double_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the octagonal shape referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the octagonal shape

These predicates may modify the octagonal shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the octagonal shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Octagonal_Shape_double_add_constraint(+Handle, +Constraint)`

Updates the octagonal shape referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_Octagonal_Shape_double_add_congruence(+Handle, +Congruence)`

Updates the octagonal shape referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Octagonal_Shape_double_add_constraints( +Handle, +Constraint_System)`

Updates the octagonal shape referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Octagonal_Shape_double_add_congruences( +Handle, +Congruence_System)`

Updates the octagonal shape referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Octagonal_Shape_double_refine_with_constraint( +Handle, +Constraint)`

Updates the octagonal shape referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Octagonal_Shape_double_refine_with_congruence( +Handle, +Congruence)`

Updates the octagonal shape referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Octagonal_Shape_double_refine_with_constraints( +Handle, +Constraint_System)`

Updates the octagonal shape referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Octagonal_Shape_double_refine_with_congruences( +Handle, +Congruence_System)`

Updates the octagonal shape referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the octagonal shape

These predicates enable transformations such as taking the topological closure (which for the domain of octagonal shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Octagonal_Shape_double_topological_closure_assign(+Handle)`

Assigns to the octagonal shape referenced by `Handle` its topological closure.

`ppl_Octagonal_Shape_double_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the octagonal shape `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Octagonal_Shape_double_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the octagonal shape `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Octagonal_Shape_double_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the octagonal shape referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Octagonal_Shape_double_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the octagonal shape referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Octagonal_Shape_double_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Octagonal_Shape_double_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Octagonal_Shape_double_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_double_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_double_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_double_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_double_wrap_assign(+Handle, +List_of_PPL_Var, +Width, +Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)`

Transforms the octagonal shape `P` referenced by `Handle` by wrapping the dimensions given by `List_of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one octagonal shape

These predicates include the binary operators which will assign to the octagonal shape referred to by the first argument its combination with the octagonal shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Octagonal_Shape_double_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the intersection of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_double_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the upper bound of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_double_difference_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the difference of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_double_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the time elapse of `P` and the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_double_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the octagonal shape `P_1` referenced by `Handle_1` with the octagonal shape referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_Octagonal_Shape_double_simplify_using_context_assign(+Handle_1,
+Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of octagonal shape `P_1` referenced by `Handle_1` and the octagonal shape `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

```
ppl_Octagonal_Shape_double_linear_partition(+Handle_1, +Handle_2,
-Handle_3, -Handle_4)
```

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Octagonal_Shape_double domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the octagonal shape referred to by the second argument has to be contained in (or equal to) the octagonal shape referred to by the first argument.

```
ppl_Octagonal_Shape_double_BHMZ05_widening_assign_with_
tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_double_BHMZ05_widening_assign(+Handle_1, +Handle_
2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_double_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_Octagonal_Shape_double_H79_widening_assign_with_tokens/4`

```
ppl_Octagonal_Shape_double_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_Octagonal_Shape_double_H79_widening_assign/2`

```
ppl_Octagonal_Shape_double_limited_BHMZ05_extrapolation_assign_with_
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_
unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_double_limited_CC76_extrapolation_assign_with_
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_
```

unsigned_2)

Assigns to the octagonal shape P_1 referenced by Handle_1 the CC76-widening of P_1 with the octagonal shape referenced by Handle_2 intersected with the constraints in Constraint_System that are satisfied by all the points of P_1. The widening with tokens delay technique is applied with C_unsigned_1 tokens; C_unsigned_2 is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_double_limited_BHMZ05_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)
```

Assigns to the octagonal shape P_1 referenced by Handle_1 the BHMZ05-widening of P_1 with the octagonal shape referenced by Handle_2 intersected with the constraints in Constraint_System that are satisfied by all the points of P_1.

```
ppl_Octagonal_Shape_double_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)
```

Assigns to the octagonal shape P_1 referenced by Handle_1 the CC76-widening of P_1 with the octagonal shape referenced by Handle_2 intersected with the constraints in Constraint_System that are satisfied by all the points of P_1.

```
ppl_Octagonal_Shape_double_CC76_narrowing_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape P_1 referenced by Handle_1 the CC76-narrowing of P_1 and the octagonal shape referenced by Handle_2.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Octagonal_Shape_double_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the octagonal shape P referenced by Handle_1 the concatenation of P and the octagonal shape referenced by Handle_2.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Octagonal_Shape_double_add_space_dimensions_and_embed(+Handle, +Dimension_Type)
```

Adds Dimension_Type new dimensions to the space enclosing the octagonal shape P referenced by Handle and and_embeds P in this space.

`ppl_Octagonal_Shape_double_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the octagonal shape `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Octagonal_Shape_double_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the octagonal shape `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Octagonal_Shape_double_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the octagonal shape `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Octagonal_Shape_double_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the octagonal shape referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Octagonal_Shape_double_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the octagonal shape referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Octagonal_Shape_double_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Pointset_Powerset_Double_Box Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of double boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of double boxes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_Double_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` *dimensions; it is empty or the universe depending on whether* `Atom` *is* `empty` *or* `universe`*, respectively.* `Handle` *is unified with the handle for $\mathcal{P}$. Thus the query*

```
?- ppl_new_Pointset_Powerset_Double_Box_from_space_dimension(3, universe, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` *bound to a valid handle for accessing it.*

`ppl_new_Pointset_Powerset_Double_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` *from* `Constraint_System`*.* `Handle` *is unified with the handle for* `P`*.*

`ppl_new_Pointset_Powerset_Double_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` *from* `Congruence_System`*.* `Handle` *is unified with the handle for* `P`*.*

Predicates that build new pointset powersets of double boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Double_Box_from_Pointset_Powerset_Double_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` *from the pointset_powerset_double_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Pointset_Powerset_Double_Box_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` *from the double_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Pointset_Powerset_Double_Box_from_Pointset_Powerset_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` *from the pointset_powerset_double_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Pointset_Powerset_Double_Box_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` *from the double_box referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Double_Box domain.

`ppl_delete_Pointset_Powerset_Double_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Double_Box_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_Double_Box_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_Double_Box_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_Double_Box_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_Double_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_Double_Box_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_Double_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Double_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Double_Box_contains_Pointset_Powerset_Double_-Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Double_Box_strictly_contains_Pointset_Powerset_-Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Double_Box_is_disjoint_from_Pointset_Powerset_-Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Double_Box_geometrically_covers_Pointset_-Powerset_Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Double_Box_geometrically_equals_Pointset_-Powerset_Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Double_Box_equals_Pointset_Powerset_Double_-Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Double_Box_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Pointset_Powerset_Double_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

`ppl_Pointset_Powerset_Double_Box_space_dimension(+Handle, ?Dimension_-Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Double_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Double_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Double_Box_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Double_Box_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Double_Box_maximize(+Handle, +Lin_Expr, ?Coeff_-1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_Double_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_Double_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_Double_Box_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Pointset_Powerset_Double_Box_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_Double_Box_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` *with the size of the external memory in bytes occupied by the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_Double_Box_size(+Handle, ?Number)
```

Unifies `Number` *with the size of the memory in bytes occupied by the pointset powerset referenced by* `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_Double_Box_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_Double_Box_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Pointset_Powerset_Double_Box_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Pointset_Powerset_Double_Box_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Pointset_Powerset_Double_Box_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Pointset_Powerset_Double_Box_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Pointset_Powerset_Double_Box_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Pointset_Powerset_Double_Box_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Pointset_Powerset_Double_Box_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Pointset_Powerset_Double_Box_topological_closure_assign(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

```
ppl_Pointset_Powerset_Double_Box_pairwise_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

```
ppl_Pointset_Powerset_Double_Box_omega_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

```
ppl_Pointset_Powerset_Double_Box_unconstrain_space_dimension(+Handle,
+PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`.

```
ppl_Pointset_Powerset_Double_Box_unconstrain_space_dimensions(+Handle,
+List_of_PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Double_Box_affine_image(+Handle, +PPL_Var,
+Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_Double_Box_affine_preimage(+Handle, +PPL_Var,
+Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`.

```
ppl_Pointset_Powerset_Double_Box_bounded_affine_image(+Handle, +PPL_
Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_Double_Box_bounded_affine_preimage(+Handle,
+PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

```
ppl_Pointset_Powerset_Double_Box_generalized_affine_image(+Handle,
+PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Pointset_Powerset_Double_Box_generalized_affine_preimage(+Handle,
+PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Pointset_Powerset_Double_Box_generalized_affine_image_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Pointset_Powerset_Double_Box_generalized_affine_preimage_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_Double_Box_intersection_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Double_Box_upper_bound_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Double_Box_difference_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the difference of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Double_Box_time_elapse_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Double_Box_upper_bound_assign_if_exact(+Handle_-
1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` *referenced by* `Handle_1` *with the pointset powerset referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails*

otherwise.

```
ppl_Pointset_Powerset_Double_Box_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_Double_Box_concatenate_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_Double_Box_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the pointset powerset* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Pointset_Powerset_Double_Box_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the pointset powerset* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

```
ppl_Pointset_Powerset_Double_Box_remove_space_dimensions(+Handle,
+List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Double_Box_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`*.*

```
ppl_Pointset_Powerset_Double_Box_expand_space_dimension(+Handle,
+PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the pointset powerset referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

```
ppl_Pointset_Powerset_Double_Box_fold_space_dimensions(+Handle,
+List_of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` *by folding the space dimensions contained in* `List_of_PPL_Vars` *into dimension* `PPL_Var`*. The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Double_Box_map_space_dimensions(+Handle, +P_-
Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_Double_Box_iterator_from_-
iterator(+Iterator_1, -Iterator_2)
```

Builds a new iterator `it` *from the iterator referenced by* `Iterator_1`*.* `Iterator_2` *is unified with the handle for* `it`*.*

```
ppl_Pointset_Powerset_Double_Box_begin_iterator(+Handle, -Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Double_Box_end_iterator(+Handle, -Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Double_Box_iterator_equals_iterator(+Iterator_1,
+Iterator_2)
```

Succeeds if and only if the iterator referenced by `Iterator_1` *is equal to the iterator referenced by* `Iterator_2`*.*

```
ppl_Pointset_Powerset_Double_Box_iterator_increment(+Iterator)
```

Increments the iterator referenced by `Iterator` *so that it "points" to the next disjunct.*

`ppl_Pointset_Powerset_Double_Box_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_Double_Box_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_Double_Box_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_Double_Box_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_Double_Box_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Double_Box_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_BD_Shape_double Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of bd shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_BD_Shape_double_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_BD_Shape_double_from_space_dimension(3, universe,
    X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_BD_Shape_double_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` *from* `Constraint_System`*.* `Handle` *is unified with the handle for* `P`*.*

`ppl_new_Pointset_Powerset_BD_Shape_double_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` *from* `Congruence_System`*.* `Handle` *is unified with the handle for* `P`*.*

Predicates that build new pointset powersets of bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_BD_Shape_double_from_Pointset_Powerset_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` *from the pointset_powerset_bd_shape_double referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Pointset_Powerset_BD_Shape_double_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` *from the bd_shape_double referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Pointset_Powerset_BD_Shape_double_from_Pointset_Powerset_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` *from the pointset_powerset_bd_shape_double referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Pointset_Powerset_BD_Shape_double_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` *from the bd_shape_double referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_BD_Shape_double domain.

`ppl_delete_Pointset_Powerset_BD_Shape_double(+Handle)`

Invalidates the handle referenced by `Handle`*: this makes sure the corresponding resources will eventually be released.*

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_BD_Shape_double_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_BD_Shape_double_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_BD_Shape_double_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_BD_Shape_double_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_BD_Shape_double_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_BD_Shape_double_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_BD_Shape_double_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_double_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_double_contains_Pointset_Powerset_BD_Shape_double(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_BD_Shape_double_strictly_contains_Pointset_Powerset_BD_Shape_double(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_BD_Shape_double_is_disjoint_from_Pointset_Powerset_BD_Shape_double(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_BD_Shape_double_geometrically_covers_Pointset_Powerset_BD_Shape_double(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_BD_Shape_double_geometrically_equals_Pointset_Powerset_BD_Shape_double(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically equals the pointset powerset referenced by* `Handle_1`.

```
ppl_Pointset_Powerset_BD_Shape_double_equals_Pointset_Powerset_BD_-
Shape_double(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_1` *is equal to the pointset powerset referenced by* `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_double_OK(+Handle)
```

Succeeds only if the pointset powerset referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_Pointset_Powerset_BD_Shape_double_constrains(+Handle, +PPL_Var)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *constrains the dimension* `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_BD_Shape_double_space_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_double_affine_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the affine dimension of the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_double_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Constraint`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_BD_Shape_double_relation_with_generator(+Handle,
+Generator, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Generator`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_BD_Shape_double_relation_with_-
congruence(+Handle, +Congruence, ?Relation_List)
```

Unifies `Relation_List` *with the list of relations the pointset powerset referenced by* `Handle` *has with* `Congruence`. *The possible relations are listed in the grammar rules above.*

```
ppl_Pointset_Powerset_BD_Shape_double_maximize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

`ppl_Pointset_Powerset_BD_Shape_double_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_BD_Shape_double_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_BD_Shape_double_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_BD_Shape_double_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the total memory in bytes occupied by the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_double_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_double_size(+Handle, ?Number)`

Unifies `Number` with the size of the memory in bytes occupied by the pointset powerset referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Pointset_Powerset_BD_Shape_double_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_BD_Shape_double_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Pointset_Powerset_BD_Shape_double_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Pointset_Powerset_BD_Shape_double_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Pointset_Powerset_BD_Shape_double_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

`ppl_Pointset_Powerset_BD_Shape_double_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

`ppl_Pointset_Powerset_BD_Shape_double_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

`ppl_Pointset_Powerset_BD_Shape_double_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

`ppl_Pointset_Powerset_BD_Shape_double_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_BD_Shape_double_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

`ppl_Pointset_Powerset_BD_Shape_double_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

`ppl_Pointset_Powerset_BD_Shape_double_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

`ppl_Pointset_Powerset_BD_Shape_double_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

`ppl_Pointset_Powerset_BD_Shape_double_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

`ppl_Pointset_Powerset_BD_Shape_double_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

`ppl_Pointset_Powerset_BD_Shape_double_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

`ppl_Pointset_Powerset_BD_Shape_double_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Pointset_Powerset_BD_Shape_double_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

`ppl_Pointset_Powerset_BD_Shape_double_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_Pointset_Powerset_BD_Shape_double_generalized_affine_-
preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` ⋈ `Lin_Expr/Coeff`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Pointset_Powerset_BD_Shape_double_generalized_affine_image_lhs_-
rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

```
ppl_Pointset_Powerset_BD_Shape_double_generalized_affine_preimage_-
lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` ⋈ `Lin_Expr_2`, where ⋈ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_BD_Shape_double_intersection_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the intersection of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_double_upper_bound_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the upper bound of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_double_difference_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the difference of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_double_time_elapse_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the time elapse of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_double_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` referenced by `Handle_1` with the pointset powerset referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_Pointset_Powerset_BD_Shape_double_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` referenced by `Handle_1` and the pointset powerset `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_BD_Shape_double_BHZ03_H79_BHMZ05_widening_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening certified by the convergence certificate for H79.

```
ppl_Pointset_Powerset_BD_Shape_double_BHZ03_H79_H79_widening_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening certified by the convergence certificate for H79.

```
ppl_Pointset_Powerset_BD_Shape_double_BGP99_BHMZ05_extrapolation_-
assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening and the cardinality threshold `C_unsigned`.

```
ppl_Pointset_Powerset_BD_Shape_double_BGP99_H79_extrapolation_-
assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_BD_Shape_double_concatenate_assign(+Handle_1,
+Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Pointset_Powerset_BD_Shape_double_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Pointset_Powerset_BD_Shape_double_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Pointset_Powerset_BD_Shape_double_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_BD_Shape_double_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Pointset_Powerset_BD_Shape_double_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Pointset_Powerset_BD_Shape_double_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Pointset_Powerset_BD_Shape_double_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

`ppl_new_Pointset_Powerset_BD_Shape_double_iterator_from_iterator(+Iterator_1, -Iterator_2)`

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

`ppl_Pointset_Powerset_BD_Shape_double_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_double_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_BD_Shape_double_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_BD_Shape_double_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_BD_Shape_double_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_BD_Shape_double_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_BD_Shape_double_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_BD_Shape_double_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_BD_Shape_double_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_BD_Shape_double_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Octagonal_Shape_double Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of octagonal shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_double_from_space_-
dimension(+Dimension_Type, +Universe_or_Empty, -Handle)
```

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_Octagonal_Shape_double_from_space_dimension(3, un
   iverse, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_double_from_-
constraints(+Constraint_System, -Handle)
```

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_double_from_-
congruences(+Congruence_System, -Handle)
```

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_double_from_Pointset_-
Powerset_Octagonal_Shape_double(+Handle_1, -Handle_2)
```

Builds a new pointset powerset `P_1` from the pointset_powerset_octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_double_from_Octagonal_-
Shape_double(+Handle_1, -Handle_2)
```

Builds a new pointset powerset `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_double_from_Pointset_-
Powerset_Octagonal_Shape_double_with_complexity(+Handle, +Complexity,
-Handle)
```

Builds a new pointset powerset `P_1` from the pointset_powerset_octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_double_from_Octagonal_-
Shape_double_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new pointset powerset `P_1` from the octagonal_shape_double referenced by handle `Handle_-1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Octagonal_Shape_double domain.

```
ppl_delete_Pointset_Powerset_Octagonal_Shape_double(+Handle)
```

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_is_empty(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_is_universe(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_is_bounded(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_contains_integer_-
point(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_is_topologically_-
closed(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_is_discrete(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_bounds_from_-
above(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from above in the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_bounds_from_-
below(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from below in the pointset powerset referenced by* `Handle`.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_contains_Pointset_-
Powerset_Octagonal_Shape_double(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in or equal to the pointset powerset referenced by* `Handle_1`.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_strictly_contains_-
Pointset_Powerset_Octagonal_Shape_double(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in but not equal to the pointset powerset referenced by* `Handle_1`.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_is_disjoint_from_-
Pointset_Powerset_Octagonal_Shape_double(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is disjoint from the pointset powerset referenced by* `Handle_1`.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_geometrically_covers_-
Pointset_Powerset_Octagonal_Shape_double(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically covers the pointset powerset referenced by* `Handle_1`.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_geometrically_equals_-
Pointset_Powerset_Octagonal_Shape_double(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically equals the pointset powerset referenced by* `Handle_1`.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_equals_Pointset_-
Powerset_Octagonal_Shape_double(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_1` *is equal to the pointset powerset referenced by* `Handle_2`.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_OK(+Handle)
```

Succeeds only if the pointset powerset referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_Pointset_Powerset_Octagonal_Shape_double_constrains(+Handle,
+PPL_Var)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *constrains the dimension* `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_space_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` *with the dimension of the vector space enclosing the pointset powerset referenced by* `Handle`.

ppl_Pointset_Powerset_Octagonal_Shape_double_affine_dimension(+Handle, ?Dimension_Type)

Unifies Dimension_Type *with the affine dimension of the pointset powerset referenced by* Handle.

ppl_Pointset_Powerset_Octagonal_Shape_double_relation_with_constraint(+Handle, +Constraint, ?Relation_List)

Unifies Relation_List *with the list of relations the pointset powerset referenced by* Handle *has with* Constraint. *The possible relations are listed in the grammar rules above.*

ppl_Pointset_Powerset_Octagonal_Shape_double_relation_with_generator(+Handle, +Generator, ?Relation_List)

Unifies Relation_List *with the list of relations the pointset powerset referenced by* Handle *has with* Generator. *The possible relations are listed in the grammar rules above.*

ppl_Pointset_Powerset_Octagonal_Shape_double_relation_with_congruence(+Handle, +Congruence, ?Relation_List)

Unifies Relation_List *with the list of relations the pointset powerset referenced by* Handle *has with* Congruence. *The possible relations are listed in the grammar rules above.*

ppl_Pointset_Powerset_Octagonal_Shape_double_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)

Succeeds if and only if pointset powerset P *referenced by* Handle *is not empty and* Lin_Expr *is bounded from above in* P.

Coeff_1 *is unified with the numerator of the supremum value and* Coeff_2 *with the denominator of the supremum value. If the supremum is also the maximum,* Boolean *is unified with the atom* true *and, otherwise, unified with the atom* false.

ppl_Pointset_Powerset_Octagonal_Shape_double_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)

Succeeds if and only if pointset powerset P *referenced by* Handle *is not empty and* Lin_Expr *is bounded from below in* P.

Coeff_1 *is unified with the numerator of the infinum value and* Coeff_2 *with the denominator of the infinum value. If the infinum is also the minimum,* Boolean *is unified with the atom* true *and, otherwise, unified with the atom* false.

ppl_Pointset_Powerset_Octagonal_Shape_double_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)

Succeeds if and only if pointset powerset P *referenced by* Handle *is not empty and* Lin_Expr *is bounded from above in* P.

Coeff_1 *is unified with the numerator of the supremum value and* Coeff_2 *with the denominator of the supremum value and* Point *with a point or closure point where* Lin_Expr *reaches this value. If the supremum is also the maximum,* Boolean *is unified with the atom* true *and, otherwise, unified with the atom* false.

ppl_Pointset_Powerset_Octagonal_Shape_double_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)

Succeeds if and only if pointset powerset P *referenced by* Handle *is not empty and* Lin_Expr *is bounded from below in* P.

Coeff_1 *is unified with the numerator of the infinum value and* Coeff_2 *with the denominator of the infinum value and* Point *with a point or closure point where* Lin_Expr *reaches this value. If the infinum is also the minimum,* Boolean *is unified with the atom* true *and, otherwise, unified with the*

atom `false`.

`ppl_Pointset_Powerset_Octagonal_Shape_double_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_double_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the external memory in bytes occupied by the pointset powerset referenced by* `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_double_size(+Handle, ?Number)`

Unifies `Number` *with the size of the memory in bytes occupied by the pointset powerset referenced by* `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Pointset_Powerset_Octagonal_Shape_double_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_Octagonal_Shape_double_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Pointset_Powerset_Octagonal_Shape_double_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

`ppl_Pointset_Powerset_Octagonal_Shape_double_add_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

`ppl_Pointset_Powerset_Octagonal_Shape_double_add_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Pointset_Powerset_Octagonal_Shape_double_refine_with_constraint( +Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_Pointset_Powerset_Octagonal_Shape_double_refine_with_congruence( +Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_Pointset_Powerset_Octagonal_Shape_double_refine_with_constraints( +Handle, +Constraint_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_Pointset_Powerset_Octagonal_Shape_double_refine_with_congruences( +Handle, +Congruence_System)`

Updates the pointset powerset referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_Pointset_Powerset_Octagonal_Shape_double_topological_closure_assign(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its topological closure.

`ppl_Pointset_Powerset_Octagonal_Shape_double_pairwise_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its pairwise reduction.

`ppl_Pointset_Powerset_Octagonal_Shape_double_omega_reduce(+Handle)`

Assigns to the pointset powerset referenced by `Handle` its omega reduction.

`ppl_Pointset_Powerset_Octagonal_Shape_double_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Pointset_Powerset_Octagonal_Shape_double_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Octagonal_Shape_double_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Octagonal_Shape_double_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Octagonal_Shape_double_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_Octagonal_Shape_double_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_Octagonal_Shape_double_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_double_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_double_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_double_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Pointset_Powerset_Octagonal_Shape_double_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the intersection of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_upper_bound_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the upper bound of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_difference_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the difference of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_time_elapse_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` referenced by `Handle_1` the time elapse of `P` and the pointset powerset referenced by `Handle_2`.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` referenced by `Handle_1` with the pointset powerset referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` referenced by `Handle_1` and the pointset powerset `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_BHZ03_H79_BHMZ05_-
widening_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening certified by the convergence certificate for H79.

```
ppl_Pointset_Powerset_Octagonal_Shape_double_BGP99_BHMZ05_-
extrapolation_assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Pointset_Powerset_Octagonal_Shape_double_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Pointset_Powerset_Octagonal_Shape_double_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Pointset_Powerset_Octagonal_Shape_double_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Pointset_Powerset_Octagonal_Shape_double_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Octagonal_Shape_double_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Pointset_Powerset_Octagonal_Shape_double_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Pointset_Powerset_Octagonal_Shape_double_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the pointset powerset referenced by `Handle` *by folding the space dimensions contained in* `List_of_PPL_Vars` *into dimension* `PPL_Var`*. The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Octagonal_Shape_double_map_space_-
dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_double_iterator_from_-
iterator(+Iterator_1, -Iterator_2)
```

Builds a new iterator `it` *from the iterator referenced by* `Iterator_1`*.* `Iterator_2` *is unified with the handle for* `it`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_double_begin_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_double_end_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_double_iterator_equals_-
iterator(+Iterator_1, +Iterator_2)
```

Succeeds if and only if the iterator referenced by `Iterator_1` *is equal to the iterator referenced by* `Iterator_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_double_iterator_-
increment(+Iterator)
```

Increments the iterator referenced by `Iterator` *so that it "points" to the next disjunct.*

```
ppl_Pointset_Powerset_Octagonal_Shape_double_iterator_-
decrement(+Iterator)
```

Decrements the iterator referenced by `Iterator` *so that it "points" to the previous disjunct.*

```
ppl_Pointset_Powerset_Octagonal_Shape_double_iterator_get_-
disjunct(+Iterator, -Handle)
```

Unifies with `Handle` *a reference to the disjunct referred to by* `Iterator_1`*.*

```
ppl_delete_Pointset_Powerset_Octagonal_Shape_double_-
iterator(+Iterator)
```

Invalidates the handle referenced by `Iterator`*: this makes sure the corresponding resources will eventually be released.*

`ppl_Pointset_Powerset_Octagonal_Shape_double_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_double_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_double_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Long_Double_Box Domain

Here we provide a short description for each of the predicates available for the domain of long double boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for long double boxes

The constructor predicates build a long double box from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Long_Double_Box_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new long double box $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Long_Double_Box_from_space_dimension(3, universe, X).
```

creates the long double box defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Long_Double_Box_from_constraints(+Constraint_System, -Handle)`

Builds a new long double box `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Long_Double_Box_from_congruences(+Congruence_System, -Handle)`

Builds a new long double box `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Long_Double_Box_from_generators(+Generator_System, -Handle)`

Builds a new long double box `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new long double boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Long_Double_Box_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Grid(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_float(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Double_Box(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_double(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_double(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new long double box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Long_Double_Box_from_Int16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new long double box `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Long_Double_Box_from_Int32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new long double box `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Long_Double_Box_from_Int64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new long double box `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Long_Double_Box_from_Uint8_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new long double box `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Long_Double_Box_from_Uint16_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new long double box `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Long_Double_Box_from_Uint32_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new long double box `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Long_Double_Box_from_Uint64_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new long double box `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Long_Double_Box_from_Z_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new long double box `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Long_Double_Box_from_Rational_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new long double box `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Long_Double_Box_from_BD_Shape_int8_t_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new long double box `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for

P_1.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Long_Double_Box_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new long double box `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Long_Double_Box domain.

`ppl_delete_Long_Double_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the long double box

Test Predicates

These predicates test the long double box for different properties and succeed or fail depending on the outcome.

`ppl_Long_Double_Box_is_empty(+Handle)`

Succeeds if and only if the long double box referenced by `Handle` is empty.

`ppl_Long_Double_Box_is_universe(+Handle)`

Succeeds if and only if the long double box referenced by `Handle` is the universe.

`ppl_Long_Double_Box_is_bounded(+Handle)`

Succeeds if and only if the long double box referenced by `Handle` is bounded.

`ppl_Long_Double_Box_contains_integer_point(+Handle)`

Succeeds if and only if the long double box referenced by `Handle` contains an integer point.

`ppl_Long_Double_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the long double box referenced by `Handle` is topologically closed.

`ppl_Long_Double_Box_is_discrete(+Handle)`

Succeeds if and only if the long double box referenced by `Handle` is discrete.

`ppl_Long_Double_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the long double box referenced by `Handle`.

`ppl_Long_Double_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the long double box referenced by `Handle`.

`ppl_Long_Double_Box_contains_Long_Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the long double box referenced by `Handle_2` is included in or equal to the long double box referenced by `Handle_1`.

`ppl_Long_Double_Box_strictly_contains_Long_Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the long double box referenced by `Handle_2` is included in but not equal to the long double box referenced by `Handle_1`.

`ppl_Long_Double_Box_is_disjoint_from_Long_Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the long double box referenced by `Handle_2` is disjoint from the long double box referenced by `Handle_1`.

`ppl_Long_Double_Box_equals_Long_Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the long double box referenced by `Handle_1` is equal to the long double box referenced by `Handle_2`.

`ppl_Long_Double_Box_OK(+Handle)`

Succeeds only if the long double box referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Long_Double_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the long double box referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the long double box

These predicates will obtain more detailed information about the long double box unifying some of their arguments with the results.

`ppl_Long_Double_Box_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the long double box referenced by `Handle`.

`ppl_Long_Double_Box_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the long double box referenced by `Handle`.

`ppl_Long_Double_Box_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the long double box referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Long_Double_Box_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the long double box referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Long_Double_Box_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the long double box referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Long_Double_Box_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the long double box referenced by `Handle`.

`ppl_Long_Double_Box_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the long double box referenced by `Handle`.

`ppl_Long_Double_Box_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the long double box referenced by `Handle`.

`ppl_Long_Double_Box_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the long double box referenced by `Handle`.

`ppl_Long_Double_Box_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if long double box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Long_Double_Box_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if long double box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Long_Double_Box_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if long double box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Long_Double_Box_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if long double box `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Long_Double_Box_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the total memory in bytes occupied by the long double box referenced by `Handle`.

`ppl_Long_Double_Box_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the long double box referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Long_Double_Box_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the long double box referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the long double box

These predicates may modify the long double box referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the long double box by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Long_Double_Box_add_constraint(+Handle, +Constraint)`

Updates the long double box referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_Long_Double_Box_add_congruence(+Handle, +Congruence)`

Updates the long double box referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Long_Double_Box_add_constraints( +Handle, +Constraint_System)`

Updates the long double box referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Long_Double_Box_add_congruences( +Handle, +Congruence_System)`

Updates the long double box referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

```
ppl_Long_Double_Box_refine_with_constraint( +Handle, +Constraint)
```

Updates the long double box referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

```
ppl_Long_Double_Box_refine_with_congruence( +Handle, +Congruence)
```

Updates the long double box referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

```
ppl_Long_Double_Box_refine_with_constraints( +Handle, +Constraint_-
System)
```

Updates the long double box referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

```
ppl_Long_Double_Box_refine_with_congruences( +Handle, +Congruence_-
System)
```

Updates the long double box referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the long double box

These predicates enable transformations such as taking the topological closure (which for the domain of long double box is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Long_Double_Box_topological_closure_assign(+Handle)
```

Assigns to the long double box referenced by `Handle` *its topological closure.*

```
ppl_Long_Double_Box_unconstrain_space_dimension(+Handle, +PPL_Var)
```

Modifies the long double box `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`*.*

```
ppl_Long_Double_Box_unconstrain_space_dimensions(+Handle, +List_of_-
PPL_Var)
```

Modifies the long double box `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Long_Double_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the long double box referenced by `Handle` *assigning the affine expression for* `Lin_-Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_Long_Double_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr,
+Coeff)
```

Transforms the long double box referenced by `Handle` *substituting the affine expression for* `Lin_-Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_Long_Double_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_-
Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to long double box `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

```
ppl_Long_Double_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_-
Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to long double box `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

```
ppl_Long_Double_Box_generalized_affine_image(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to long double box `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

```
ppl_Long_Double_Box_generalized_affine_preimage(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to long double box `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

```
ppl_Long_Double_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_-
Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to long double box `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

```
ppl_Long_Double_Box_generalized_affine_preimage_lhs_rhs(+Handle,
+Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to long double box `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

```
ppl_Long_Double_Box_wrap_assign(+Handle, +List_of_PPL_Var, +Width,
+Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)
```

Transforms the long double box `P` referenced by `Handle` by wrapping the dimensions given by `List_-of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one long double box

These predicates include the binary operators which will assign to the long double box referred to by the first argument its combination with the long double box referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Long_Double_Box_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the long double box `P` referenced by `Handle_1` the intersection of `P` and the long double box referenced by `Handle_2`.

```
ppl_Long_Double_Box_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the long double box `P` referenced by `Handle_1` the upper bound of `P` and the long double box referenced by `Handle_2`.

```
ppl_Long_Double_Box_difference_assign(+Handle_1, +Handle_2)
```

Assigns to the long double box `P` referenced by `Handle_1` the difference of `P` and the long double box referenced by `Handle_2`.

```
ppl_Long_Double_Box_time_elapse_assign(+Handle_1, +Handle_2)
```

Assigns to the long double box `P` referenced by `Handle_1` the time elapse of `P` and the long double box referenced by `Handle_2`.

```
ppl_Long_Double_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the long double box `P_1` referenced by `Handle_1` with the long double box referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_Long_Double_Box_simplify_using_context_assign(+Handle_1, +Handle_-
2, ?Boolean)
```

Succeeds if and only if the intersection of long double box `P_1` referenced by `Handle_1` and the long double box `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

```
ppl_Long_Double_Box_linear_partition(+Handle_1, +Handle_2, -Handle_3,
-Handle_4)
```

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Long_Double_Box domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the long double box referred to by the second argument has to be contained in (or equal to) the long double box referred to by the first argument.

```
ppl_Long_Double_Box_CC76_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the long double box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the long double box referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_-unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Long_Double_Box_CC76_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the long double box `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the long double box referenced by `Handle_2`.

```
ppl_Long_Double_Box_widening_assign_with_tokens(+Handle_1, +Handle_2,
+C_unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_Long_Double_Box_CC76_widening_assign_with_tokens/4`

```
ppl_Long_Double_Box_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_Long_Double_Box_CC76_widening_assign/2`

```
ppl_Long_Double_Box_limited_CC76_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the long double box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the long double box referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*. The widening with tokens delay technique is applied with* `C_-unsigned_1` *tokens;* `C_unsigned_2` *is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Long_Double_Box_limited_CC76_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

Assigns to the long double box `P_1` *referenced by* `Handle_1` *the CC76-widening of* `P_1` *with the long double box referenced by* `Handle_2` *intersected with the constraints in* `Constraint_System` *that are satisfied by all the points of* `P_1`*.*

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the long double box referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Long_Double_Box_concatenate_assign(+Handle_1, +Handle_2)
```

Assigns to the long double box `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the long double box referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the long double box referred to in the first argument. These predicates enable the modification of the vector space of the long double box referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Long_Double_Box_add_space_dimensions_and_embed(+Handle,
+Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the long double box* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Long_Double_Box_add_space_dimensions_and_project(+Handle,
+Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the long double box* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

```
ppl_Long_Double_Box_remove_space_dimensions(+Handle, +List_of_PPL_-
Vars)
```

Removes from the vector space enclosing the long double box `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Long_Double_Box_remove_higher_space_dimensions(+Handle,
+Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the long double box `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`*.*

```
ppl_Long_Double_Box_expand_space_dimension(+Handle, +PPL_Var,
+Dimension_Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the long double box referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

```
ppl_Long_Double_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars,
+PPL_Var)
```

Modifies the long double box referenced by `Handle` *by folding the space dimensions contained in* `List_of_PPL_Vars` *into dimension* `PPL_Var`*. The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_Long_Double_Box_map_space_dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for the BD_Shape_long_double Domain

Here we provide a short description for each of the predicates available for the domain of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for bd shapes

The constructor predicates build a bd shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_BD_Shape_long_double_from_space_dimension(+Dimension_Type,
+Universe_or_Empty, -Handle)
```

Builds a new bd shape $\mathcal{P}$ *with* `Dimension_Type` *dimensions; it is empty or the universe depending on whether* `Atom` *is* `empty` *or* `universe`*, respectively.* `Handle` *is unified with the handle for* $\mathcal{P}$*. Thus the query*

```
?- ppl_new_BD_Shape_long_double_from_space_dimension(3, universe, X).
```

creates the bd shape defining the 3-dimensional vector space $\mathbb{R}^3$ *with* `X` *bound to a valid handle for accessing it.*

```
ppl_new_BD_Shape_long_double_from_constraints(+Constraint_System,
-Handle)
```

Builds a new bd shape `P` *from* `Constraint_System`*.* `Handle` *is unified with the handle for* `P`*.*

`ppl_new_BD_Shape_long_double_from_congruences(+Congruence_System, -Handle)`

Builds a new bd shape `P` *from* `Congruence_System`*.* `Handle` *is unified with the handle for* `P`*.*

`ppl_new_BD_Shape_long_double_from_generators(+Generator_System, -Handle)`

Builds a new bd shape `P` *from* `Generator_System`*.* `Handle` *is unified with the handle for* `P`*.*

Predicates that build new bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_BD_Shape_long_double_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` *from the int8_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_BD_Shape_long_double_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` *from the int16_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_BD_Shape_long_double_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` *from the int32_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_BD_Shape_long_double_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` *from the int64_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_BD_Shape_long_double_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` *from the uint8_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_BD_Shape_long_double_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` *from the uint16_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_BD_Shape_long_double_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` *from the uint32_box referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_BD_Shape_long_double_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Z_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Rational_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_BD_Shape_int8_t(+Handle_1, -Handle_-
2)
```

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_BD_Shape_int16_t(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_BD_Shape_int32_t(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_BD_Shape_int64_t(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_BD_Shape_mpz_class(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_BD_Shape_mpq_class(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Octagonal_Shape_int8_t(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Octagonal_Shape_int16_t(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Octagonal_Shape_int32_t(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Octagonal_Shape_int64_t(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Octagonal_Shape_mpz_class(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Octagonal_Shape_mpq_class(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_C_Polyhedron(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_NNC_Polyhedron(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Grid(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Float_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_BD_Shape_float(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Octagonal_Shape_float(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Double_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_BD_Shape_double(+Handle_1, -Handle_-
2)
```

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Octagonal_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Long_Double_Box(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_BD_Shape_long_double(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)
```

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Int8_Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Int16_Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Int32_Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Int64_Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Uint8_Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Uint16_Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Uint32_Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Uint64_Box_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Z_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Rational_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_BD_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_BD_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_BD_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_BD_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_BD_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_BD_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Octagonal_Shape_int8_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Octagonal_Shape_int16_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Octagonal_Shape_int32_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Octagonal_Shape_int64_t_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Octagonal_Shape_mpz_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Octagonal_Shape_mpq_class_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_C_Polyhedron_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_NNC_Polyhedron_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Grid_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_Float_Box_with_complexity(+Handle,
+Complexity, -Handle)
```

Builds a new bd shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_BD_Shape_long_double_from_BD_Shape_float_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new bd shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_long_double_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_long_double_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_long_double_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_long_double_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_long_double_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_long_double_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_BD_Shape_long_double_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new bd shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the BD_Shape_long_double domain.

`ppl_delete_BD_Shape_long_double(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the bd shape

Test Predicates

These predicates test the bd shape for different properties and succeed or fail depending on the outcome.

`ppl_BD_Shape_long_double_is_empty(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is empty.

`ppl_BD_Shape_long_double_is_universe(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is the universe.

`ppl_BD_Shape_long_double_is_bounded(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is bounded.

`ppl_BD_Shape_long_double_contains_integer_point(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` contains an integer point.

`ppl_BD_Shape_long_double_is_topologically_closed(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is topologically closed.

`ppl_BD_Shape_long_double_is_discrete(+Handle)`

Succeeds if and only if the bd shape referenced by `Handle` is discrete.

`ppl_BD_Shape_long_double_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the bd shape referenced by `Handle`.

`ppl_BD_Shape_long_double_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the bd shape referenced by `Handle`.

`ppl_BD_Shape_long_double_contains_BD_Shape_long_double(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is included in or equal to the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_long_double_strictly_contains_BD_Shape_long_double(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is included in but not equal to the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_long_double_is_disjoint_from_BD_Shape_long_double(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_2` is disjoint from the bd shape referenced by `Handle_1`.

`ppl_BD_Shape_long_double_equals_BD_Shape_long_double(+Handle_1, +Handle_2)`

Succeeds if and only if the bd shape referenced by `Handle_1` is equal to the bd shape referenced by `Handle_2`.

`ppl_BD_Shape_long_double_OK(+Handle)`

Succeeds only if the bd shape referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_BD_Shape_long_double_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the bd shape referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the bd shape

These predicates will obtain more detailed information about the bd shape unifying some of their arguments with the results.

`ppl_BD_Shape_long_double_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the bd shape referenced by `Handle`.

`ppl_BD_Shape_long_double_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the bd shape referenced by `Handle`.

`ppl_BD_Shape_long_double_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_BD_Shape_long_double_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_BD_Shape_long_double_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the bd shape referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_BD_Shape_long_double_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_long_double_get_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_long_double_get_minimized_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` with the constraints (in the form of a list) in the minimized constraint system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_long_double_get_minimized_congruences(+Handle, ?Congruence_System)`

Unifies `Congruence_System` with the congruences (in the form of a list) in the minimized congruence system satisfied by the bd shape referenced by `Handle`.

`ppl_BD_Shape_long_double_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_BD_Shape_long_double_minimize(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean)
```

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_BD_Shape_long_double_maximize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_BD_Shape_long_double_minimize_with_point(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if bd shape `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_BD_Shape_long_double_external_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the total memory in bytes occupied by the bd shape referenced by `Handle`.

```
ppl_BD_Shape_long_double_total_memory_in_bytes(+Handle, ?Number)
```

Unifies `Number` with the size of the external memory in bytes occupied by the bd shape referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_BD_Shape_long_double_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the bd shape referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the bd shape

These predicates may modify the bd shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the bd shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual*

for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_BD_Shape_long_double_add_constraint(+Handle, +Constraint)`

Updates the bd shape referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_BD_Shape_long_double_add_congruence(+Handle, +Congruence)`

Updates the bd shape referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_BD_Shape_long_double_add_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_BD_Shape_long_double_add_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_BD_Shape_long_double_refine_with_constraint( +Handle, +Constraint)`

Updates the bd shape referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.

`ppl_BD_Shape_long_double_refine_with_congruence( +Handle, +Congruence)`

Updates the bd shape referenced by `Handle` to one obtained by refining its congruence system with `Congruence`.

`ppl_BD_Shape_long_double_refine_with_constraints( +Handle, +Constraint_System)`

Updates the bd shape referenced by `Handle` to one obtained by refining its constraint system with the constraints in `Constraint_System`.

`ppl_BD_Shape_long_double_refine_with_congruences( +Handle, +Congruence_System)`

Updates the bd shape referenced by `Handle` to one obtained by refining its congruence system with the congruences in `Congruence_System`.

Predicates that transform the bd shape

These predicates enable transformations such as taking the topological closure (which for the domain of bd shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

`ppl_BD_Shape_long_double_topological_closure_assign(+Handle)`

Assigns to the bd shape referenced by `Handle` its topological closure.

`ppl_BD_Shape_long_double_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the bd shape `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_BD_Shape_long_double_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the bd shape `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_BD_Shape_long_double_affine_image(+Handle, +PPL_Var, +Lin_Expr,
+Coeff)
```

Transforms the bd shape referenced by `Handle` *assigning the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_BD_Shape_long_double_affine_preimage(+Handle, +PPL_Var, +Lin_Expr,
+Coeff)
```

Transforms the bd shape referenced by `Handle` *substituting the affine expression for* `Lin_Expr/Coeff` *to* `PPL_Var`*.*

```
ppl_BD_Shape_long_double_bounded_affine_image(+Handle, +PPL_Var,
+Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_BD_Shape_long_double_bounded_affine_preimage(+Handle, +PPL_Var,
+Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$*.*

```
ppl_BD_Shape_long_double_generalized_affine_image(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_BD_Shape_long_double_generalized_affine_preimage(+Handle, +PPL_
Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_BD_Shape_long_double_generalized_affine_image_lhs_rhs(+Handle,
+Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_BD_Shape_long_double_generalized_affine_preimage_lhs_rhs(+Handle,
+Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

Assigns to bd shape `P` *referenced by* `Handle` *the generalized preimage with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$*, where* $\bowtie$ *is the symbol represented by* `Relation_Symbol`*.*

```
ppl_BD_Shape_long_double_wrap_assign(+Handle, +List_of_PPL_Var,
+Width, +Representation, +Overflow, +Constraint_System, +C_unsigned,
+Boolean)
```

Transforms the bd shape `P` *referenced by* `Handle` *by wrapping the dimensions given by* `List_of_PPL_Vars` *while respecting the specified* `Width`*,* `Representation` *and* `Overflow` *behavior of all these variables. The* `Constraint_System` *represents the conditional or looping construct guard with*

respect to which wrapping is performed. The non-negative integer `C_unsigned` *and* `Boolean` *allow control of the complexity/precision ratio; higher values for* `C_unsigned` *will lead to possibly greater precision while a true value for* `Boolean` *indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.*

Predicates whose results depend on more than one bd shape

These predicates include the binary operators which will assign to the bd shape referred to by the first argument its combination with the bd shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_BD_Shape_long_double_intersection_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` *referenced by* `Handle_1` *the intersection of* `P` *and the bd shape referenced by* `Handle_2`*.*

```
ppl_BD_Shape_long_double_upper_bound_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the bd shape referenced by* `Handle_2`*.*

```
ppl_BD_Shape_long_double_difference_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` *referenced by* `Handle_1` *the difference of* `P` *and the bd shape referenced by* `Handle_2`*.*

```
ppl_BD_Shape_long_double_time_elapse_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the bd shape referenced by* `Handle_2`*.*

```
ppl_BD_Shape_long_double_upper_bound_assign_if_exact(+Handle_1,
+Handle_2)
```

Succeeds if the least upper bound of the bd shape `P_1` *referenced by* `Handle_1` *with the bd shape referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_BD_Shape_long_double_simplify_using_context_assign(+Handle_1,
+Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of bd shape `P_1` *referenced by* `Handle_1` *and the bd shape* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

```
ppl_BD_Shape_long_double_linear_partition(+Handle_1, +Handle_2,
-Handle_3, -Handle_4)
```

`Handle_1` *and* `Handle_2` *are handles for elements* `P_1` *and* `P_2` *in the BD_Shape_long_double domain. The predicate unifies handle* `Handle_3` *to a reference to the intersection of* `P_1` *and* `P_2` *and* `Handle_4` *to a reference to a pointset powerset of nnc polyhedra* `P_4`*; where* `P_4` *is the linear partition of* `P_1` *with respect to* `P_2`*. This predicate is only provided if the class* `Pointset_Powerset_NNC_Polyhedron` *has been enabled when configuring the library.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to

behave as specified the bd shape referred to by the second argument has to be contained in (or equal to) the bd shape referred to by the first argument.

```
ppl_BD_Shape_long_double_BHMZ05_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_long_double_H79_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_long_double_BHMZ05_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_long_double_H79_widening_assign(+Handle_1, +Handle_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2`.

```
ppl_BD_Shape_long_double_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_BD_Shape_long_double_H79_widening_assign_with_tokens/4`

```
ppl_BD_Shape_long_double_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_BD_Shape_long_double_H79_widening_assign/2`

```
ppl_BD_Shape_long_double_limited_BHMZ05_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_long_double_limited_H79_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_BD_Shape_long_double_limited_CC76_extrapolation_assign_with_-
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_-
unsigned_2)
```

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

`ppl_BD_Shape_long_double_limited_BHMZ05_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

`ppl_BD_Shape_long_double_limited_H79_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the H79-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

`ppl_BD_Shape_long_double_limited_CC76_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the bd shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

`ppl_BD_Shape_long_double_CC76_narrowing_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P_1` referenced by `Handle_1` the CC76-narrowing of `P_1` and the bd shape referenced by `Handle_2`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_BD_Shape_long_double_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the bd shape `P` referenced by `Handle_1` the concatenation of `P` and the bd shape referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the bd shape referred to in the first argument. These predicates enable the modification of the vector space of the bd shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_BD_Shape_long_double_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_BD_Shape_long_double_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the bd shape `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_BD_Shape_long_double_remove_space_dimensions(+Handle, +List_of_-
PPL_Vars)
```

Removes from the vector space enclosing the bd shape `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

```
ppl_BD_Shape_long_double_remove_higher_space_dimensions(+Handle,
+Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the bd shape `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

```
ppl_BD_Shape_long_double_expand_space_dimension(+Handle, +PPL_Var,
+Dimension_Type)
```

Expands the `PPL_Var-th` dimension of the vector space enclosing the bd shape referenced by `Handle` to `Dimension_Type` new space dimensions.

```
ppl_BD_Shape_long_double_fold_space_dimensions(+Handle, +List_of_PPL_-
Vars, +PPL_Var)
```

Modifies the bd shape referenced by `Handle` by folding the space dimensions contained in `List_of_-PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

```
ppl_BD_Shape_long_double_map_space_dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, which has `n` entries. The result is undefined if `P_Func` does not encode a partial function.

Predicates for the Octagonal_Shape_long_double Domain

Here we provide a short description for each of the predicates available for the domain of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for octagonal shapes

The constructor predicates build a octagonal shape from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Octagonal_Shape_long_double_from_space_dimension(+Dimension_-
Type, +Universe_or_Empty, -Handle)
```

Builds a new octagonal shape $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Octagonal_Shape_long_double_from_space_dimension(3, universe, X).
```

creates the octagonal shape defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Octagonal_Shape_long_double_from_constraints(+Constraint_System, -Handle)`

Builds a new octagonal shape `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Octagonal_Shape_long_double_from_congruences(+Congruence_System, -Handle)`

Builds a new octagonal shape `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Octagonal_Shape_long_double_from_generators(+Generator_System, -Handle)`

Builds a new octagonal shape `P` from `Generator_System`. `Handle` is unified with the handle for `P`.

Predicates that build new octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Octagonal_Shape_long_double_from_Int8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Int16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Int32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Int64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Uint8_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Uint16_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Uint32_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Uint64_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Z_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Rational_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_BD_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_BD_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_BD_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_BD_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_BD_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_BD_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_int8_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_int16_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_int32_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_int64_t(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_mpz_class(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_mpq_class(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_C_Polyhedron(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_NNC_Polyhedron(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Grid(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Float_Box(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_BD_Shape_float(+Handle_1, -Handle_2)`

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_-
float(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_long_double_from_Double_Box(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_long_double_from_BD_Shape_double(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_-
double(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_long_double_from_Long_Double_Box(+Handle_1,
-Handle_2)
```

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_long_double_from_BD_Shape_long_-
double(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_long_-
double(+Handle_1, -Handle_2)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_long_double_from_Int8_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_long_double_from_Int16_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_long_double_from_Int32_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_long_double_from_Int64_Box_with_-
complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the int64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Uint8_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the uint8_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Uint16_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the uint16_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Uint32_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the uint32_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Uint64_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the uint64_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Z_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the z_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Rational_Box_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the rational_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_BD_Shape_int8_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_BD_Shape_int16_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_BD_Shape_int32_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_BD_Shape_int64_t_with_-complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_BD_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_BD_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_int8_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int8_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_int16_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int16_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_int32_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int32_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_int64_t_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_int64_t referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_mpz_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_mpz_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_mpq_class_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_mpq_class referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_C_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the polyhedron referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Grid_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the grid referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Float_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the float_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_BD_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_float_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_float referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_BD_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the octagonal_shape_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Octagonal_Shape_long_double_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new octagonal shape `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

```
ppl_new_Octagonal_Shape_long_double_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)
```

Builds a new octagonal shape `P_1` from the octagonal_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Octagonal_Shape_long_double domain.

```
ppl_delete_Octagonal_Shape_long_double(+Handle)
```

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the octagonal shape

Test Predicates

These predicates test the octagonal shape for different properties and succeed or fail depending on the outcome.

```
ppl_Octagonal_Shape_long_double_is_empty(+Handle)
```

Succeeds if and only if the octagonal shape referenced by `Handle` is empty.

```
ppl_Octagonal_Shape_long_double_is_universe(+Handle)
```

Succeeds if and only if the octagonal shape referenced by `Handle` is the universe.

```
ppl_Octagonal_Shape_long_double_is_bounded(+Handle)
```

Succeeds if and only if the octagonal shape referenced by `Handle` is bounded.

```
ppl_Octagonal_Shape_long_double_contains_integer_point(+Handle)
```

Succeeds if and only if the octagonal shape referenced by `Handle` contains an integer point.

```
ppl_Octagonal_Shape_long_double_is_topologically_closed(+Handle)
```

Succeeds if and only if the octagonal shape referenced by `Handle` is topologically closed.

```
ppl_Octagonal_Shape_long_double_is_discrete(+Handle)
```

Succeeds if and only if the octagonal shape referenced by `Handle` is discrete.

```
ppl_Octagonal_Shape_long_double_bounds_from_above(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` is bounded from above in the octagonal shape referenced by `Handle`.

```
ppl_Octagonal_Shape_long_double_bounds_from_below(+Handle, +Lin_Expr)
```

Succeeds if and only if `Lin_Expr` is bounded from below in the octagonal shape referenced by `Handle`.

```
ppl_Octagonal_Shape_long_double_contains_Octagonal_Shape_long_double(+Handle_1, +Handle_2)
```

Succeeds if and only if the octagonal shape referenced by `Handle_2` is included in or equal to the octagonal shape referenced by `Handle_1`.

`ppl_Octagonal_Shape_long_double_strictly_contains_Octagonal_Shape_long_double(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` is included in but not equal to the octagonal shape referenced by `Handle_1`.

`ppl_Octagonal_Shape_long_double_is_disjoint_from_Octagonal_Shape_long_double(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_2` is disjoint from the octagonal shape referenced by `Handle_1`.

`ppl_Octagonal_Shape_long_double_equals_Octagonal_Shape_long_double(+Handle_1, +Handle_2)`

Succeeds if and only if the octagonal shape referenced by `Handle_1` is equal to the octagonal shape referenced by `Handle_2`.

`ppl_Octagonal_Shape_long_double_OK(+Handle)`

Succeeds only if the octagonal shape referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Octagonal_Shape_long_double_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the octagonal shape referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the octagonal shape

These predicates will obtain more detailed information about the octagonal shape unifying some of their arguments with the results.

`ppl_Octagonal_Shape_long_double_space_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_long_double_affine_dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_long_double_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the octagonal shape referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Octagonal_Shape_long_double_relation_with_generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the octagonal shape referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Octagonal_Shape_long_double_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the octagonal shape referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Octagonal_Shape_long_double_get_constraints(+Handle, ?Constraint_System)`

Unifies `Constraint_System` *with the constraints (in the form of a list) in the constraint system satisfied by the octagonal shape referenced by* `Handle`.

```
ppl_Octagonal_Shape_long_double_get_congruences(+Handle, ?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the congruence system satisfied by the octagonal shape referenced by* `Handle`.

```
ppl_Octagonal_Shape_long_double_get_minimized_constraints(+Handle, ?Constraint_System)
```

Unifies `Constraint_System` *with the constraints (in the form of a list) in the minimized constraint system satisfied by the octagonal shape referenced by* `Handle`.

```
ppl_Octagonal_Shape_long_double_get_minimized_congruences(+Handle, ?Congruence_System)
```

Unifies `Congruence_System` *with the congruences (in the form of a list) in the minimized congruence system satisfied by the octagonal shape referenced by* `Handle`.

```
ppl_Octagonal_Shape_long_double_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if octagonal shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Octagonal_Shape_long_double_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if octagonal shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Octagonal_Shape_long_double_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if octagonal shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from above in* `P`.

`Coeff_1` *is unified with the numerator of the supremum value and* `Coeff_2` *with the denominator of the supremum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the supremum is also the maximum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

```
ppl_Octagonal_Shape_long_double_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if octagonal shape `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

`ppl_Octagonal_Shape_long_double_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the total memory in bytes occupied by the octagonal shape referenced by `Handle`.

`ppl_Octagonal_Shape_long_double_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the octagonal shape referenced by `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Octagonal_Shape_long_double_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the octagonal shape referenced by `Handle` on the standard output.

Space-dimension preserving predicates that may change the octagonal shape

These predicates may modify the octagonal shape referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the octagonal shape by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Octagonal_Shape_long_double_add_constraint(+Handle, +Constraint)`

Updates the octagonal shape referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.

`ppl_Octagonal_Shape_long_double_add_congruence(+Handle, +Congruence)`

Updates the octagonal shape referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.

`ppl_Octagonal_Shape_long_double_add_constraints( +Handle, +Constraint_System)`

Updates the octagonal shape referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.

`ppl_Octagonal_Shape_long_double_add_congruences( +Handle, +Congruence_System)`

Updates the octagonal shape referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.

`ppl_Octagonal_Shape_long_double_refine_with_constraint( +Handle, +Constraint)`

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`.

```
ppl_Octagonal_Shape_long_double_refine_with_congruence( +Handle,
+Congruence)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`.

```
ppl_Octagonal_Shape_long_double_refine_with_constraints( +Handle,
+Constraint_System)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`.

```
ppl_Octagonal_Shape_long_double_refine_with_congruences( +Handle,
+Congruence_System)
```

Updates the octagonal shape referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`.

Predicates that transform the octagonal shape

These predicates enable transformations such as taking the topological closure (which for the domain of octagonal shape is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Octagonal_Shape_long_double_topological_closure_assign(+Handle)
```

Assigns to the octagonal shape referenced by `Handle` *its topological closure.*

```
ppl_Octagonal_Shape_long_double_unconstrain_space_dimension(+Handle,
+PPL_Var)
```

Modifies the octagonal shape `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`.

```
ppl_Octagonal_Shape_long_double_unconstrain_space_dimensions(+Handle,
+List_of_PPL_Var)
```

Modifies the octagonal shape `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Octagonal_Shape_long_double_affine_image(+Handle, +PPL_Var, +Lin_-
Expr, +Coeff)
```

Transforms the octagonal shape referenced by `Handle` *assigning the affine expression for* `Lin_-Expr/Coeff` *to* `PPL_Var`.

```
ppl_Octagonal_Shape_long_double_affine_preimage(+Handle, +PPL_Var,
+Lin_Expr, +Coeff)
```

Transforms the octagonal shape referenced by `Handle` *substituting the affine expression for* `Lin_-Expr/Coeff` *to* `PPL_Var`.

```
ppl_Octagonal_Shape_long_double_bounded_affine_image(+Handle, +PPL_-
Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

Assigns to octagonal shape `P` *referenced by* `Handle` *the generalized image with respect to the generalized affine transfer relation* $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Octagonal_Shape_long_double_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

`ppl_Octagonal_Shape_long_double_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_long_double_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_long_double_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_long_double_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to octagonal shape `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Octagonal_Shape_long_double_wrap_assign(+Handle, +List_of_PPL_Var, +Width, +Representation, +Overflow, +Constraint_System, +C_unsigned, +Boolean)`

Transforms the octagonal shape `P` referenced by `Handle` by wrapping the dimensions given by `List_of_PPL_Vars` while respecting the specified `Width`, `Representation` and `Overflow` behavior of all these variables. The `Constraint_System` represents the conditional or looping construct guard with respect to which wrapping is performed. The non-negative integer `C_unsigned` and `Boolean` allow control of the complexity/precision ratio; higher values for `C_unsigned` will lead to possibly greater precision while a true value for `Boolean` indicates that the space dimensions should be wrapped individually. See Section Wrapping Operator for a more detailed description of this operator.

Predicates whose results depend on more than one octagonal shape

These predicates include the binary operators which will assign to the octagonal shape referred to by the first argument its combination with the octagonal shape referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Octagonal_Shape_long_double_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the octagonal shape `P` referenced by `Handle_1` the intersection of `P` and the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_long_double_upper_bound_assign(+Handle_1,
+Handle_2)
```

Assigns to the octagonal shape `P` referenced by `Handle_1` the upper bound of `P` and the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_long_double_difference_assign(+Handle_1, +Handle_-
2)
```

Assigns to the octagonal shape `P` referenced by `Handle_1` the difference of `P` and the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_long_double_time_elapse_assign(+Handle_1,
+Handle_2)
```

Assigns to the octagonal shape `P` referenced by `Handle_1` the time elapse of `P` and the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_long_double_upper_bound_assign_if_exact(+Handle_1,
+Handle_2)
```

Succeeds if the least upper bound of the octagonal shape `P_1` referenced by `Handle_1` with the octagonal shape referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

```
ppl_Octagonal_Shape_long_double_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of octagonal shape `P_1` referenced by `Handle_1` and the octagonal shape `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

```
ppl_Octagonal_Shape_long_double_linear_partition(+Handle_1, +Handle_2,
-Handle_3, -Handle_4)
```

`Handle_1` and `Handle_2` are handles for elements `P_1` and `P_2` in the Octagonal_Shape_long_double domain. The predicate unifies handle `Handle_3` to a reference to the intersection of `P_1` and `P_2` and `Handle_4` to a reference to a pointset powerset of nnc polyhedra `P_4`; where `P_4` is the linear partition of `P_1` with respect to `P_2`. This predicate is only provided if the class `Pointset_Powerset_NNC_-Polyhedron` has been enabled when configuring the library.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the octagonal shape referred to by the second argument has to be contained in (or equal to) the octagonal shape referred to by the first argument.

```
ppl_Octagonal_Shape_long_double_BHMZ05_widening_assign_with_-
tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_-unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_long_double_BHMZ05_widening_assign(+Handle_1,
+Handle_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2`.

```
ppl_Octagonal_Shape_long_double_widening_assign_with_tokens(+Handle_1,
+Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

Same as predicate `ppl_Octagonal_Shape_long_double_H79_widening_assign_with_tokens/4`

```
ppl_Octagonal_Shape_long_double_widening_assign(+Handle_1, +Handle_2)
```

Same as predicate `ppl_Octagonal_Shape_long_double_H79_widening_assign/2`

```
ppl_Octagonal_Shape_long_double_limited_BHMZ05_extrapolation_assign_
with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1,
?C_unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_long_double_limited_CC76_extrapolation_assign_
with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1,
?C_unsigned_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.

```
ppl_Octagonal_Shape_long_double_limited_BHMZ05_extrapolation_
assign(+Handle_1, +Handle_2, +Constraint_System)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the BHMZ05-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_Octagonal_Shape_long_double_limited_CC76_extrapolation_
assign(+Handle_1, +Handle_2, +Constraint_System)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the CC76-widening of `P_1` with the octagonal shape referenced by `Handle_2` intersected with the constraints in `Constraint_System` that are satisfied by all the points of `P_1`.

```
ppl_Octagonal_Shape_long_double_CC76_narrowing_assign(+Handle_1,
+Handle_2)
```

Assigns to the octagonal shape `P_1` referenced by `Handle_1` the CC76-narrowing of `P_1` and the octagonal shape referenced by `Handle_2`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Octagonal_Shape_long_double_concatenate_assign(+Handle_1,
+Handle_2)
```

Assigns to the octagonal shape `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the octagonal shape referenced by* `Handle_2`*.*

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. These predicates enable the modification of the vector space of the octagonal shape referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Octagonal_Shape_long_double_add_space_dimensions_and_-
embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the octagonal shape* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Octagonal_Shape_long_double_add_space_dimensions_and_-
project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the octagonal shape* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

```
ppl_Octagonal_Shape_long_double_remove_space_dimensions(+Handle,
+List_of_PPL_Vars)
```

Removes from the vector space enclosing the octagonal shape `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Octagonal_Shape_long_double_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the octagonal shape `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`*.*

```
ppl_Octagonal_Shape_long_double_expand_space_dimension(+Handle, +PPL_-
Var, +Dimension_Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the octagonal shape referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

```
ppl_Octagonal_Shape_long_double_fold_space_dimensions(+Handle, +List_-
of_PPL_Vars, +PPL_Var)
```

Modifies the octagonal shape referenced by `Handle` *by folding the space dimensions contained in* `List_of_PPL_Vars` *into dimension* `PPL_Var`*. The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_Octagonal_Shape_long_double_map_space_dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for the Pointset_Powerset_Long_Double_Box Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of long double boxes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of long double boxes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Pointset_Powerset_Long_Double_Box_from_space_-
dimension(+Dimension_Type, +Universe_or_Empty, -Handle)
```

Builds a new pointset powerset $\mathcal{P}$ *with* `Dimension_Type` *dimensions; it is empty or the universe depending on whether* `Atom` *is* `empty` *or* `universe`*, respectively.* `Handle` *is unified with the handle for* $\mathcal{P}$*. Thus the query*

```
?- ppl_new_Pointset_Powerset_Long_Double_Box_from_space_dimension(3, universe,
    X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ *with* `X` *bound to a valid handle for accessing it.*

```
ppl_new_Pointset_Powerset_Long_Double_Box_from_constraints(+Constraint_-
System, -Handle)
```

Builds a new pointset powerset `P` *from* `Constraint_System`*.* `Handle` *is unified with the handle for* `P`*.*

```
ppl_new_Pointset_Powerset_Long_Double_Box_from_congruences(+Congruence_-
System, -Handle)
```

Builds a new pointset powerset `P` *from* `Congruence_System`*.* `Handle` *is unified with the handle for* `P`*.*

Predicates that build new pointset powersets of long double boxes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Long_Double_Box_from_Pointset_Powerset_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Long_Double_Box_from_Long_Double_Box(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the long_double_box referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Long_Double_Box_from_Pointset_Powerset_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_Long_Double_Box_from_Long_Double_Box_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the long_double_box referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Long_Double_Box domain.

`ppl_delete_Pointset_Powerset_Long_Double_Box(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Long_Double_Box_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_Long_Double_Box_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_Long_Double_Box_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_Long_Double_Box_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_Long_Double_Box_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_Long_Double_Box_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_Long_Double_Box_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Long_Double_Box_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Long_Double_Box_contains_Pointset_Powerset_Long_Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Long_Double_Box_strictly_contains_Pointset_Powerset_Long_Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Long_Double_Box_is_disjoint_from_Pointset_Powerset_Long_Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Long_Double_Box_geometrically_covers_Pointset_Powerset_Long_Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Long_Double_Box_geometrically_equals_Pointset_Powerset_Long_Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Long_Double_Box_equals_Pointset_Powerset_Long_Double_Box(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Long_Double_Box_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Pointset_Powerset_Long_Double_Box_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_Long_Double_Box_space_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_Long_Double_Box_affine_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_Long_Double_Box_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_Long_Double_Box_relation_with_generator(+Handle,
+Generator, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_Long_Double_Box_relation_with_-
congruence(+Handle, +Congruence, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_Long_Double_Box_maximize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_Long_Double_Box_minimize(+Handle, +Lin_Expr,
?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_Long_Double_Box_maximize_with_point(+Handle,
+Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the

atom `false`.

`ppl_Pointset_Powerset_Long_Double_Box_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

`ppl_Pointset_Powerset_Long_Double_Box_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`.

`ppl_Pointset_Powerset_Long_Double_Box_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the external memory in bytes occupied by the pointset powerset referenced by* `Handle`.

`ppl_Pointset_Powerset_Long_Double_Box_size(+Handle, ?Number)`

Unifies `Number` *with the size of the memory in bytes occupied by the pointset powerset referenced by* `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Pointset_Powerset_Long_Double_Box_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_Long_Double_Box_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Pointset_Powerset_Long_Double_Box_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

```
ppl_Pointset_Powerset_Long_Double_Box_add_constraints( +Handle,
+Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

```
ppl_Pointset_Powerset_Long_Double_Box_add_congruences( +Handle,
+Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

```
ppl_Pointset_Powerset_Long_Double_Box_refine_with_constraint( +Handle,
+Constraint)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

```
ppl_Pointset_Powerset_Long_Double_Box_refine_with_congruence( +Handle,
+Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

```
ppl_Pointset_Powerset_Long_Double_Box_refine_with_constraints(
+Handle, +Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

```
ppl_Pointset_Powerset_Long_Double_Box_refine_with_congruences(
+Handle, +Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Pointset_Powerset_Long_Double_Box_topological_closure_-
assign(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

```
ppl_Pointset_Powerset_Long_Double_Box_pairwise_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

```
ppl_Pointset_Powerset_Long_Double_Box_omega_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

`ppl_Pointset_Powerset_Long_Double_Box_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Pointset_Powerset_Long_Double_Box_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Long_Double_Box_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Long_Double_Box_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_Long_Double_Box_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_Long_Double_Box_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_Long_Double_Box_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Long_Double_Box_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Long_Double_Box_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Long_Double_Box_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Pointset_Powerset_Long_Double_Box_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the intersection of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Long_Double_Box_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the upper bound of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Long_Double_Box_difference_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the difference of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Long_Double_Box_time_elapse_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the time elapse of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Long_Double_Box_upper_bound_assign_if_exact(+Handle_1, +Handle_2)`

Succeeds if the least upper bound of the pointset powerset `P_1` referenced by `Handle_1` with the pointset powerset referenced by `Handle_2` is exact; in which case the least upper bound is assigned to `P_1`; fails otherwise.

`ppl_Pointset_Powerset_Long_Double_Box_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)`

Succeeds if and only if the intersection of pointset powerset `P_1` referenced by `Handle_1` and the pointset powerset `P_2` referenced by `Handle_2` is non-empty. Assigns to `P_1` its meet-preserving simplification with respect to `P_2`.

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Pointset_Powerset_Long_Double_Box_concatenate_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Pointset_Powerset_Long_Double_Box_add_space_dimensions_and_embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Pointset_Powerset_Long_Double_Box_add_space_dimensions_and_project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

`ppl_Pointset_Powerset_Long_Double_Box_remove_space_dimensions(+Handle, +List_of_PPL_Vars)`

Removes from the vector space enclosing the pointset powerset `P` referenced by `Handle` the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Long_Double_Box_remove_higher_space_dimensions(+Handle, +Dimension_Type)`

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` referenced by `Handle` so that, upon successful return, the new space dimension is `Dimension_Type`.

`ppl_Pointset_Powerset_Long_Double_Box_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)`

Expands the `PPL_Var-th` dimension of the vector space enclosing the pointset powerset referenced by `Handle` to `Dimension_Type` new space dimensions.

`ppl_Pointset_Powerset_Long_Double_Box_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)`

Modifies the pointset powerset referenced by `Handle` by folding the space dimensions contained in `List_of_PPL_Vars` into dimension `PPL_Var`. The presence of duplicates in `List_of_PPL_Vars` is a waste but an innocuous one.

`ppl_Pointset_Powerset_Long_Double_Box_map_space_dimensions(+Handle, +P_Func)`

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_Long_Double_Box_iterator_from_-
iterator(+Iterator_1, -Iterator_2)
```

Builds a new iterator `it` *from the iterator referenced by* `Iterator_1`*.* `Iterator_2` *is unified with the handle for* `it`*.*

```
ppl_Pointset_Powerset_Long_Double_Box_begin_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Long_Double_Box_end_iterator(+Handle, -Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_Long_Double_Box_iterator_equals_-
iterator(+Iterator_1, +Iterator_2)
```

Succeeds if and only if the iterator referenced by `Iterator_1` *is equal to the iterator referenced by* `Iterator_2`*.*

```
ppl_Pointset_Powerset_Long_Double_Box_iterator_increment(+Iterator)
```

Increments the iterator referenced by `Iterator` *so that it "points" to the next disjunct.*

```
ppl_Pointset_Powerset_Long_Double_Box_iterator_decrement(+Iterator)
```

Decrements the iterator referenced by `Iterator` *so that it "points" to the previous disjunct.*

```
ppl_Pointset_Powerset_Long_Double_Box_iterator_get_disjunct(+Iterator,
-Handle)
```

Unifies with `Handle` *a reference to the disjunct referred to by* `Iterator_1`*.*

```
ppl_delete_Pointset_Powerset_Long_Double_Box_iterator(+Iterator)
```

Invalidates the handle referenced by `Iterator`*: this makes sure the corresponding resources will eventually be released.*

```
ppl_Pointset_Powerset_Long_Double_Box_add_disjunct(+Handle_1,
+Handle_2)
```

Adds to the pointset powerset referenced by `Handle_1` *a disjunct referred to by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Long_Double_Box_drop_disjunct(+Handle,
+Iterator)
```

If `it` *is the iterator referred to by* `Iterator`*, drops from the pointset powerset referenced by* `Handle` *the disjunct pointed to by* `it` *and assigns to* `it` *an iterator to the next disjunct.*

`ppl_Pointset_Powerset_Long_Double_Box_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_BD_Shape_long_double Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of bd shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of bd shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

`ppl_new_Pointset_Powerset_BD_Shape_long_double_from_space_dimension(+Dimension_Type, +Universe_or_Empty, -Handle)`

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` dimensions; it is empty or the universe depending on whether `Atom` is `empty` or `universe`, respectively. `Handle` is unified with the handle for $\mathcal{P}$. Thus the query

```
?- ppl_new_Pointset_Powerset_BD_Shape_long_double_from_space_dimension(3, univ
   erse, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` bound to a valid handle for accessing it.

`ppl_new_Pointset_Powerset_BD_Shape_long_double_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` from `Constraint_System`. `Handle` is unified with the handle for `P`.

`ppl_new_Pointset_Powerset_BD_Shape_long_double_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` from `Congruence_System`. `Handle` is unified with the handle for `P`.

Predicates that build new pointset powersets of bd shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_BD_Shape_long_double_from_Pointset_Powerset_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the pointset_powerset_bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_BD_Shape_long_double_from_BD_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` from the bd_shape_long_double referenced by handle `Handle_1`. `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_BD_Shape_long_double_from_Pointset_Powerset_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the pointset_powerset_bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

`ppl_new_Pointset_Powerset_BD_Shape_long_double_from_BD_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` from the bd_shape_long_double referenced by handle `Handle_1` using an algorithm whose complexity does not exceed `Complexity`; `Handle_2` is unified with the handle for `P_1`.

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_BD_Shape_long_double domain.

`ppl_delete_Pointset_Powerset_BD_Shape_long_double(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_BD_Shape_long_double_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_BD_Shape_long_double_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_BD_Shape_long_double_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_BD_Shape_long_double_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` *contains an integer point.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_is_topologically_-
closed(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is topologically closed.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_is_discrete(+Handle)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *is discrete.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_bounds_from_above(+Handle,
+Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from above in the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_bounds_from_below(+Handle,
+Lin_Expr)
```

Succeeds if and only if `Lin_Expr` *is bounded from below in the pointset powerset referenced by* `Handle`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_contains_Pointset_-
Powerset_BD_Shape_long_double(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in or equal to the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_strictly_contains_-
Pointset_Powerset_BD_Shape_long_double(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is included in but not equal to the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_is_disjoint_from_Pointset_-
Powerset_BD_Shape_long_double(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *is disjoint from the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_geometrically_covers_-
Pointset_Powerset_BD_Shape_long_double(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically covers the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_geometrically_equals_-
Pointset_Powerset_BD_Shape_long_double(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_2` *geometrically equals the pointset powerset referenced by* `Handle_1`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_equals_Pointset_Powerset_-
BD_Shape_long_double(+Handle_1, +Handle_2)
```

Succeeds if and only if the pointset powerset referenced by `Handle_1` *is equal to the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_OK(+Handle)
```

Succeeds only if the pointset powerset referenced by `Handle` *is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_constrains(+Handle, +PPL_-
Var)
```

Succeeds if and only if the pointset powerset referenced by `Handle` *constrains the dimension* `PPL_Var`*.*

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

```
ppl_Pointset_Powerset_BD_Shape_long_double_space_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_long_double_affine_dimension(+Handle,
?Dimension_Type)
```

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

```
ppl_Pointset_Powerset_BD_Shape_long_double_relation_with_-
constraint(+Handle, +Constraint, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_BD_Shape_long_double_relation_with_-
generator(+Handle, +Generator, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_BD_Shape_long_double_relation_with_-
congruence(+Handle, +Congruence, ?Relation_List)
```

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

```
ppl_Pointset_Powerset_BD_Shape_long_double_maximize(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_BD_Shape_long_double_minimize(+Handle, +Lin_-
Expr, ?Coeff_1, ?Coeff_2, ?Boolean)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

```
ppl_Pointset_Powerset_BD_Shape_long_double_maximize_with_-
point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)
```

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the

atom `false`.

`ppl_Pointset_Powerset_BD_Shape_long_double_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if pointset powerset `P` *referenced by* `Handle` *is not empty and* `Lin_Expr` *is bounded from below in* `P`.

`Coeff_1` *is unified with the numerator of the infinum value and* `Coeff_2` *with the denominator of the infinum value and* `Point` *with a point or closure point where* `Lin_Expr` *reaches this value. If the infinum is also the minimum,* `Boolean` *is unified with the atom* `true` *and, otherwise, unified with the atom* `false`.

`ppl_Pointset_Powerset_BD_Shape_long_double_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the total memory in bytes occupied by the pointset powerset referenced by* `Handle`.

`ppl_Pointset_Powerset_BD_Shape_long_double_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` *with the size of the external memory in bytes occupied by the pointset powerset referenced by* `Handle`.

`ppl_Pointset_Powerset_BD_Shape_long_double_size(+Handle, ?Number)`

Unifies `Number` *with the size of the memory in bytes occupied by the pointset powerset referenced by* `Handle`.

Ascii dump predicate

This output predicate is useful for debugging.

`ppl_Pointset_Powerset_BD_Shape_long_double_ascii_dump(+Handle)`

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

`ppl_Pointset_Powerset_BD_Shape_long_double_add_constraint(+Handle, +Constraint)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

`ppl_Pointset_Powerset_BD_Shape_long_double_add_congruence(+Handle, +Congruence)`

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_add_constraints( +Handle,
+Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_add_congruences( +Handle,
+Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_refine_with_constraint(
+Handle, +Constraint)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_refine_with_congruence(
+Handle, +Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_refine_with_constraints(
+Handle, +Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_refine_with_congruences(
+Handle, +Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`*.*

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Pointset_Powerset_BD_Shape_long_double_topological_closure_-
assign(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_pairwise_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_omega_reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

`ppl_Pointset_Powerset_BD_Shape_long_double_unconstrain_space_dimension(+Handle, +PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimension `PPL_Var`.

`ppl_Pointset_Powerset_BD_Shape_long_double_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)`

Modifies the pointset powerset `P` referenced by `Handle` by unconstraining the space dimensions that are specified in `List_of_PPL_Var`. The presence of duplicates in `List_of_PPL_Var` is a waste but an innocuous one.

`ppl_Pointset_Powerset_BD_Shape_long_double_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` assigning the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_BD_Shape_long_double_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)`

Transforms the pointset powerset referenced by `Handle` substituting the affine expression for `Lin_Expr/Coeff` to `PPL_Var`.

`ppl_Pointset_Powerset_BD_Shape_long_double_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_BD_Shape_long_double_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1/Coeff} \leq \texttt{PPL_Var} \leq \texttt{Lin_Expr_2/Coeff}$.

`ppl_Pointset_Powerset_BD_Shape_long_double_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_BD_Shape_long_double_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{PPL_Var} \bowtie \texttt{Lin_Expr/Coeff}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_BD_Shape_long_double_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_BD_Shape_long_double_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation $\texttt{Lin_Expr_1} \bowtie \texttt{Lin_Expr_2}$, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Pointset_Powerset_BD_Shape_long_double_intersection_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the intersection of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_upper_bound_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the upper bound of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_difference_assign(+Handle_-
1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the difference of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_time_elapse_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_upper_bound_assign_if_-
exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` *referenced by* `Handle_1` *with the pointset powerset referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_simplify_using_context_-
assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_BD_Shape_long_double_BHZ03_H79_BHMZ05_widening_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` *referenced by* `Handle_1` *the BHZ03-widening between* `P_1` *and the pointset powerset referenced by* `Handle_2`*, using the BHMZ05-widening certified by the convergence certificate for H79.*

`ppl_Pointset_Powerset_BD_Shape_long_double_BHZ03_H79_H79_widening_-assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening certified by the convergence certificate for H79.

`ppl_Pointset_Powerset_BD_Shape_long_double_BGP99_BHMZ05_-extrapolation_assign(+Handle_1, +Handle_2, C_unsigned)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHMZ05-widening and the cardinality threshold `C_unsigned`.

`ppl_Pointset_Powerset_BD_Shape_long_double_BGP99_H79_extrapolation_-assign(+Handle_1, +Handle_2, C_unsigned)`

Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening and the cardinality threshold `C_unsigned`.

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

`ppl_Pointset_Powerset_BD_Shape_long_double_concatenate_-assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the concatenation of `P` and the pointset powerset referenced by `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

`ppl_Pointset_Powerset_BD_Shape_long_double_add_space_dimensions_and_-embed(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_embeds `P` in this space.

`ppl_Pointset_Powerset_BD_Shape_long_double_add_space_dimensions_and_-project(+Handle, +Dimension_Type)`

Adds `Dimension_Type` new dimensions to the space enclosing the pointset powerset `P` referenced by `Handle` and and_projects `P` in this space.

```
ppl_Pointset_Powerset_BD_Shape_long_double_remove_space_-
dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`*. The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_remove_higher_space_-
dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_expand_space_-
dimension(+Handle, +PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the pointset powerset referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_fold_space_-
dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` *by folding the space dimensions contained in* `List_of_PPL_Vars` *into dimension* `PPL_Var`*. The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_map_space_-
dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`*, which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_BD_Shape_long_double_iterator_from_-
iterator(+Iterator_1, -Iterator_2)
```

Builds a new iterator `it` *from the iterator referenced by* `Iterator_1`*.* `Iterator_2` *is unified with the handle for* `it`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_begin_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_end_iterator(+Handle,
-Iterator)
```

Unifies `Iterator` *with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by* `Handle`*.*

```
ppl_Pointset_Powerset_BD_Shape_long_double_iterator_equals_-
iterator(+Iterator_1, +Iterator_2)
```

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

```
ppl_Pointset_Powerset_BD_Shape_long_double_iterator_-
increment(+Iterator)
```

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

```
ppl_Pointset_Powerset_BD_Shape_long_double_iterator_-
decrement(+Iterator)
```

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

```
ppl_Pointset_Powerset_BD_Shape_long_double_iterator_get_-
disjunct(+Iterator, -Handle)
```

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

```
ppl_delete_Pointset_Powerset_BD_Shape_long_double_iterator(+Iterator)
```

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

```
ppl_Pointset_Powerset_BD_Shape_long_double_add_disjunct(+Handle_1,
+Handle_2)
```

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

```
ppl_Pointset_Powerset_BD_Shape_long_double_drop_disjunct(+Handle,
+Iterator)
```

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

```
ppl_Pointset_Powerset_BD_Shape_long_double_drop_disjuncts(+Handle,
+Iterator_1, +Iterator_2)
```

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

Predicates for the Pointset_Powerset_Octagonal_Shape_long_double Domain

Here we provide a short description for each of the predicates available for the domain of pointset powersets of octagonal shapes.

Constructor, copy, conversion and destructor predicates

Constructor predicates for pointset powersets of octagonal shapes

The constructor predicates build a pointset powerset from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs.

```
ppl_new_Pointset_Powerset_Octagonal_Shape_long_double_from_space_-
dimension(+Dimension_Type, +Universe_or_Empty, -Handle)
```

Builds a new pointset powerset $\mathcal{P}$ with `Dimension_Type` *dimensions; it is empty or the universe depending on whether* `Atom` *is* `empty` *or* `universe`*, respectively.* `Handle` *is unified with the handle for $\mathcal{P}$. Thus the query*

```
?- ppl_new_Pointset_Powerset_Octagonal_Shape_long_double_from_space_dimension(
   3, universe, X).
```

creates the pointset powerset defining the 3-dimensional vector space $\mathbb{R}^3$ with `X` *bound to a valid handle for accessing it.*

`ppl_new_Pointset_Powerset_Octagonal_Shape_long_double_from_constraints(+Constraint_System, -Handle)`

Builds a new pointset powerset `P` *from* `Constraint_System`*.* `Handle` *is unified with the handle for* `P`*.*

`ppl_new_Pointset_Powerset_Octagonal_Shape_long_double_from_congruences(+Congruence_System, -Handle)`

Builds a new pointset powerset `P` *from* `Congruence_System`*.* `Handle` *is unified with the handle for* `P`*.*

Predicates that build new pointset powersets of octagonal shapes by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class .

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

`ppl_new_Pointset_Powerset_Octagonal_Shape_long_double_from_Pointset_Powerset_Octagonal_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` *from the pointset_powerset_octagonal_shape_long_double referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Pointset_Powerset_Octagonal_Shape_long_double_from_Octagonal_Shape_long_double(+Handle_1, -Handle_2)`

Builds a new pointset powerset `P_1` *from the octagonal_shape_long_double referenced by handle* `Handle_1`*.* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Pointset_Powerset_Octagonal_Shape_long_double_from_Pointset_Powerset_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` *from the pointset_powerset_octagonal_shape_long_double referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

`ppl_new_Pointset_Powerset_Octagonal_Shape_long_double_from_Octagonal_Shape_long_double_with_complexity(+Handle, +Complexity, -Handle)`

Builds a new pointset powerset `P_1` *from the octagonal_shape_long_double referenced by handle* `Handle_1` *using an algorithm whose complexity does not exceed* `Complexity`*;* `Handle_2` *is unified with the handle for* `P_1`*.*

Destructor predicate

Below is the destructor predicate for the Pointset_Powerset_Octagonal_Shape_long_double domain.

`ppl_delete_Pointset_Powerset_Octagonal_Shape_long_double(+Handle)`

Invalidates the handle referenced by `Handle`: this makes sure the corresponding resources will eventually be released.

Predicates that do not change the pointset powerset

Test Predicates

These predicates test the pointset powerset for different properties and succeed or fail depending on the outcome.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_is_empty(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is empty.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_is_universe(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is the universe.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_is_bounded(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is bounded.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_contains_integer_point(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` contains an integer point.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_is_topologically_closed(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is topologically closed.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_is_discrete(+Handle)`

Succeeds if and only if the pointset powerset referenced by `Handle` is discrete.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_bounds_from_above(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from above in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_bounds_from_below(+Handle, +Lin_Expr)`

Succeeds if and only if `Lin_Expr` is bounded from below in the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_contains_Pointset_Powerset_Octagonal_Shape_long_double(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in or equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_strictly_contains_Pointset_Powerset_Octagonal_Shape_long_double(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is included in but not equal to the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_is_disjoint_from_-Pointset_Powerset_Octagonal_Shape_long_double(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` is disjoint from the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_geometrically_-covers_Pointset_Powerset_Octagonal_Shape_long_double(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_geometrically_-equals_Pointset_Powerset_Octagonal_Shape_long_double(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_equals_Pointset_-Powerset_Octagonal_Shape_long_double(+Handle_1, +Handle_2)`

Succeeds if and only if the pointset powerset referenced by `Handle_1` is equal to the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_OK(+Handle)`

Succeeds only if the pointset powerset referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_constrains(+Handle, +PPL_Var)`

Succeeds if and only if the pointset powerset referenced by `Handle` constrains the dimension `PPL_Var`.

Predicates that return information about the pointset powerset

These predicates will obtain more detailed information about the pointset powerset unifying some of their arguments with the results.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_space_-dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the dimension of the vector space enclosing the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_affine_-dimension(+Handle, ?Dimension_Type)`

Unifies `Dimension_Type` with the affine dimension of the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_relation_with_-constraint(+Handle, +Constraint, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_relation_with_-generator(+Handle, +Generator, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_relation_with_congruence(+Handle, +Congruence, ?Relation_List)`

Unifies `Relation_List` with the list of relations the pointset powerset referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from above in `P`.

`Coeff_1` is unified with the numerator of the supremum value and `Coeff_2` with the denominator of the supremum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the supremum is also the maximum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

Succeeds if and only if pointset powerset `P` referenced by `Handle` is not empty and `Lin_Expr` is bounded from below in `P`.

`Coeff_1` is unified with the numerator of the infinum value and `Coeff_2` with the denominator of the infinum value and `Point` with a point or closure point where `Lin_Expr` reaches this value. If the infinum is also the minimum, `Boolean` is unified with the atom `true` and, otherwise, unified with the atom `false`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_external_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the total memory in bytes occupied by the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_total_memory_in_bytes(+Handle, ?Number)`

Unifies `Number` with the size of the external memory in bytes occupied by the pointset powerset referenced by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_size(+Handle, ?Number)`

Unifies `Number` *with the size of the memory in bytes occupied by the pointset powerset referenced by* `Handle`*.*

Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_ascii_dump(+Handle)
```

Dumps an ascii representation of the PPL internal state for the pointset powerset referenced by `Handle` *on the standard output.*

Space-dimension preserving predicates that may change the pointset powerset

These predicates may modify the pointset powerset referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

Predicates that may change the pointset powerset by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_add_-
constraint(+Handle, +Constraint)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Constraint` *to its constraint system. For a C polyhedron,* `Constraint` *must be an equality or a non-strict inequality.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_add_-
congruence(+Handle, +Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding* `Congruence` *to its congruence system. For a C polyhedron,* `Congruence` *must be an equality.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_add_constraints(
+Handle, +Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its constraint system the constraints in* `Constraint_System`*. For a C polyhedron,* `Constraints` *must be a list of equalities and non-strict inequalities.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_add_congruences(
+Handle, +Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by adding to its congruence system the congruences in* `Congruence_System`*. For a C polyhedron,* `Congruences` *must be a list of equalities.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_refine_with_-
constraint( +Handle, +Constraint)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with* `Constraint`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_refine_with_-
congruence( +Handle, +Congruence)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with* `Congruence`.

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_refine_with_-
constraints( +Handle, +Constraint_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its constraint system with the constraints in* `Constraint_System`.

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_refine_with_-
congruences( +Handle, +Congruence_System)
```

Updates the pointset powerset referenced by `Handle` *to one obtained by refining its congruence system with the congruences in* `Congruence_System`.

Predicates that transform the pointset powerset

These predicates enable transformations such as taking the topological closure (which for the domain of pointset powerset is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_topological_-
closure_assign(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its topological closure.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_pairwise_-
reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its pairwise reduction.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_omega_-
reduce(+Handle)
```

Assigns to the pointset powerset referenced by `Handle` *its omega reduction.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_unconstrain_space_-
dimension(+Handle, +PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimension* `PPL_Var`.

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_unconstrain_space_-
dimensions(+Handle, +List_of_PPL_Var)
```

Modifies the pointset powerset `P` *referenced by* `Handle` *by unconstraining the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_affine_-
image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *assigning the affine expression for* `Lin_-Expr/Coeff to PPL_Var`.

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_affine_-
preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

Transforms the pointset powerset referenced by `Handle` *substituting the affine expression for* `Lin_-Expr/Coeff to PPL_Var`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1/Coeff` $\leq$ `PPL_Var` $\leq$ `Lin_Expr_2/Coeff`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_generalized_affine_preimage(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `PPL_Var` $\bowtie$ `Lin_Expr/Coeff`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized image with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_1, +Relation_Symbol, +Lin_Expr_2)`

Assigns to pointset powerset `P` referenced by `Handle` the generalized preimage with respect to the generalized affine transfer relation `Lin_Expr_1` $\bowtie$ `Lin_Expr_2`, where $\bowtie$ is the symbol represented by `Relation_Symbol`.

Predicates whose results depend on more than one pointset powerset

These predicates include the binary operators which will assign to the pointset powerset referred to by the first argument its combination with the pointset powerset referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_intersection_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the intersection of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_upper_bound_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` referenced by `Handle_1` the upper bound of `P` and the pointset powerset referenced by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_difference_assign(+Handle_1, +Handle_2)`

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the difference of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_time_elapse_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the time elapse of* `P` *and the pointset powerset referenced by* `Handle_2`*.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_upper_bound_assign_-
if_exact(+Handle_1, +Handle_2)
```

Succeeds if the least upper bound of the pointset powerset `P_1` *referenced by* `Handle_1` *with the pointset powerset referenced by* `Handle_2` *is exact; in which case the least upper bound is assigned to* `P_1`*; fails otherwise.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_simplify_using_-
context_assign(+Handle_1, +Handle_2, ?Boolean)
```

Succeeds if and only if the intersection of pointset powerset `P_1` *referenced by* `Handle_1` *and the pointset powerset* `P_2` *referenced by* `Handle_2` *is non-empty. Assigns to* `P_1` *its meet-preserving simplification with respect to* `P_2`*.*

Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the pointset powerset referred to by the second argument has to be contained in (or equal to) the pointset powerset referred to by the first argument.

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_BHZ03_H79_BHMZ05_-
widening_assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P_1` *referenced by* `Handle_1` *the BHZ03-widening between* `P_1` *and the pointset powerset referenced by* `Handle_2`*, using the BHMZ05-widening certified by the convergence certificate for H79.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_BGP99_BHMZ05_-
extrapolation_assign(+Handle_1, +Handle_2, C_unsigned)
```

Assigns to the pointset powerset `P_1` *referenced by* `Handle_1` *the result of applying the BGP99 extrapolation operator between* `P_1` *and the pointset powerset referenced by* `Handle_2`*, using the BHMZ05-widening and the cardinality threshold* `C_unsigned`*.*

Predicates that may modify the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument.

Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_concatenate_-
assign(+Handle_1, +Handle_2)
```

Assigns to the pointset powerset `P` *referenced by* `Handle_1` *the concatenation of* `P` *and the pointset powerset referenced by* `Handle_2`.

Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. These predicates enable the modification of the vector space of the pointset powerset referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_add_space_-
dimensions_and_embed(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the pointset powerset* `P` *referenced by* `Handle` *and and_embeds* `P` *in this space.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_add_space_-
dimensions_and_project(+Handle, +Dimension_Type)
```

Adds `Dimension_Type` *new dimensions to the space enclosing the pointset powerset* `P` *referenced by* `Handle` *and and_projects* `P` *in this space.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_remove_space_-
dimensions(+Handle, +List_of_PPL_Vars)
```

Removes from the vector space enclosing the pointset powerset `P` *referenced by* `Handle` *the space dimensions that are specified in* `List_of_PPL_Var`. *The presence of duplicates in* `List_of_PPL_Var` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_remove_higher_-
space_dimensions(+Handle, +Dimension_Type)
```

Removes the higher dimensions from the vector space enclosing the pointset powerset `P` *referenced by* `Handle` *so that, upon successful return, the new space dimension is* `Dimension_Type`.

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_expand_space_-
dimension(+Handle, +PPL_Var, +Dimension_Type)
```

Expands the `PPL_Var-th` *dimension of the vector space enclosing the pointset powerset referenced by* `Handle` *to* `Dimension_Type` *new space dimensions.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_fold_space_-
dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)
```

Modifies the pointset powerset referenced by `Handle` *by folding the space dimensions contained in* `List_of_PPL_Vars` *into dimension* `PPL_Var`. *The presence of duplicates in* `List_of_PPL_Vars` *is a waste but an innocuous one.*

```
ppl_Pointset_Powerset_Octagonal_Shape_long_double_map_space_-
dimensions(+Handle, +P_Func)
```

Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the `P_Func`, *which has* `n` *entries. The result is undefined if* `P_Func` *does not encode a partial function.*

Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

`ppl_new_Pointset_Powerset_Octagonal_Shape_long_double_iterator_from_iterator(+Iterator_1, -Iterator_2)`

Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_begin_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_end_iterator(+Handle, -Iterator)`

Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_iterator_equals_iterator(+Iterator_1, +Iterator_2)`

Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_iterator_increment(+Iterator)`

Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_iterator_decrement(+Iterator)`

Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_iterator_get_disjunct(+Iterator, -Handle)`

Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.

`ppl_delete_Pointset_Powerset_Octagonal_Shape_long_double_iterator(+Iterator)`

Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_add_disjunct(+Handle_1, +Handle_2)`

Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_drop_disjunct(+Handle, +Iterator)`

If `it` is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by `it` and assigns to `it` an iterator to the next disjunct.

`ppl_Pointset_Powerset_Octagonal_Shape_long_double_drop_disjuncts(+Handle, +Iterator_1, +Iterator_2)`

If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).

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This document describes the Parma Polyhedra Library (PPL).

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Compilation and Installation

When the Parma Polyhedra Library is configured, it tests for the existence of each supported Prolog system. If a supported Prolog system is correctly installed in a standard location, things are arranged so that the corresponding interface is built and installed.

The Prolog interface files are all installed in the directory `prefix/lib/ppl`. Since this includes shared and dynamically loaded libraries, you must make your dynamic linker/loader aware of this fact. If you use a GNU/Linux system, try the commands `man ld.so` and `man ldconfig` for more information.

As an option, the Prolog interface can track the creation and disposal of polyhedra. In fact, differently from native Prolog data, PPL polyhedra must be explicitly disposed and forgetting to do so is a very common mistake. To enable this option, configure the library adding `-DPROLOG_TRACK_ALLOCATION` to the options passed to the C++ compiler. Your configure command would then look like

```
path/to/configure --with-cxxflags="-DPROLOG_TRACK_ALLOCATION" ...
```

System-Dependent Features

CIAO Prolog

The Ciao Prolog interface to the PPL is available both as 'PPL enhanced' Ciao Prolog interpreter and as a library that can be linked to Ciao Prolog programs. Only Ciao Prolog versions 1.10 #5 and later are supported.

So that it can be used with the Ciao Prolog PPL interface, the Ciao Prolog installation must be configured with the `--disable-regs` option.

The `ppl_ciao` Executable

If an appropriate version of Ciao Prolog is installed on the machine on which you compiled the library, the command `make install` will install the executable `ppl_ciao` in the directory `prefix/bin`. The `ppl_ciao` executable is simply the Ciao Prolog interpreter with the Parma Polyhedra Library linked in.

The only thing you should do to use the library is to call `ppl_initialize/0` before any other PPL predicate and to call `ppl_finalize/0` when you are done with the library.

Linking the Library To Ciao Prolog Programs

In order to allow linking Ciao Prolog programs to the PPL, the following files are installed in the directory `prefix/lib/ppl`: `ppl_ciao.pl` contains the required foreign declarations; `libppl_ciao.*` contain the executable code for the Ciao Prolog interface in various formats (static library, shared library, libtool library). If your Ciao Prolog program is constituted by, say, `source1.pl` and `source2.pl` and you want to create the executable `myprog`, your compilation command may look like

```
ciaoc -o myprog prefix/lib/ppl/ppl_ciao.pl ciao_pl_check.pl \
  -L '-Lprefix/lib/ppl -lppl_ciao -Lprefix/lib -lppl -lgmpxx -lgmp -lstdc++'
```

GNU Prolog

The GNU Prolog interface to the PPL is available both as a 'PPL enhanced' GNU Prolog interpreter and as a library that can be linked to GNU Prolog programs. The only GNU Prolog version that is known to work is a patched version of the 'unstable version' tagged `20040608` (which unpacks to a directory called `gprolog-1.2.18`). The patch is contained in the `interfaces/Prolog/GNU/README` file of the PPL's distribution.

So that it can be used with the GNU Prolog PPL interface (and, for that matter, with any foreign code), the GNU Prolog installation must be configured with the `--disable-regs` option.

The `ppl_gprolog` Executable

If an appropriate version of GNU Prolog is installed on the machine on which you compiled the library, the command `make install` will install the executable `ppl_gprolog` in the directory `prefix/bin`. The `ppl_gprolog` executable is simply the GNU Prolog interpreter with the Parma Polyhedra Library linked in. The only thing you should do to use the library is to call `ppl_initialize/0` before any other PPL predicate and to call `ppl_finalize/0` when you are done with the library.

Linking the Library To GNU Prolog Programs

In order to allow linking GNU Prolog programs to the PPL, the following files are installed in the directory `prefix/lib/ppl`: `ppl_gprolog.pl` contains the required foreign declarations; `libppl_gprolog.*` contain the executable code for the GNU Prolog interface in various formats (static library, shared library, libtool library). If your GNU Prolog program is constituted by, say, `source1.pl` and `source2.pl` and you want to create the executable `myprog`, your compilation command may look like

```
gplc -o myprog prefix/lib/ppl/ppl_gprolog.pl source1.pl source2.pl \
  -L '-Lprefix/lib/ppl -lppl_gprolog -Lprefix/lib -lppl -lgmpxx -lgmp -lstdc++'
```

GNU Prolog uses several stacks to execute a Prolog program each with a pre-defined default size. If the size of a stack is too small for the application an overflow will occur. To change the default size of a stack, the user has to set the value of the relevant environment variable; in particular, to execute some of the tests, we found it necessary to increase the size of GLOBALSZ. Thus, for the above example, the compilation command would be

```
GLOBALSZ=32768 gplc -o myprog prefix/lib/ppl/ppl_gprolog.pl source1.pl source2.pl
       \
  -L '-Lprefix/lib/ppl -lppl_gprolog -Lprefix/lib -lppl -lgmpxx -lgmp -lstdc++'
```

More information on adjusting the size of the stacks can be found in Section 3.3 in the GNU Prolog Manual

SICStus Prolog

The SICStus Prolog interface to the PPL is available both as a statically linked module or as a dynamically linked one. Only SICStus Prolog versions 3.9.0 and later are supported.

The Statically Linked `ppl_sicstus` Executable

If an appropriate version of SICStus Prolog is installed on the machine on which you compiled the library, the command `make install` will install the executable `ppl_sicstus` in the directory `prefix/bin`. The `ppl_sicstus` executable is simply the SICStus Prolog system with the Parma Polyhedra Library statically linked. The only thing you should do to use the library is to load `prefix/lib/ppl/ppl_sicstus.pl`.

Loading the SICStus Interface Dynamically

In order to dynamically load the library from SICStus Prolog you should simply load `prefix/lib/ppl/ppl_sicstus.pl`. Notice that, for dynamic linking to work, you should have configured the library with the `--enable-shared` option.

SWI-Prolog

The SWI-Prolog interface to the PPL is available both as a statically linked module or as a dynamically linked one. Only SWI-Prolog version 5.6.0 and later versions are supported.

The `ppl_pl` Executable

If an appropriate version of SWI-Prolog is installed on the machine on which you compiled the library, the command `make install` will install the executable `ppl_pl` in the directory `prefix/bin`. The `ppl_pl` executable is simply the SWI-Prolog shell with the Parma Polyhedra Library statically linked: from within `ppl_pl` all the services of the library are available without further action.

Loading the SWI-Prolog Interface Dynamically

In order to dynamically load the library from SWI-Prolog you should simply load `prefix/lib/ppl/ppl_swiprolog.pl`. This will invoke `ppl_initialize/0` and `ppl_finalize/0` automatically. Alternatively, you can load the library directly with

```
:- load_foreign_library('prefix/lib/ppl/libppl_swiprolog').
```

This will call `ppl_initialize/0` automatically. Analogously,

```
:- unload_foreign_library('prefix/lib/ppl/libppl_swiprolog').
```

will, as part of the unload process, invoke `ppl_finalize/0`.

Notice that, for dynamic linking to work, you should have configured the library with the `--enable-shared` option.

XSB

The XSB Prolog interface to the PPL is available as a dynamically linked module. Only some CVS versions of XSB starting from 2 July 2005 are known to work. CVS versions starting from 11 November 2005 are known not to work.

In order to dynamically load the library from XSB you should load the `ppl_xsb` module and import the predicates you need. For things to work, you may have to copy the files `prefix/lib/ppl/ppl_xsb.xwam` and `prefix/lib/ppl/ppl_xsb.so` in your current directory or in one of the XSB library directories.

YAP

The YAP Prolog interface to the PPL is available as a dynamically linked module. Only YAP versions following 5.1.0 and CVS HEAD versions starting from 4 January 2006 are supported. Notice that support for unbounded integers in YAP is young and may have errors that could affect programs using the PPL (see, e.g., http://www.cs.unipr.it/pipermail/ppl-devel/2006-January/007780.html).

In order to dynamically load the library from YAP you should simply load `prefix/lib/ppl/ppl_yap.pl`. This will invoke `ppl_initialize/0` automatically; it is the programmer's responsibility to call `ppl_finalize/0` when the PPL library is no longer needed. Notice that, for dynamic linking to work, you should have configured the library with the `--enable-shared` option.

2 GNU General Public License

Version 3, 29 June 2007

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4 Module Index

4.1 Modules

Here is a list of all modules:

Prolog Language Interface 953

5 Module Documentation

5.1 Prolog Language Interface

The Parma Polyhedra Library comes equipped with an interface for the Prolog language.

Index

Prolog Language Interface, 953